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Origin of resonances in chiral dynamics

Tetsuo Hyodo*
Department of Physics, Tokyo Institute of Technology, Meguro 152-8551, Japan
E-mail: hyodo@th.phys.titech.ac.jp

Daisuke Jido
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Atsushi Hosaka
Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan

The nature of baryon resonances is studied in the dynamical chiral coupled-channel approach for meson-baryon scattering. In general, origin of resonances in two-body scattering can be classified into two categories: dynamically generated states and genuine elementary particles. We demonstrate that the genuine contribution in the loop function can be excluded by adopting a natural renormalization scheme. The origin of resonances can be studied by looking at the effective interaction in the natural renormalization scheme, which is deduced from the phenomenological amplitude fitted to experimental data. Applying this method to the baryon resonances, we find that the dominant component for the $\Lambda(1405)$ resonance is dynamical, while a genuine contribution plays a substantial role for the structure of the $N(1535)$.

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1. Introduction

One of the great successes of current algebra in 1960's is the celebrated Weinberg-Tomozawa theorem [1, 2], which tells us that the chiral symmetry highly constrains the low energy *s*-wave scattering of the Nambu-Goldstone boson with a target hadron. The establishment of power counting [3] enables one to sort out the effective chiral Lagrangian and amplitude, leading to the systematic computation of the higher order correction to current algebra, which results in the chiral perturbation theory [4]. In recent years, implementation of the chiral low energy interaction into the dynamical framework of hadron scattering was turned out to be a powerful tool to study resonance physics. This chiral coupled-channel approach has been providing fairly successful description of baryon resonances in the scattering of an octet pseudoscalar meson and an octet ground state baryon [5, 6, 7, 8].

Once a good description of a resonance is obtained, we may next consider what structure it has. Excited baryons can consist of several components, such as three-quark state, meson-baryon molecule, and more complicated structures. Quantum theory tells us that the physical state must be realized as a superposition of all possible components, as far as they have the same quantum numbers. Hence, the clarification of the *dominant* component among others should help our intuitive understanding of the structure.

In chiral dynamics, the excited baryons are described as resonances in the meson-baryon scattering amplitude. In this case, any components other than dynamical two-body state (meson-baryon molecule) are expressed by the Castillejo-Dalitz-Dyson (CDD) pole contribution [9]. Therefore, the estimation of the size of the CDD pole contribution in the amplitude will shed light on the origin of resonances in this approach. We would like to report our recent study on the origin of baryon resonances in chiral dynamics, paying special attention to the renormalization procedure [10].

2. Chiral dynamics for meson-baryon scattering

We first write down the general form of the amplitude based on the N/D method [7], for *s*-wave single-channel meson-baryon scattering at total energy $\sqrt{s}$:

$$T(\sqrt{s};a) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s};a)}, \tag{2.1}$$

where $V(\sqrt{s})$ is the kernel interaction constrained by chiral symmetry, which is a real function expressing dynamical contributions other than the *s*-channel unitarity. The function $G(\sqrt{s};a)$ is the once subtracted dispersion integral of the phase-space function $\rho(\sqrt{s})$, with which the unitarity of the amplitude is maintained through the optical theorem: $\mathrm{Im}\, T^{-1}(\sqrt{s};a) = -\mathrm{Im}\, G(\sqrt{s};a) = \rho(\sqrt{s})/2$. While the imaginary part of $G(\sqrt{s};a)$ is given by the phase space, the real part depends on the subtraction constant a.

We may identify the dispersion integral $G(\sqrt{s};a)$ as the loop function with dimensional regularization. In this case, Eq. (2.1) is considered to be the solution of the algebraic Bethe-Salpeter equation and the subtraction constant a plays a similar role with the cutoff parameter in the loop function. Through the order by order matching, the interaction kernel $V(\sqrt{s})$ is determined by chiral perturbation theory [7]. At leading order, $V(\sqrt{s})$ is given by the *s*-wave interaction of the

Weinberg-Tomozawa (WT) term

$$V(\sqrt{s}) = V_{\rm WT}(\sqrt{s}) = -\frac{C}{2f^2}(\sqrt{s}-M_T), \tag{2.2}$$

where C, M_T and f are the group theoretical factor, the baryon mass, and the meson decay constant, respectively. With the leading order WT term, this framework is almost equivalent to the old coupled-channel works with vector meson exchange potential [11]. Based on chiral perturbation theory, it is now possible to include higher order correction systematically [5, 8].

In the framework of N/D method, the CDD pole contribution should be included in the kernel interaction $V(\sqrt{s})$, except for the poles at infinity. Indeed, it is known that the contribution from the genuine states can be introduced in the interaction kernel $V(\sqrt{s})$, *via* an explicit resonance propagator [12] or the contracted resonance contribution from the higher order Lagrangian [13]. In the following, we demonstrate that the CDD pole contribution can be embedded also in the loop function $G(\sqrt{s})$, and propose a method to extract such a hidden contribution by using a different renormalization scheme.

3. CDD pole contribution in the loop function

In the standard phenomenological studies, the interaction kernel $V(\sqrt{s})$ is determined first, and then the subtraction constant a has been fitted to reproduce experimental data. To illustrate the role of the subtraction constant in this approach, we argue the phenomenological amplitude in a schematic manner. Suppose that we have the scattering amplitude $T_{\rm exp}$ with enough experimental data. We may try to calculate this amplitude in chiral approach with the leading order kernel $V^{(1)}$ and with the next-to-leading order kernel included $V^{(1)}+V^{(2)}$:

$$T^{(1)}(a^{(1)}) = \frac{1}{[V^{(1)}]^{-1}-G(a^{(1)})}, \tag{3.1}$$

$$T^{(2)}(a^{(2)}) = \frac{1}{[V^{(1)}+V^{(2)}]^{-1}-G(a^{(2)})}. \tag{3.2}$$

The subtraction constant should be chosen independently in each scheme for a good description of the resulting scattering amplitude. If we achieve the complete description $T^{(1)}=T^{(2)}=T_{\rm exp}$, the amplitudes in two schemes should become equivalent. In Eq. (3.1), the effect of the different interaction kernels must be compensated by the difference of the subtraction constant.

The leading order interaction $V^{(1)}$ is the WT term in Eq. (2.2), so it is clear that this term has no s-channel resonance contribution. On the other hand, the higher order term $V^{(2)}$ can have the CDD pole contribution from the contracted resonance propagator, as is known by the studies of chiral perturbation theory. If this is the case, the subtraction constant $a^{(1)}$ effectively contains the CDD pole contribution in the loop function for the scheme of Eq. (3.1).

The above exercise implies the existence of the CDD pole contribution in the loop function. As far as phenomenological aspects of the model are concerned, the existence of the CDD pole contribution in the loop function is not a problem at all. However, in order to move one step forward to study the origin of resonances in this approach, we should follow a different strategy to make the CDD pole contribution in the model under control.

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4. Natural renormalization scheme

For this purpose, we propose the "natural renormalization scheme," in which the hidden CDD pole contribution in the loop function $G(\sqrt{s};a)$ is visualized in the interaction kernel $V(\sqrt{s})$. This situation can be achieved by requiring

- no state exists below the meson-baryon threshold, and
- the amplitude T matches with the interaction kernel V at certain low energy scale.

The latter condition is based on the validity of chiral expansion for low energy kinematics. These conditions uniquely determine the subtraction constant in the natural renormalization scheme a_{natural} such that the loop function should satisfy [10]

$$G(\sqrt{s};a_{\text{natural}}) = 0 \quad \text{at} \quad \sqrt{s} = M_T. \tag{4.1}$$

The condition (4.1) was already proposed in a different context; in Ref. [8] the matching with the u-channel scattering amplitude was emphasized, and the matching with chiral low energy amplitude was argued in Refs. [14, 12]. Our point is to regard this condition as the exclusion of the CDD pole in the loop function, based on the consistency with the negativeness of the loop function.

5. Energy scale of the natural renormalization scheme

Let us discuss the typical energy scale for the condition (4.1). In Ref. [7], a "natural" value for the subtraction constant was estimated to be $a(\mu) \sim -2$, through the comparison of the loop function of dimensional regularization with that of three-momentum cutoff q_{max} under nonrelativistic expansion, putting $q_{\text{max}} = \mu = 630$ MeV and the average target baryon mass $M_T = 1.15$ GeV.

This is different from our value of a_{natural}, practically and conceptually. The "natural" value in Ref. [7] is obtained by fixing the typical energy scale of the meson-baryon scattering to be 630 MeV. In the present context, the condition (4.1) is obtained by excluding the CDD poles the loop function, without introducing an explicit energy scale (such as $\sim$ 630 MeV). Therefore, it is not guaranteed that we obtain the "natural size" $a(\mu) \sim -2$ in our natural renormalization scheme.

In this respect, it is instructive to study the typical energy scale given in Eq. (4.1). As discussed in Ref. [7], it is not possible to match the real part of the loop functions in different regularization in whole energy region, since they have different $\sqrt{s}$ dependence. Here we estimate the scale of the loop function by matching it with three-momentum cutoff regularization at threshold:

$$G^{3d}(\sqrt{s} = M_T + m; q_{\text{max}}) = G^{\text{dim}}(\sqrt{s} = M_T + m, a_{\text{natural}}). \tag{5.1}$$

Since a_{natural} is given by Eq. (4.1), this equation determines the value of q_{max} for given m and M_T. The results are shown in Fig. 5. As seen in the figure, for $\bar{K}N$ scattering ($m \sim 496$ MeV, $M_T \sim 939$ MeV), the typical scale is around 630 MeV in accordance with the estimation in Ref. [7]. On the other hand, when the pion is scattered as in $\pi\Sigma$ channel ($m \sim 138$ MeV, $M_T \sim 1193$ MeV), the scale of the natural renormalization is as small as 200 MeV. One should keep in mind that the natural renormalization scheme may introduce an energy scale different from the scale of our interest.

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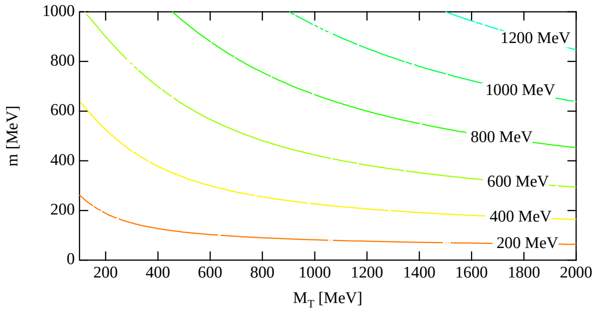

Figure 1: Contour plot of the typical energy scale q_{max} corresponding to the natural subtraction constant a_{natural} is shown as functions of the masses M_T and m.

6. Interpretation of phenomenological model

Using the natural renormalization scheme, we can study the origin of the resonances in chiral dynamics. Let us assume that we have enough experimental data for the system of interest from the low-energy to the resonance-energy region. In the conventional phenomenological approaches, we choose the interaction kernel V as the leading order WT term,

$$T(\sqrt{s};a_{\text{pheno}}) = \frac{1}{V_{\text{WT}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{pheno}})}, \tag{6.1}$$

where the subtraction constant a_{pheno} in the loop function G is a free parameter to reproduce experimental data and takes care of the contributions that are not included in the interaction kernel V_{WT}. We call this procedure the phenomenological renormalization scheme.

On the other hand, the natural renormalization scheme fixes the subtraction constant such that in the resulting loop function there is no contribution from states below the threshold. To achieve the equivalent scattering amplitude, a different interaction kernel V_{natural} will be required:

$$T(\sqrt{s};a_{\text{natural}}) = \frac{1}{V_{\text{natural}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{natural}})}, \tag{6.2}$$

with the subtraction constant a_{natural}.

The scattering amplitude T should equivalently be reproduced by both renormalization schemes. Thus, equating the denominators of Eqs. (6.1) and (6.2)

$$V_{\text{natural}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{natural}}) = V_{\text{WT}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{pheno}}) \ , \tag{6.3}$$

we obtain the effective interaction kernel V_{natural} in the natural renormalization scheme as

$$V_{\text{natural}}(\sqrt{s}) = -\frac{C}{2f^2}(\sqrt{s}-M_T) + \frac{C}{2f^2}\frac{(\sqrt{s}-M_T)^2}{\sqrt{s}-M_{\text{eff}}}, \tag{6.4}$$

with an effective mass

$$M_{\text{eff}} \equiv M_T - \frac{16\pi^2 f^2}{CM_T\Delta a}, \quad \Delta a = a_{\text{pheno}} - a_{\text{natural}}. \tag{6.5}$$

The expression (6.4) indicates that the interaction kernel $V_{\rm natural}(\sqrt{s})$ can have a pole in the s-channel scattering region with an attractive interaction $C > 0$ and a negative value for Δa. Note that the energy dependence of each term of Eq. (6.4) is consistent with the chiral expansion, since the pole term is quadratic in powers of the meson energy $\omega \sim \sqrt{s} - M_T$.

The relevance of the second term of Eq. (6.4) depends on the scale of the effective mass $M_{\rm eff}$, which is obtained by the difference of the phenomenological and natural subtraction constants Δa. If Δa is small, the effective pole mass $M_{\rm eff}$ becomes large. In this case, the second term of Eq. (6.4) can be neglected or gives smooth energy dependence in the resonance energy region $\sqrt{s} \sim M_T + m \ll M_{\rm eff}$. If the difference Δa is large, the effective mass $M_{\rm eff}$ gets closer to the threshold and the pole contribution is no longer negligible. This means that the use of a negative Δa with large absolute value is equivalent to the introduction of a pole in the chiral Lagrangian. We therefore consider that the pole in the effective interaction (6.4) is a source of the physical resonances in this case. In this way, we find a possible seed of the resonance in the loop function, even if we use the leading order chiral interaction.

In the phenomenological scheme (6.1), the interaction kernel $V_{\rm WT}$ does not include the CDD pole contribution, while in the natural scheme (6.2) the loop function G does not contain the CDD pole, as discussed in the previous section. Therefore, when the physical amplitude contains the CDD pole contribution, the effect is attributed to $G(\sqrt{s}; a_{\rm pheno})$ in the phenomenological scheme, while to $V_{\rm natural}(\sqrt{s})$ in the natural scheme. Indeed, we have demonstrated that $V_{\rm natural}(\sqrt{s})$ contains a resonance propagator. In the limit $\Delta a \to 0$, the two schemes agree with each other, which corresponds to the amplitude compatible with the meson-baryon picture of resonances, as explained in Sec. 4.

7. Numerical analysis

Let us apply the above method to physical baryon resonances in meson-baryon scattering. We consider meson-baryon scatterings in $S = -1$ and $I = 0$ channel and $S = 0$ and $I = 1/2$ channel, where the $\Lambda(1405)$ and the $N(1535)$ resonances are generated, respectively. For these channels, the phenomenological subtraction constants $a_{{\rm pheno},i}$ can be found in Refs. [17, 18] which are based on the results in Refs. [16, 15]. In these models, the scattering observables such as cross sections and phase shifts are well reproduced by the interaction kernel of the WT term. At the same time, according to Eq. (4.1), we obtain the natural values of the subtraction constants $a_{{\rm natural},i}$ by setting $G(M_N) = 0$ for all channels.

We evaluate the effective interaction in the natural renormalization scheme and extract the pole positions in the kernel. The nearest pole of the effective interaction in each channel is given by[1]

$$z_{\rm eff}^{N^*} = 1693 \pm 37i \text{ MeV}, \quad z_{\rm eff}^{\Lambda^*} \sim 7.9 \text{ GeV}. \tag{7.1}$$

It is observed that the pole for the $N(1535)$ lies in the energy region of resonance, while the pole for the $\Lambda(1405)$ is obviously out of the scale of the physics of the resonance. The influence of the pole (7.1) can be clearly seen in the diagonal components of the second term of Eq. (6.4) (Fig. 7). We observe that the contributions are small for the $S = -1$ channels, whereas there is a bump

[1] With n-coupled channels, the effective interaction has n poles, and a pair of complex poles can also appear [10].

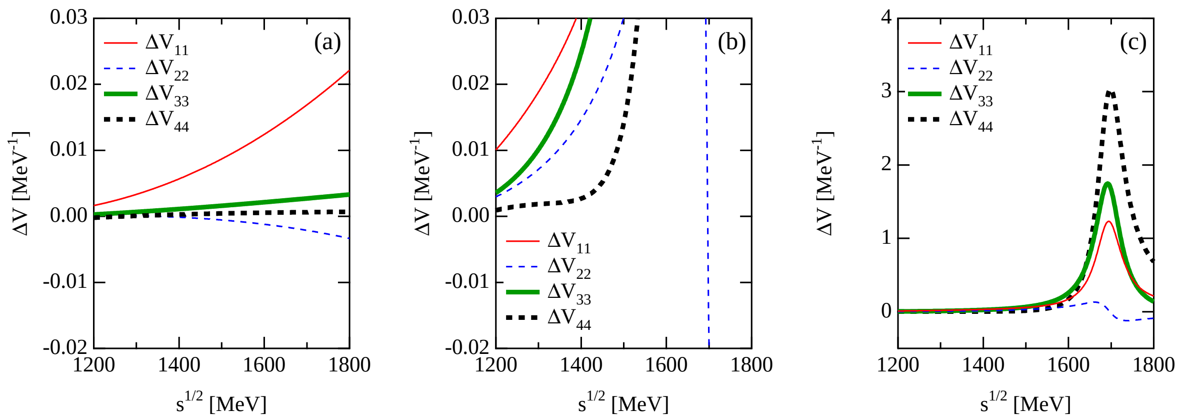

Figure 2: Deviations of the effective interactions from the Weinberg-Tomozawa term, (a) $S=-1$ channels, (b) enlargement of panel (c), (c) $S=0$ channels. The channels 1–4 correspond to $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, and $K\Xi$ for $S=-1$ channels, and to πN, ηN, $K\Lambda$, and $K\Sigma$ for $S=0$ channels, respectively.

structure at around 1700 MeV in the $S=0$ channel, originate from the pole (7.1) [Fig. 7(c)]. This result indicates that the $\Lambda(1405)$ is largely dominated by the component of the dynamical meson and baryon, while the $N(1535)$ may require some CDD pole contribution.

Next we consider the idealized purely dynamical components, which can be obtained by adopting the WT term for the interaction kernel and the natural subtraction constant. In this case, there is no free parameter in the model and we can calculate the pole positions:

$$z^{N^*} = 1582 - 61i \text{ MeV}, \tag{7.2}$$

$$z_1^{\Lambda^*} = 1417 - 19i \text{ MeV}, \quad z_2^{\Lambda^*} = 1402 - 72i \text{ MeV}. \tag{7.3}$$

Note that in chiral dynamics the $\Lambda(1405)$ is described as the two poles in the complex energy plane [19, 20]. These can be compared with the pole positions in the phenomenological amplitude:

$$z^{N^*} = 1493 - 31i \text{ MeV}, \tag{7.4}$$

$$z_1^{\Lambda^*} = 1429 - 14i \text{ MeV}, \quad z_2^{\Lambda^*} = 1397 - 73i \text{ MeV}, \tag{7.5}$$

which corresponds to the physical resonances. We plot the pole positions in Fig. 3. The poles for the $\Lambda(1405)$ appear in the close positions for the dynamical component (7.3) and the physical one (7.5). This again indicates the dominance of the meson-baryon component in the $\Lambda(1405)$. On the other hand, the pole for the $N(1535)$ moves to the higher energy when we use the natural values. Although the dynamical component generates a state by itself, the physical $N(1535)$ requires some more contributions, which is expressed as the pole in the effective interaction (7.1) in the natural scheme. The comparison in Fig. 3 also indicates the dynamical nature of the $\Lambda(1405)$ and the sizable CDD pole contribution for the $N(1535)$.

8. Conclusions

In this report, we have discussed the origin of the resonances in chiral dynamics. From the viewpoint of the renormalization, we point out that the CDD pole contribution can be accommo-

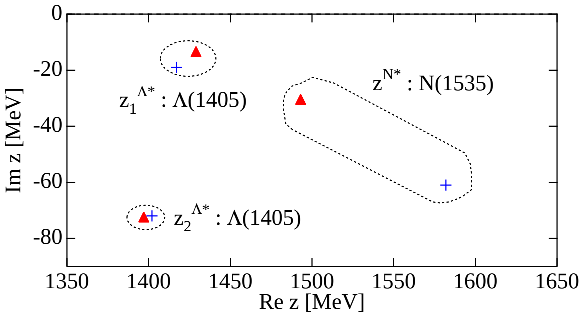

Figure 3: Pole positions of the meson-baryon scattering amplitudes. The crosses denote the pole positions in the natural renormalization with the WT interaction (dynamical component); the triangles stand for the pole positions with the phenomenological amplitude. $z_1^{\Lambda^*}$ and $z_2^{\Lambda^*}$ are the poles for the $\Lambda(1405)$ in the $S=-1$ scattering amplitude, and z^{N^*} is the pole for the $N(1535)$ in the $S=0$ amplitude.

dated in the loop function, whose effect was not clear in the standard phenomenological fitting scheme. To avoid this kind of ambiguity of the interaction kernel, we construct the "natural renormalization" scheme for the loop function in which the CDD pole contribution is excluded. We show that it is possible to visualize the CDD pole contribution in the interaction kernel, from which the information of the origin of the resonances can be clearly extracted.

We analyze the $S=-1$ and $S=0$ meson-baryon scatterings in which the $\Lambda(1405)$ and the $N(1535)$ are dynamically generated. We find that the physical $\Lambda(1405)$ can be well reproduced when the leading order WT interaction is used as for the kernel of the scattering equation, while the $N(1535)$ requires a substantial correction in addition to the WT term, especially a pole singularity at around 1700 MeV. These facts indicate that the $\Lambda(1405)$ can be mainly described by a dynamical state of the meson-baryon scattering, which is consistent with the analysis of the N_c scaling [21, 22] and the estimation of the electromagnetic size [23]. On the other hand, the $N(1535)$ may have an appreciable component beyond the present model space of meson-baryon two-body coupled channels. This could be, for instance, conventional three-quark state, correlated five-quark state, chiral partner of the ground state nucleon, or dynamical vector-meson plus baryon channels. Further investigation is called for the clarification of the structure of the $N(1535)$ resonance.

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