PROCEEDINGS OF SCIENCE

Lattice Study of Dense Two Color Matter

Simon Hands[*†]
Department of Physics, Swansea University,
Singleton Park, Swansea SA2 8PP, U.K.
E-mail: `s.hands@swan.ac.uk`

I present results from lattice simulations of Two Color QCD with two quark flavors in the presence of a quark chemical potential μ. In particular, the equation of state, conformal anomaly, superfluid order parameter and Polyakov line are all discussed. I argue that the transition from hadronic to quark matter, and that from confined to deconfined matter occur at distinct values of μ, lending support to the existence of a quarkyonic phase.

International Workshop on QCD Green's Functions, Confinement and Phenomenology
September 7-11 , 2009
ECT* Trento, Italy

PoS(QCD-TNT09)018

[*]Speaker.

[†]It is a pleasure to thank my principal collaborators Seyong Kim and Jon-Ivar Skullerud

In this talk I will review recent results obtained via lattice simulations of Two Color QCD (QC$_2$D), that is, QCD in which the gauge group is SU(2) rather than SU(3), in the presence of a non-vanishing quark chemical potential. The goal of this project is to gain non-perturbative insight into cold, dense baryonic matter, ie. the region of the (T,μ) plane conventionally placed at the lower right of the QCD phase diagram. This is a region of tremendous importance for the understanding of the interiors of compact astrophysical objects such as neutron stars; it has been the focus of intense theoretical interest in recent years, with the discussion of possible exotic color superconducting phases, in which color-carrying degrees of freedom such as quarks or gluons are all gapped via a Higgs-Meissner mechanism, and which may also be superfluid or even crystalline. However even basic questions, such as the maximum stable mass of a neutron star, require quantitative input about the equation of state of ultradense matter which needs a controlled non-perturbative calculation.

The most reliable source of such information, lattice QCD, is in general inoperable in this regime for the following reason. In Euclidean metric the QCD Lagrangian reads

$$\mathscr{L}_{QCD} = \bar{\psi}(M+m)\psi + \frac{1}{4}F_{\mu\nu}F_{\mu\nu}, \text{ with } M(\mu) = D\!\!\!/[A] + \mu\gamma_0. \tag{1}$$

It is straightforward to show $\gamma_5 M(\mu)\gamma_5 \equiv M^\dagger(-\mu)$, implying $\det M(\mu) = (\det M(-\mu))^*$. This implies that the path integral measure is not real and positive for $\mu \neq 0$, the fundamental reason being traced to the explicit breaking of symmetry under time reversal. Therefore Monte Carlo importance sampling, the mainstay of numerical lattice QCD, is ineffective. It is helpful to ask what goes wrong if the real positive measure factor $\det M^\dagger M$, eg. as implemented in the hybrid Monte Carlo (HMC) algorithm, is used. It turns out that in QCD, while M describes a color triplet of quark fields q, the $M^\dagger$ factor describes a color antitriplet "conjugate quarks" q^c. Gauge singlet bound states of the form qq^c resemble mesons, but carry non-zero baryon charge $B > 0$. The lightest such state is degenerate with the pseudo-Goldstone π-meson; hence HMC simulations with $\mu \neq 0$ predict an unphysical "onset" transition from the vacuum to a state with quark density $n_q > 0$ at $\mu_o \simeq \frac{1}{2}m_\pi$. The resulting ground state is a Bose-Einstein condensate (BEC) of diquark baryons, and bears no resemblance to nuclear matter, which phenomenologically we know forms at $\mu_o \approx \frac{1}{N_c}m_{nucleon}$. The physical transition can only be found if the correct complex path integral measure $\det^2 M$ is used, and must result from extremely non-trivial cancellations between configurations with differing phases – this has come to be known as the *Silver Blaze* problem [1].

In QC$_2$D this is not a bug, but a feature. Because q and $\bar{q}$ live in equivalent representations of the color group, it is possible to show that $\det M$ is real and therefore the theory has a positive measure for N_f even [2]. Physically this is expressed through the presence of both $q\bar{q}$ mesons and qq, $\overline{qq}$ baryons in the same hadron multiplets. For light quarks the scale hierarchy $m_\pi \ll m_\rho$ permits the use of chiral perturbation theory (χPT) in studying the response of the lightest multiplet to $\mu \neq 0$ [3]. The key result is that for $\mu \geq \mu_o \equiv \frac{1}{2}m_\pi$ a non-zero baryon charge density $n_q > 0$ does develop, along with a gauge-invariant superfluid order parameter which for $N_f = 2$ reads $\langle qq\rangle \sim \langle \psi^{tr} C\gamma_5\tau_2\varepsilon_{ab}\psi\rangle \neq 0$, where τ_2 acts on color indices and $\varepsilon_{ab} = -\varepsilon_{ba}$ on flavor. The resulting system is a BEC composed of weakly interacting qq baryons with $J^P = 0^+$.

Quantitatively, for $\mu \geq \mu_o$ χPT predicts [3]

$$\frac{\langle\bar\psi\psi(\mu)\rangle}{\langle\bar\psi\psi\rangle_0} = \frac{\mu_o^2}{\mu^2}; \quad \frac{\langle qq(\mu)\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1-\frac{\mu_o^4}{\mu^4}}; \quad n_q(\mu) = 8N_f f_\pi^2 \mu \left(1-\frac{\mu_o^4}{\mu^4}\right). \tag{2}$$

This has been confirmed by several simulations using staggered lattice fermions, eg. [2, 4]. However, it is possible to use the last of these predictions to develop the thermodynamics of the system at $T=0$ more fully:

$$\text{pressure} \;\; p_{\chi PT} = \int_{\mu_o}^{\mu} n_q d\mu \; = \; 4N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right); \tag{3}$$

$$\text{energy density} \;\; \varepsilon_{\chi PT} = -p + \mu n_q \; = \; 4N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right); \tag{4}$$

$$\text{conformal anomaly} \;\; (T_{\mu\mu})_{\chi PT} = \varepsilon - 3p \; = \; 8N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right). \tag{5}$$

Note that $(T_{\mu\mu})_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$.

These results should be contrasted with those of another paradigm for cold dense matter, namely a degenerate system of weakly interacting (thus deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$:

$$n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4; \quad (T_{\mu\mu})_{SB} = 0. \tag{6}$$

In this system superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface; hence

$$\langle qq\rangle \propto \Delta\mu^2. \tag{7}$$

Fig. 1 plots the ratios $n_{\chi PT}/n_{SB}$, $p_{\chi PT}/p_{SB}$ and $\varepsilon_{\chi PT}/\varepsilon_{SB}$ as functions of μ for the choice $f_\pi^2 = N_c/6\pi^2$. Since pressure is just minus free energy density, by equating pressures we predict a phase transition between the BEC phase and free quark matter at $\mu_Q \approx 2.3\mu_o$[1]. Because n and ε are discontinuous at this point, this naive treatment predicts the resulting deconfining transition is first order.

This simple-minded argument has motivated us to pursue lattice simulations of QC_2D beyond the BEC regime, using $N_f = 2$ flavors of Wilson fermion. Since Wilson fermions do not have a manifest chiral symmetry, we have little to say about this aspect of the physics, which at high quark density should anyway be of secondary importance for phenomena near the Fermi surface; they do however carry a conserved baryon charge, which is crucial. Our initial runs were on a $8^3\times16$ system with lattice spacing $a = 0.23$fm, $m_\pi a = 0.79(1)$ and $m_\pi/m_\rho = 0.779(4)$ [5]. In this talk I will present as yet unpublished data from runs on an approximately matched $12^3\times24$ lattice with $a = 0.18$fm, $m_\pi a = 0.68(1)$ and $m_\pi/m_\rho = 0.80(1)$. Scales are set by equating the observed string tension to (440MeV)^3. In both cases the physical temperature $T \approx 50$MeV, although in fact the second lattice is cooler. We used a standard HMC algorithm – the only modification to orthodox

[1]the point of equality at $\mu/\mu_o \approx 1.4$ can be ignored because presumably at small densities a bag constant contribution to the BEC pressure can no longer be ignored.

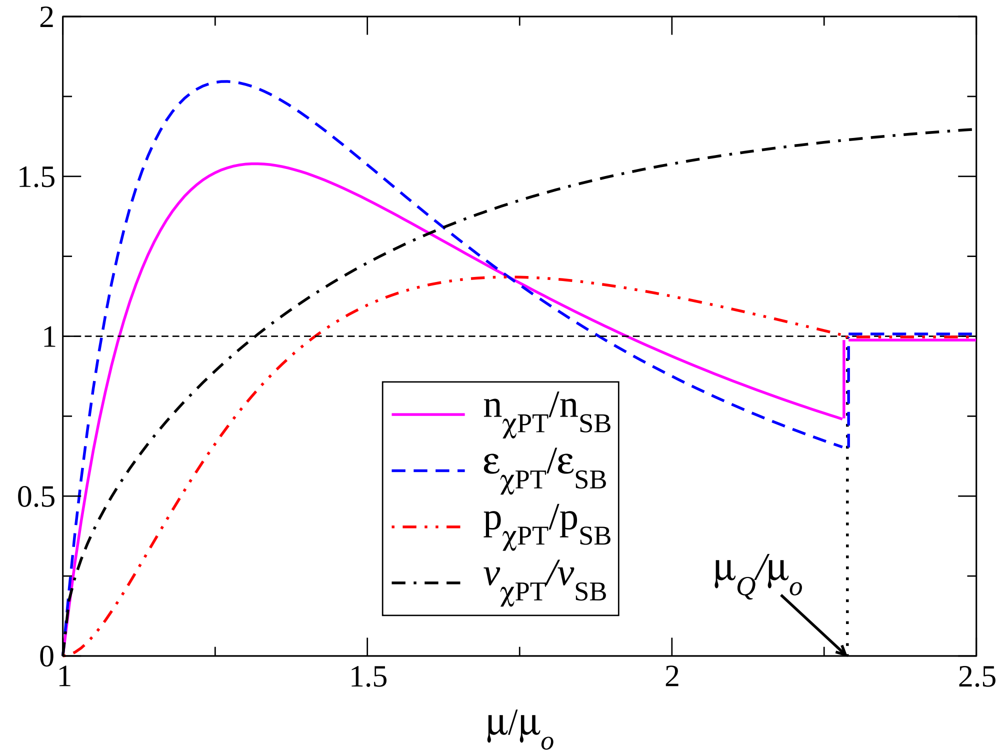

Figure 1: Comparison of thermodynamics at $T = 0$ for QC$_2$D between χPT and free degenerate quarks [5].

lattice QCD is to include a diquark source term of the form $j\kappa(-\bar\psi_1 C\gamma_5\tau_2\bar\psi_2^{tr} + \psi_2^{tr}C\gamma_5\tau_2\psi_1)$; this mitigates the impact of IR fluctuations in the superfluid regime and also enables the algorithm to change the sign of det$M$ for a single flavor, thus maintaining ergodicity. Ultimately the physical limt $j \to 0$ must be taken.

Fig. 2 shows results for quark density and pressure, plotted in the same way as in Fig. 1. The same gross features are present, namely a sharp rise from $\mu a \approx \mu_o a = 0.32$ up to a maximum, then a fall to a plateau beginning at $\mu_Q a \approx 0.5$, which continues until $\mu_D a \approx 0.8$. If following Fig. 1 we associate the plateau with the beginning of a degenerate matter phase then we identify a BEC/BCS crossover at $\mu_Q \approx 560$MeV, corresponding to a quark density $n_q \approx 6\text{fm}^{-3}$, ie. roughly 13 times nuclear density.

In contrast to χPT, the quark contribution to ε exceeds the free field value by almost a factor of 20 for $\mu \gtrsim \mu_o$, as shown in Fig. 3(a); it should be remarked here that unlike n_q and p, ε is subject to a quantum correction known as a Karsch coefficient which is still to be calculated for this system, though its renormalised value is unlikely to differ by more than 50%. In any case, since the Karsch coefficient is μ-independent, the shape of the curve will remain the same. Because of this unexpected behaviour at small μ, the energy per quark ε_q/n_q exhibits a shallow but robust minimum for $\mu > \mu_Q$, as shown in Fig. 3(b), a feature completely absent in the model governed by eqns. (2-6).

Fig. 4 plots quark and gluon contributions to the conformal anomaly $T_{\mu\mu}$. Once again, there are unknown additive and multiplicative renormalisation factors, but the shapes of the curves will remain unaltered. In panel (a) the vertical scales have been cunningly chosen to highlight the

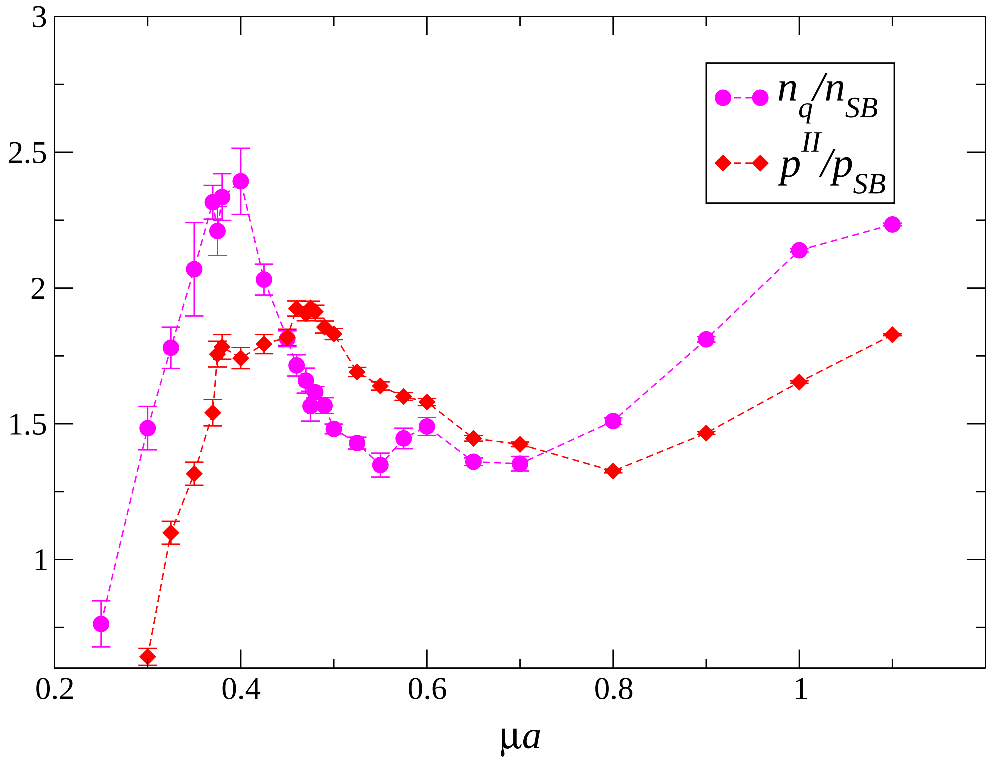

Figure 2: Equivalent to Fig. 1 for lattice QC$_2$D with $\beta = 1.9$, $\kappa = 0.168$, $ja = 0.04$ on $12^3 \times 24$.

similarity of quark and gluon components, but their behaviour diverges sharply above $\mu \approx \mu_Q$. In panel (b) which covers a wider μ-range, we see that the gluon data is to good approximation parabolic, suggesting that $\varepsilon_g < 3p$ as $\mu \to \infty$, and hence that conformal symmetry is not recovered in this limit (although of course the dimensionless combination $(\varepsilon - 3p)/\mu^4 \propto \mu^{-2}$). The negative value of $T_{\mu\mu}$ at large μ has also been predicted using a χPT treatment in which asymptotic freedom is taken into account [6]. Another very striking feature of Fig. 4(b) is the sharp change of behaviour in the quark component of $T_{\mu\mu}$ at $\mu_D a \approx 0.8$.

Next consider the gluonic contribution to the energy density. Fig. 5 plots the dimensionless combination ε_g/μ^4 against μ; of course this quantity is not predicted either in χPT or the free quark gas. While we have no quantitative theory of the gluonic contribution to QC$_2$D thermodynamics at $\mu \neq 0$, we would expect its relative importance to increase across a deconfining transition. In fact, the ratio is remarkably constant over a wide range of μ, as would be predicted by dimensional analysis; in particular there is no singular bahaviour at $\mu = \mu_Q$, although there is some hint of a systematic rise for $\mu \gtrsim \mu_D$.

In Fig. 6 we plot quantities which give information about the nature and symmetries of the ground state. In the limit $j \to 0$, the diquark condensate $\langle qq \rangle$ is an order parameter for the spontaneous breaking of U(1)$_B$ symmetry leading to baryon number superfluidity. Although the data of Fig. 6 is taken with $j \neq 0$, implying some care must be taken with the extrapolation $j \to 0$ at small μ [5], we are confident that this symmetry is broken for all $\mu > \mu_o$. The approximate flatness of the curve for $\mu_Q \lesssim \mu \lesssim \mu_D$ is then evidence for a scaling $\langle qq \rangle \propto \mu^2$ similar to (7). We take this as an

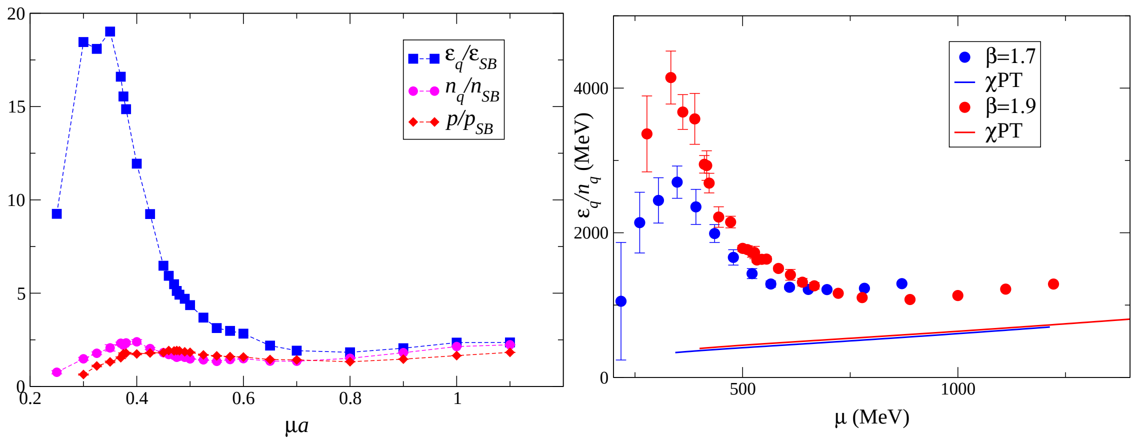

Figure 3: (a) same as Fig. 2, but now including ε_q; (b) energy per quark ε_q/n_q versus μ for two different lattice spacings

indication that in this region the system consists of degenerate quark matter with a Fermi surface disrupted by a BCS instability.

The Polyakov line is an order parameter for deconfinement in the limit of infinitely massive quarks – away from this limit it continues to yield information on the free energy of an isolated color source. Fig. 6 shows that QC$_2$D remains confined for $\mu < \mu_D$, but that there appears to be a transition to a deconfined state for chemical potentials in excess of this value. In physical units $\mu_D \approx 900$MeV, corresponding to a quark density $n_q \approx 35\text{fm}^{-3}$, some 80 times nuclear density.

In conclusion, the simulations suggest that QC$_2$D has three distinct transitions (or at least crossovers): the first at $\mu = \mu_o$ is a second order phase transition (in the limit $j \to 0$) from vacuum to a BEC superfluid, and is described accurately for the most part by χPT (the quark energy density ε_q looks to be an important exception); the second at $\mu = \mu_Q$ is a BEC/BCS crossover to form a ground state whose thermodynamic behaviour suggests it is formed of degenerate quark matter with a well-defined Fermi sphere, albeit one whose surface is disrupted by a BCS condensate; the third at $\mu = \mu_D$ is signalled by a discontinuity in gluon energy density, the quark contribution to the conformal anomaly, and a non-zero Polyakov loop, and corresponds to deconfined quark matter. Although the low density BEC phase is clearly unphysical (since to first approximation nuclear matter in our world is a degenerate system of nucleons), it may well be that for $\mu \gtrsim \mu_Q$ QC$_2$D has important and relevant lessons for quark matter in real QCD.

In particular, between μ_Q and μ_D the system looks to resemble the "quarkyonic matter" phase recently postulated on the basis of large-N_c arguments [7]; namely it is a state in which matter is degenerate so that there is a well-defined Fermi momentum scale, but also confined so that excitations above the ground state remain color singlet. A recent study of a Two Color quarkyonic phase using the PNJL model has appeared in [8]. An interesting issue is whether QC$_2$D is special in that

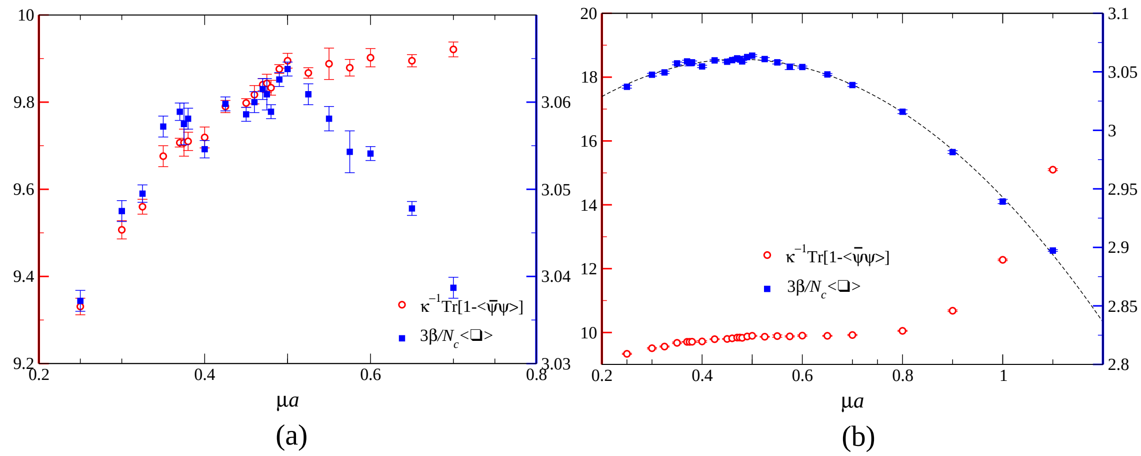

Figure 4: (a) Quark and gluon contributions to $T_{\mu\mu}$ vs. μ; (b) same as (a) with a scale extending to larger μ.

the 2-body bound states required by color confinement are also preferred by more general renormalisation group arguments [9]. Whatever the outcome, to our mind the study of deconfinement in this hitherto-unexplored physical regime promises to be fascinating.

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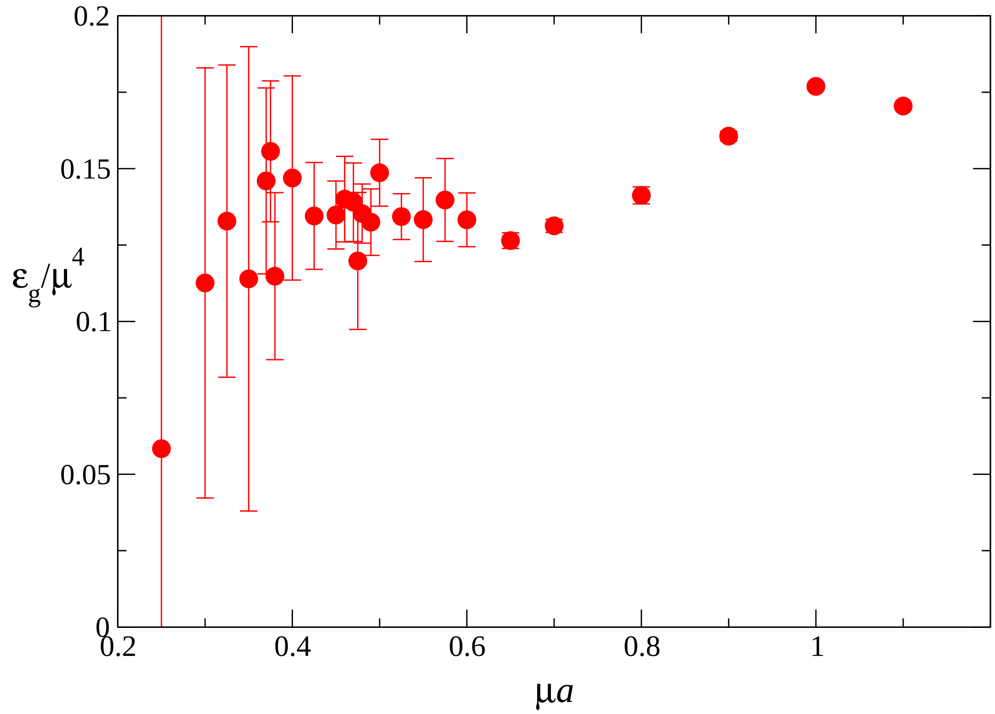

Figure 5: Gluon contribution to the energy density ε_g/μ^4 versus μ.

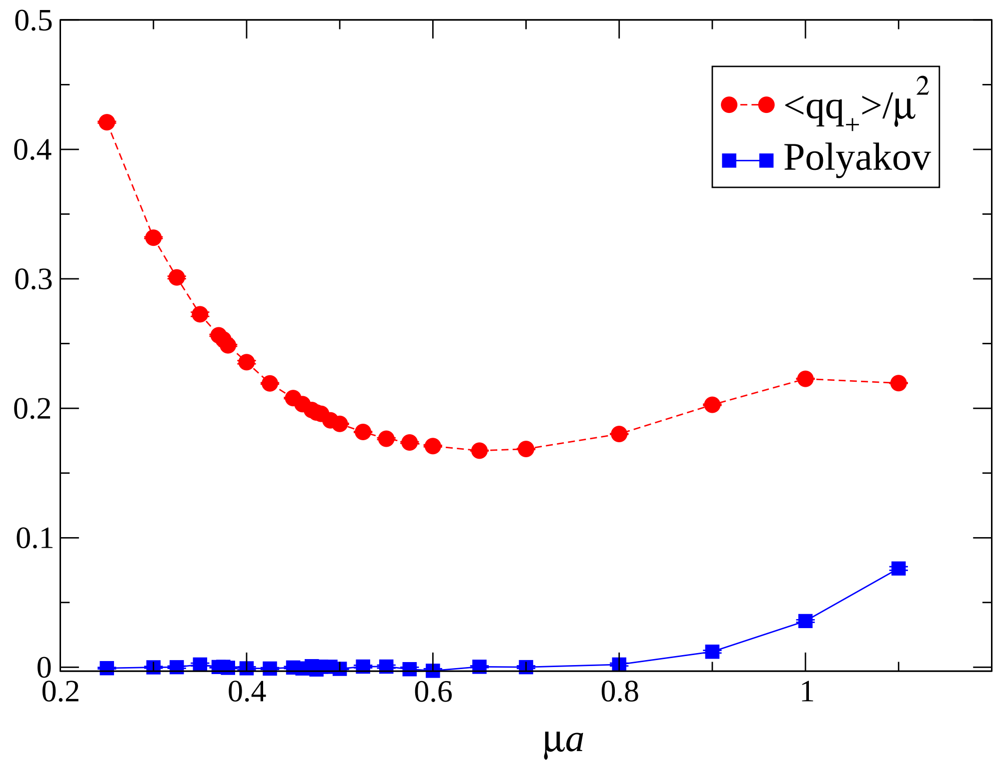

Figure 6: Superfluid order parameter $\langle qq\rangle/\mu^2$ and Polyakov line versus μ.