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The top-quark's running mass

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We discuss the direct determination of the running top-quark mass from measurements of the total cross section of hadronic top-quark pair-production. The theory predictions in the $\overline{\text{MS}}$ scheme are very stable under scale variations and show rapid apparent convergence of the perturbative expansion. These features are explained by studying the underlying parton dynamics.

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1. Introduction

The top-quark is the heaviest known elementary particle and it plays a prominent role in the physics program of Tevatron and the Large Hadron Collider (LHC) (see e.g. [1]). The top-quark mass is a very important parameter in fits constraining the Standard Model (SM), i.e. giving rise to indirect limits on the mass of the Higgs boson (see e.g. [2]). Currently, a value of $m_t = 173.1^{+1.3}_{-1.3}$ GeV is quoted for the mass of the top-quark [3]. This amounts to an experimental uncertainty of less than 1%. Due to the high mass the top-quark's width is so large that it typically decays before it can hadronize [4] so that mass measurements proceed via kinematic reconstruction from the decay products and comparison to Monte Carlo simulations. Thus, there is no immediate interpretation of the measured quantity in terms of a parameter of the SM Lagrangian in a specific renormalization scheme.

In order to address this issue, we have chosen the following approach. We start from the total cross section for hadronic top-quark pair production, i.e. a quantity with well-defined scheme dependence which is known to sufficient accuracy in perturbative Quantum Chromodynamics (QCD). Its dependence on the top-quark mass is commonly given in the on-shell scheme, although it is well-known that the concept of the pole mass has intrinsic theoretical limitations leading, for instance, to a poorly behaved perturbative series. This typically implies a strong dependence of the extracted value for the top-quark mass on the order of perturbation theory. So-called short distance masses offer a solution to this problem. As we compute the total cross section as a function of the top-quark mass in the $\overline{\mathrm{MS}}$ scheme [5] we demonstrate stability of the perturbative expansion and good properties of apparent convergence [6]. In particular, this allows for the direct determination of the top-quark's running mass from Tevatron measurements for the total cross section [7], which is of importance for global analyses of electro-weak precision data.

2. The total cross section for top-quark-pair production

We start by recalling the relevant formulae for the total cross section $\sigma_{pp\to t\bar{t}X}$ of top-quark hadro-production within perturbative QCD,

$$\sigma_{pp\to t\bar{t}X}(S,m_t^2) = \sum_{i,j=q,\bar{q},g} \int\limits_{4m_t^2}^{S} ds\, L_{ij}(s,S,\mu_f^2)\, \hat{\sigma}_{ij}(s,m_t^2,\mu_f^2)\,, \tag{2.1}$$

$$L_{ij}(s,S,\mu_f^2) = \frac{1}{S}\int\limits_{s}^{S} \frac{d\hat{s}}{\hat{s}} \phi_{i/p}\left(\frac{\hat{s}}{S},\mu_f^2\right) \phi_{j/p}\left(\frac{s}{\hat{s}},\mu_f^2\right)\,, \tag{2.2}$$

where S denotes the hadronic center-of-mass energy squared and m_t the top-quark mass (taken to be the pole mass here). The standard definition for the parton luminosity L_{ij} convolutes the two parton distributions (PDFs) $\phi_{i/p}$ at the factorization scale μ_f, while the partonic cross sections $\hat{\sigma}_{ij}$ parameterize the hard partonic scattering process. $\hat{\sigma}_{ij}$ depends only on dimensionless ratios of m_t, μ_f and the partonic center-of-mass energy squared s.

The QCD radiative corrections for the total cross section in Eq. (2.1) as an expansion in the strong coupling constant α_s are currently known completely at next-to-leading order (NLO) [8] and, as approximation, at next-to-next-to-leading order (NNLO) [9]. The latter result is based on the known threshold corrections to the partonic cross section $\hat{\sigma}_{ij}$, i.e. the complete tower of Sudakov logarithms in $\beta = \sqrt{1-4m_t{}^2/s}$ and the two-loop Coulomb corrections, i.e. powers $1/\beta^k$ (see also [10] for some recent improvements). It also includes the complete dependence on μ_f and the renormalization scale μ_r, both being known from a renormalization group analysis.

The parton luminosity L_{ij} in Eq. (2.2) is fully known to NNLO accuracy from global fits (e.g. [11, 12]). For a fixed collider energy S, it is a steeply falling function of s. Thus, in the convolution Eq. (2.1) L_{ij} dominantly samples the threshold region of the underlying hard parton scattering $\hat{\sigma}_{ij}$, which justifies the use of threshold approximations for the latter quantity. As an upshot, the presently available perturbative corrections through NNLO lead to accurate predictions for the total hadronic cross section of top-quark pairs with a small associated theoretical uncertainty [6, 9] (see also e.g. [13] for related theory improvements through threshold resummation).

3. The top-quark mass in the $\overline{\mathrm{MS}}$ scheme

Colored particles in QCD are not asymptotic states of the S-matrix due to confinement. Therefore the pole mass for quarks is a poor scheme choice since its definition implies intrinsic uncertainties of the order of Λ_{QCD}, a fact that is often referred to in perturbation theory as the infrared renormalon problem. It is well-known that short distance masses impose renormalization conditions which avoid this problem. In a perturbative expansion in α_s the pole mass m_t can be related to the running mass $m(\mu_r)$ in the $\overline{\mathrm{MS}}$ scheme,

$$m_t = m(\mu_r)\left(1+\alpha_s(\mu_r)d^{(1)}(\mu_r)+\dots\right)\,, \tag{3.1}$$

where the coefficients $d^{(l)}$ are actually known to three-loop order [5]. The basic idea for the direct determination of a $\overline{\mathrm{MS}}$ mass is to use the manifest dependence of the total cross section $\sigma_{pp\to t\bar{t}X}$ on the top-quark mass to estimate the parameter from the data for the measured cross section. For the pole mass m_t we have

$$\sigma_{pp\to t\bar{t}X} = \alpha_s^2\,\sigma^{(0)}(m_t)+\alpha_s^3\,\sigma^{(1)}(m_t)+\dots\,, \tag{3.2}$$

which we can convert with Eq. (3.1) to the $\overline{\mathrm{MS}}$ mass $m(m)$ (for simplicity abbreviated as $\overline{m}$) according to

$$\sigma_{pp\to t\bar{t}X} = \alpha_s^2\,\sigma^{(0)}(\overline{m})+\alpha_s^3\left(\sigma^{(1)}(\overline{m})+\overline{m}\,d^{(1)}\partial_m\sigma^{(0)}(m)\Big|_{m=\overline{m}}\right)+\dots\,, \tag{3.3}$$

where the coefficients $d^{(l)}$ have to be evaluated for $\mu_r=\overline{m}$ (corresponding to the scale of α_s). In Eqs. (3.1)–(3.3) we have confined ourselves here for brevity to NLO (see [6] for the formalism through NNLO).

Eq. (3.3) gives a direct handle on the running mass at large scales. To illustrate the phenomenological implications for predictions at hadron colliders, we plot in Fig. 1 the scale dependence of the total cross section at the various orders in perturbation theory. For Tevatron with $\sqrt{S}=1.96\,\mathrm{TeV}$ (and using the MSTW 2008 PDF set [11]), we compare the on-shell scheme with a pole mass of $m_t = 173\,\mathrm{GeV}$ with the corresponding predictions for a running mass with a value of $\overline{m}=163\,\mathrm{GeV}$. For the computation of the total cross section in the on-shell scheme, we choose three (fixed) values

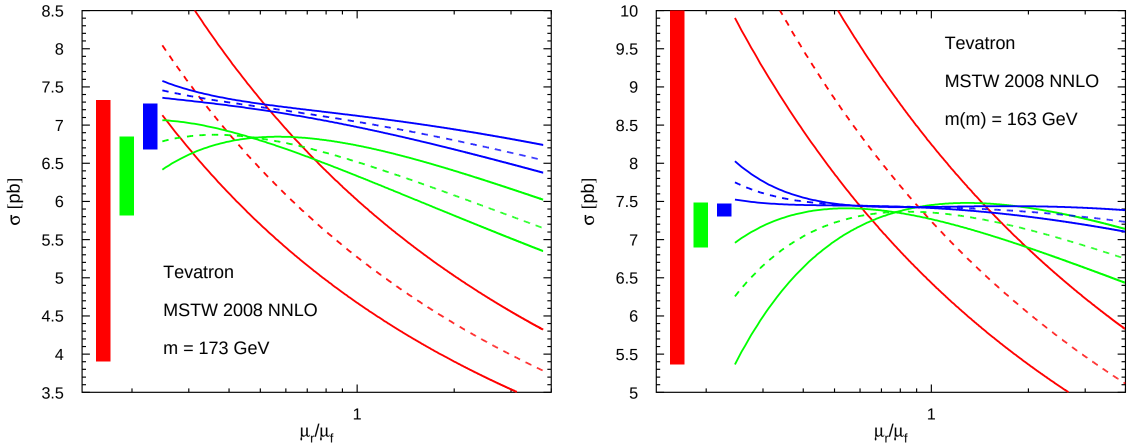

Figure 1: The scale dependence of the total cross section at Tevatron with $\sqrt{S} = 1.96\,\mathrm{TeV}$ with MSTW 2008 PDF set [11]. The top-quark mass is taken in the on-shell scheme at $m_t = 173\,\mathrm{GeV}$ (left) and in the $\overline{\mathrm{MS}}$ scheme at $\overline{m} = 163\,\mathrm{GeV}$ (right) at LO (red), NLO (green) and approximate NNLO (blue). The dashed lines denote the choice $\mu_f = m_t$ (left) and $\mu_f = \overline{m}$ (right) for the factorization scale, the solid lines the maximal deviations for $\mu_r \in [m_t/2, 2m_t]$ and $\mu_f = m_t/2, m_t$ and $2m_t$ (left) and $\mu_r \in [\overline{m}/2, 2\overline{m}]$ and $\mu_f = \overline{m}/2, \overline{m}$ and $2\overline{m}$ (right). The vertical bars indicate the size of the scale variation in the standard range $[m_t/2, 2m_t]$ (left) and $[\overline{m}/2, 2\overline{m}]$ (right).

for the factorization scale $\mu_f = m_t/2, m_t$ and $2m_t$ and, likewise $\mu_f = \overline{m}/2, \overline{m}$ and $2\overline{m}$ for the $\overline{\mathrm{MS}}$ scheme. The vertical bands on the left in Fig. 1 denote the maximum and the minimum values for a variation of $\mu_r \in [m_t/2, 2m_t]$ (and, respectively, $\mu_r \in [\overline{m}/2, 2\overline{m}]$) for the three choices of μ_f.

In general, we observe in both schemes a reduced scale dependence as we increase the order of perturbation theory, i.e. a reduced theoretical uncertainty. Also, we do observe apparent convergence of the expansion upon including successive orders in α_s. For the on-shell scheme, however, the higher order corrections are quite sizable, $\mathscr{O}(30\%)$ at NLO and another $\mathscr{O}(10\%)$ at NNLO at the central value $\mu_r = \mu_f = m_t$. For the running $\overline{\mathrm{MS}}$ mass on the other hand both NLO and NNLO corrections are negligible for the choice $\mu_r = \mu_f = \overline{m}$. Remarkably, in the $\overline{\mathrm{MS}}$ scheme we do find even greater stability with respect to scale variations, which at NLO and NNLO is reduced by more than a factor of two compared to the results in the pole mass scheme. Similar results and conclusions have been found for top-quark pair production at LHC, see [6], although the improvement is slightly less distinct than at Tevatron.

In order to address the underlying parton dynamics of relevance for the two mass schemes it is instructive to consider the total parton cross sections $\hat{\sigma}_{ij}$, i.e. the equivalent expression of Eq. (3.3) for the individual partonic channels. As a matter of fact, it turns out, that a result completely analogous to Eq. (3.3) can be derived. To NLO this is true because the boundary term in the conversion $m_t \to \overline{m}$ from the convolution integral in Eq. (2.1) vanishes, so that we can apply Eq. (3.3) with the simple replacement $\sigma \to \hat{\sigma}_{ij}$.

In Fig. 2 we plot $\hat{\sigma}_{ij}$ in both schemes, i.e. the on-shell scheme with $m_t = 173\,\mathrm{GeV}$ and the $\overline{\mathrm{MS}}$ scheme with a running mass $\overline{m} = 163\,\mathrm{GeV}$ as a function of the partonic center-of-mass energy s. The energy range is selected to match the discussion for the Tevatron around Fig. 1. Of course,

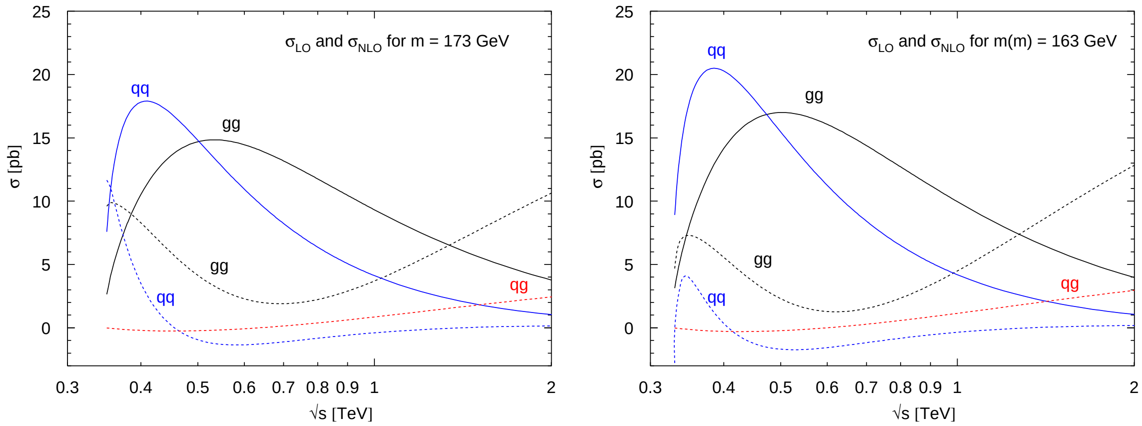

Figure 2: The parton cross section for the channels $q\bar{q}$, gg and qg at the scale $\mu_f = \mu_r = m$ in the on-shell scheme for $m_t = 173\,\text{GeV}$ (left) and in the $\overline{\text{MS}}$ scheme for $\overline{m} = 163\,\text{GeV}$. Solid lines denote the $\mathscr{O}(\alpha_s^2)$ (LO) and dashed lines the $\mathscr{O}(\alpha_s^3)$ contributions (NLO). The energy range corresponds to Tevatron with $\sqrt{S} = 1.96\,\text{TeV}$ and the value of α_s to MSTW 2008 PDF set [11].

the Born cross sections remain largely unchanged the only difference in the $\overline{\text{MS}}$ case being the slightly smaller numerical value of the mass (hence, larger cross sections). At NLO, the perturbative corrections in the on-shell scheme for the channels $q\bar{q}$ and gg clearly display the well-known large logarithmic corrections near threshold. This is not the case for the $\overline{\text{MS}}$ scheme, which exhibits a much reduced sensitivity to the threshold region. Due to the terms $\sim \partial_m \hat{\sigma}_{ij}^{(0)}$ in the partonic equivalent of Eq. (3.3), the NLO corrections are sizably reduced and the Sudakov logarithms are numerically compensated to a large extent. The qg-channel is new at NLO, thus it does not receive any modification under scheme transformations at this order.

The parton cross sections of Fig. 2 enter the convolution with the parton luminosity L_{ij} as given in Eq. (2.1). To that end, recall that the hadronic cross section at Tevatron almost saturates already for partonic center-of-mass energies $\sqrt{s} \lesssim 600\,\text{GeV}$. A detailed treatment of the threshold region e.g. in Fig. 2 also needs to incorporate $t\bar{t}$ bound state effects which requires the application of non-relativistic QCD including an all-order resummation of Coulomb corrections, see [14].

As an upshot, the parton level studies of the $\overline{\text{MS}}$ case in Fig. 2 provide us with a detailed understanding of the excellent apparent convergence and scale stability seen in Fig. 1. In a direct comparison to data [7], this leads to very stable results for the extracted mass parameter. At LO, NLO, and NNLO values of $\overline{m} = 159.2^{+3.5}_{-3.4}\,\text{GeV}$, $\overline{m} = 159.8^{+3.3}_{-3.3}\,\text{GeV}$ and $\overline{m} = 160.0^{+3.3}_{-3.2}\,\text{GeV}$ are determined in [6], where the errors reflect the quoted experimental uncertainty for the total cross section. In contrast, the on-shell scheme predictions would return rather different results at the higher orders. Converting the best estimate for the running mass (i.e. the NNLO value) back to the on-shell mass by inverting Eq. (3.1) leads to a pole mass value of $m_t = 168.9^{+3.5}_{-3.4}\,\text{GeV}$. Within errors, the result is consistent with the direct measurements, although as mentioned above, concerns have been raised to interpret the quoted value [3] of $m_t = 173.1^{+1.3}_{-1.3}\,\text{GeV}$ as a pole mass. Since the experimental analysis is based to large extend on leading-order Monte Carlo prescriptions, additional efforts are needed to study the detailed scheme dependence, see e.g. [15] for the renormalization group flow for heavy quark masses.

4. Summary

We have computed the total cross section for top-quark pair production with the $\overline{\rm MS}$ mass definition for the top-quark [6]. The approximate NNLO predictions exhibit a greatly improved pattern of apparent convergence for the perturbative expansion and very good stability with respect to scale variations. Comparison with experimental data has lead to a best estimate for the running mass of $\overline{m} = 160.0^{+3.3}_{-3.2}\,$GeV, which is the first direct determination of $m(m)$ from top-quark pair-production. The corresponding value for the pole mass of $m_t = 168.9^{+3.5}_{-3.4}\,$GeV is consistent with current world average [3], $m_t = 173.1^{+1.3}_{-1.3}\,$GeV.

Altogether, our approach [6, 9] provides reliable approximate NNLO predictions for the total cross section for top-quark pair production and stable values for the top-quark's running mass.

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