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$B \to K\eta^{(\prime)}$ decays and NLO contributions in the pQCD approach

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We report our calculations of the partial NLO QCD corrections to the $B \to K\eta^{(\prime)}$ decays in the perturbative QCD (pQCD) factorization approach. The NLO contributions can provide a 70% enhancement (a 30% reduction) to the leading order pQCD predictions for the branching ratios of $B \to K\eta'$ ($B \to K\eta$) decays. Such NLO contributions play the key role in understanding the observed pattern of branching ratios. The pQCD predictions for the CP asymmetries of $B \to K\eta^{(\prime)}$ decays are also consistent with currently available data.

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1. Introduction

The unexpectedly large branching ratios for $B \to K\eta'$ decays were firstly reported in 1997 by CLEO Collaboration [1]. 12 years later, three of the four $B \to K\eta^{(\prime)}$ decays have been measured with high precision [2]. Besides the branching ratios, the CP violating asymmetries for $B^{\pm} \to K^{\pm}\eta^{(\prime)}$ and $B^0 \to K^0\eta^{(\prime)}$ decays have been measured very recently [2, 3].

In the SM the decay $B \to K\eta^{(\prime)}$ is believed to proceed dominantly through gluonic penguin processes[4] and has been evaluated by employing various methods [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Although great progress have been made during the past decade, but the predictions for $Br(B \to K\eta')$ from both the QCD factorization (QCDF) approach [13, 16] and the perturbative QCD (pQCD) approach [15, 17] are still smaller than the data.

Furthermore, there is a large disparity between the measured branching ratios: $Br(B \to K\eta') \gg Br(B \to K\eta)$. Many efforts have been made to interpret this pattern, which include, for example,

(a) Conventional $b \to sq\bar{q}$ with constructive (destructive) interference between the $u\bar{u}, d\bar{d}$ and $s\bar{s}$ components of η' (η) [4];

(b) Large intrinsic charm content of η' through the chain $b \to sc\bar{c} \to s\eta'$ [6] or through $b \to sc\bar{c} \to sg^*g^* \to s(\eta, \eta')$ due to the QCD anomaly [7];

(c) The spectator hard-scattering mechanism through the anomalous coupling of $gg \to \eta'$ [8, 9, 10];

(d) A significant flavor-singlet contribution [9, 13];

(e) A strong penguin $b \to sg$ enhanced by new physics [11, 12].

In Ref. [15], the authors calculated the branching ratios of $B \to K\eta^{(\prime)}$ decays by employing the pQCD approach at leading order. They considered the large corrections from $SU(3)$ flavor symmetry breaking as well as the possible gluonic component of η' meson, but their prediction for $Br(B^0 \to K^0\eta')$ ($Br(B^0 \to K^0\eta)$) is much smaller (larger) than the measured value. A sizable gluonic content in η' meson may provide a large enhancement to the decay rate of $B \to K\eta'$. But the calculation in Ref. [18] showed that such contribution is numerically very small and can be neglected safely.

Besides the possible mechanisms mentioned above, we here consider a new and natural solution: the effects of the next-to-leading order (NLO) contributions in the pQCD approach. The NLO contributions considered here include: QCD vertex corrections, the quark-loops and the chromo-magnetic penguins. We expect that they are the major part of the full NLO contributions in pQCD approach [19].

In the pQCD approach, the decay amplitude is separated into soft (Φ_{M_i}), hard ($H(k_i,t)$), and harder($C(M_W)$) dynamics characterized by different energy scales ($\Lambda_{QCD}, t, m_b, M_W$) [17]. The decay amplitude $\mathscr{A}(B \to M_2M_3)$ can be written conceptually as the convolution,

$$\mathscr{A}(B \to M_2M_3) \sim \int d^4k_1 d^4k_2 d^4k_3 \,\mathrm{Tr}\left[C(t)\Phi_B(k_1)\Phi_{M_2}(k_2)\Phi_{M_3}(k_3)H(k_1,k_2,k_3,t)\right], \tag{1.1}$$

where k_i's are momenta of light quarks included in each meson, and Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient evaluated at scale t. The hard kernel $H(k_1,k_2,k_3,t)$

PoS(FPCP2009)055

describes the hard dynamics, and therefore can be perturbatively calculated. The function Φ_{M_i} is the wave function.

Since the b quark inside the B meson is rather heavy, we consider the B meson at rest for simplicity. It is then convenient to use light-cone coordinate $(p^+, p^-, \mathbf{p}_\mathrm{T})$ to describe the meson's momenta: $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $\mathbf{p}_\mathrm{T} = (p^1, p^2)$.

For the studied $B \to K\eta^{(\prime)}$ decays, the weak effective Hamiltonian H_{eff} for $b \to s$ transition can be written as [20]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qs}^* \left\{ \left[C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu) \right] + \sum_{i=3}^{10} C_i(\mu)\, O_i(\mu) \right\} . \tag{1.2}$$

where $G_F = 1.16639 \times 10^{-5} GeV^{-2}$ is the Fermi constant, and V_{ij} is the CKM matrix element, $C_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale μ and $O_i(\mu)$ are the four-fermion operators.

In PQCD approach, the energy scale "t" is chosen as the largest energy scale in the hard kernel $H(x_i, b_i, t)$ of a given Feynman diagram, in order to suppress the higher order corrections and improve the reliability of the perturbative calculation. Here, the scale "t" may be larger or smaller than the m_b scale. In the range of $t < m_b$ or $t \geq m_b$, the number of active quarks is $N_f = 4$ or $N_f = 5$, respectively. The explicit expressions of the LO and NLO $C_i(m_W)$ can be found easily, for example, in Refs. [21, 20]. For the expressions of the wave functions of B meson and the relevant distribution functions of the K and (η, η') mesons, one can see Ref. [21, 22]. The Gegenbauer moments are the following [23]:

$$a_1^K = 0.2, \quad a_2^K = 0.25, \quad a_4^K = -0.015. \tag{1.3}$$

The values of other parameters are $\eta_3 = 0.015$ and $\omega = -3.0$.

For the mixing of the $\eta - \eta'$ system, we use the the quark-flavor mixing scheme, where the physical states η and η' are related to the flavor states $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ through a single mixing angle ϕ,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} F_1(\phi)(u\bar{u} + d\bar{d}) + F_2(\phi)\, s\bar{s} \\ F_1'(\phi)(u\bar{u} + d\bar{d}) + F_2'(\phi)\, s\bar{s} \end{pmatrix} \tag{1.4}$$

with $F_1(\phi) = \cos\phi/\sqrt{2}$, $F_2(\phi) = -\sin\phi$, $F_1'(\phi) = \sin\phi/\sqrt{2}$ and $F_2'(\phi) = \cos\phi$. The distribution amplitudes $\phi_{\eta_q}^{A,P,T}$ represent the axial vector, pseudoscalar and tensor component of the wave function respectively [23], and can be found in Ref.[22].

2. Decay amplitudes at leading order

At the leading order in pQCD approach, the Feynman diagrams as shown in Fig. 1 may contribute to $B \to K\eta^{(\prime)}$ decays. From the factorizable emission diagrams 1(a) and 1(b), the corresponding form factors can be extracted by perturbative calculation. For Fig.1(a) and 1(b) with the $B \to K$ transition, the operators $O_{1,2}$, $O_{3,4}$ and $O_{9,10}$ are $(V-A)(V-A)$ currents, the sum of the

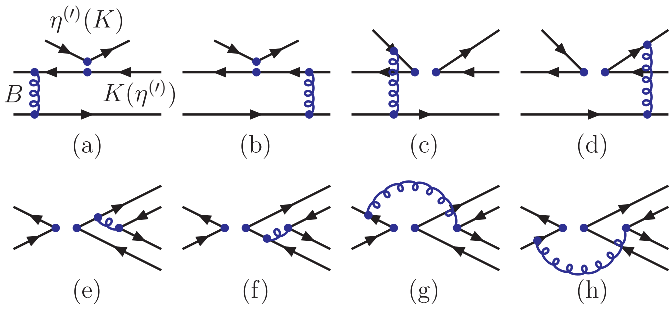

Figure 1: Feynman diagrams which may contribute to the $B \to K\eta^{(\prime)}$ decays at leading order.

individual amplitudes is given as

$$\begin{aligned} F_{eK} &= \frac{8}{\sqrt{2}}\pi G_F C_F m_B^4 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1,b_1) \\ &\quad \times \left\{ \left[(1+x_2)\phi_K^A(\bar{x}_2) + (1-2x_2) r_K (\phi_K^P(\bar{x}_2) - \phi_K^T(\bar{x}_2))\right] \cdot E_e(t_a) h_e(x_1,x_2,b_1,b_2) \right. \\ &\quad \left. + 2 r_K \phi_K^P(\bar{x}_2) \cdot E_e(t_a') h_e(x_2,x_1,b_2,b_1) \right\}, \end{aligned} \tag{2.1}$$

where $r_K = m_0^K/m_B$ with m_0^K is the chiral scale; $C_F = 4/3$ is a color factor, and $\bar{x}_2 = 1 - x_2$. The evolution function $E_e(t)$ and hard function h_e can be found in Ref. [22]. Also from diagrams 1(a) and 1(b), the decay amplitudes corresponding to the $(V-A)(V+A)$ and/or $(S-P)(S+P)$ currents are the following

$$F_{eK}^{P1} = -F_{eK}\,, \tag{2.2}$$

$$\begin{aligned} F_{eK}^{P2} &= \frac{16}{\sqrt{2}}\pi G_F C_F m_B^4 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1) \\ &\quad \times \left\{ r_\eta \left[\phi_K^A(\bar{x}_2) + r_K((2+x_2)\phi_K^P(\bar{x}_2) + x_2 \phi_K^T(\bar{x}_2))\right] \cdot E_e(t_a) h_e(x_1,x_2,b_1,b_2) \right. \\ &\quad \left. + 2 r_K r_\eta \phi_K^P(\bar{x}_2) \cdot E_e(t_a') h_e(x_2,x_1,b_2,b_1) \right\}\,. \end{aligned} \tag{2.3}$$

From Fig. 1, one can find the corresponding decay amplitudes: $(M_{eK}, M_{eK}^{P1,P2})$ (1(c) and 1(d)), $(M_{aK}, M_{aK}^{P1,P2})$ (1(e) and 1(f)) and $(F_{aK}, F_{aK}^{P1,P2})$ (1(g) and 1(h)) [22]. By exchanging position of the K and $\eta^{(\prime)}$ in Fig. 1, one can find the corresponding decay amplitudes for the new diagrams easily[22]: $(F_{e\eta}, F_{e\eta}^{P1,P2})$, $(M_{e\eta}, M_{e\eta}^{P1,P2})$, $(M_{a\eta}, M_{a\eta}^{P1,P2})$, and $(F_{a\eta}, F_{a\eta}^{P1,P2})$.

For the two $B \to K\eta$ decays, the total decay amplitude with the inclusion of the corresponding Wilson coefficients can be finally written as

$$\begin{aligned} \mathcal{M}(K^0\eta) &= <K^0\eta|H_{eff}|B^0> = F_{eK}\left\{ \left[\xi_u a_2 - \xi_t\left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9\right)\right] f_\eta^q \right. \\ &\quad \left. -\xi_t\left(a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right) f_\eta^s \right\} - F_{e\eta}\xi_t\left(a_4 - \frac{1}{2}a_{10}\right) f_K F_1(\phi) \\ &\quad - \left[F_{eK}^{P_2} f_\eta^s + F_{e\eta}^{P_2} f_K F_1(\phi)\right] \xi_t \left(a_6 - \frac{1}{2}a_8\right) - \left[F_{ak}F_2(\phi) + F_{a\eta}F_1(\phi)\right]\, \xi_t \left(a_4 - \frac{1}{2}a_{10}\right) \\ &\quad + \left[F_{aK}^{P_2} F_2(\phi) + F_{a\eta}^{P_2} F_1(\phi)\right]\, \xi_t \left(a_6 - \frac{1}{2}a_8\right)\, f_B \end{aligned}$$

$$+M_{eK}\left\{\left[\xi_u C_2-\xi_t\cdot\left(2C_4+\frac{1}{2}C_{10}\right)\right]F_1(\phi)-\xi_t\left(C_3+C_4-\frac{1}{2}C_9-\frac{1}{2}C_{10}\right)F_2(\phi)\right\}$$
$$-M_{e\eta}\xi_t\left(C_3-\frac{1}{2}C_9\right)F_1(\phi)-\left[M_{eK}^{P1}F_2(\phi)+M_{e\eta}^{P1}F_1(\phi)\right]\xi_t\left(C_5-\frac{1}{2}C_7\right)$$
$$-M_{eK}^{P_2}\xi_t\left[\left(2C_6+\frac{1}{2}C_8\right)F_1(\phi)+(C_6-\frac{1}{2}C_8)F_2(\phi)\right], \tag{2.4}$$

$$\mathscr{M}(K^+\eta) = < K^+\eta|H_{eff}|B^0 >= F_{eK}\left\{\left[\xi_u a_2-\xi_t\left(2a_3-2a_5-\frac{1}{2}a_7+\frac{1}{2}a_9\right)\right]f_\eta^q\right.$$
$$\left.-\xi_t\left(a_3+a_4-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9-\frac{1}{2}a_{10}\right)f_\eta^s\right\}+\{F_{e\eta}F_1(\phi)f_K+[F_{a\eta}F_1(\phi)$$
$$+F_{aK}F_2(\phi)]f_B\}\,\xi_u a_1-[F_{e\eta}F_1(\phi)f_K+(F_{a\eta}F_1(\phi)+F_{aK}F_2(\phi))\,f_B]\,\xi_t\,(a_4+a_{10})$$
$$-\left[F_{e\eta}^{P2}F_1(\phi)f_K+\left(F_{a\eta}^{P2}F_1(\phi)+F_{aK}^{P2}F_2(\phi)\right)f_B\right]\xi_t\,(a_6+a_8)-F_{eK}^{P2}f_\eta^s\xi_t\left(a_6-\frac{1}{2}a_8\right)$$
$$-M_{eK}^{P1}\xi_t\left(C_5-\frac{1}{2}C_7\right)+M_{eK}\left\{\left[\xi_u C_2-\xi_t\left(2C_4+\frac{1}{2}C_{10}\right)\right]F_1(\phi)\right.$$
$$\left.-\xi_t\left(C_3+C_4-\frac{1}{2}C_9-\frac{1}{2}C_{10}\right)F_2(\phi)\right\}-\left[M_{aK}^{P1}F_2(\phi)+\left(M_{e\eta}^{P1}+M_{a\eta}^{P1}\right)F_1(\phi)\right]$$
$$\times\xi_t(C_5+C_7)+[M_{aK}F_2(\phi)+(M_{e\eta}+M_{a\eta})F_1(\phi)]\,[\xi_u C_1-\xi_t\,(C_3+C_9)]$$
$$-M_{eK}^{P2}\xi_t\left[\left(2C_6+\frac{1}{2}C_8\right)F_1(\phi)+\left(C_6-\frac{1}{2}C_8\right)F_2(\phi)\right]. \tag{2.5}$$

where $\xi_u=V_{ub}^*V_{us}$, $\xi_t=V_{tb}^*V_{ts}$, the coefficients a_i are the combinations of the Wilson coefficients C_i, and have been defined as usual

$$a_{1,2}=C_{2,1}+\frac{C_{1,2}}{3};\quad a_i=C_i+\frac{C_{i+1}}{3}, i=3,5,7,9;\quad a_i=C_i+\frac{C_{i-1}}{3}, i=4,6,8,10. \tag{2.6}$$

The total decay amplitudes for the two $B\to K\eta'$ decays can be obtained easily from Eqs.(2.4) and (2.5) by the following replacements

$$f_\eta^d\to f_{\eta'}^d,\quad f_\eta^s\to f_{\eta'}^s,\quad F_1(\phi)\to F_1'(\phi),\quad F_2(\phi)\to F_2'(\phi). \tag{2.7}$$

3. NLO contributions in pQCD approach

At the NLO level, the following changes or contributions should be considered:

- The NLO Wilson coefficients $C_i(m_W)$, the NLO RG evolution matrix and the $\alpha_s(t)$ at two-loop level should be used.
- The NLO hard kernel $H^{(1)}(\alpha_s^2)$ should be included. All the Feynman diagrams, which may contribute to the hard kernel H at the order of α_s^2, as illustrated by Figs. 2-5, should be considered.

At present, the calculations for the vertex corrections, the quark-loops and chromo-magnetic penguins, in relevant with $B\to K\eta^{(\prime)}$ decays as shown in Fig. 2, have been done in Ref. [22]. For

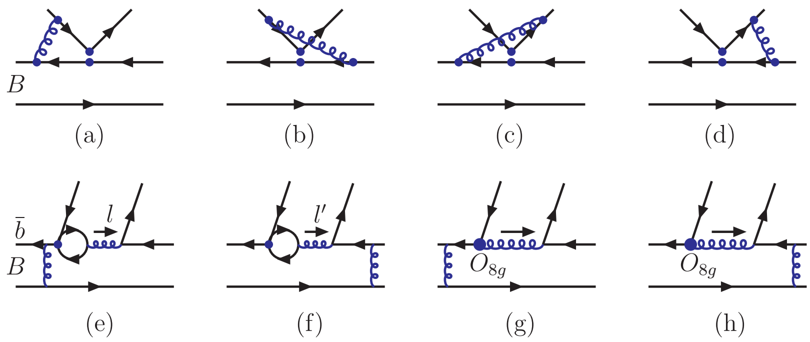

Figure 2: The Feynman diagrams in relevant with the NLO vertex QCD corrections, the quark-loops and charomo-magnetic penguins for $B \to K\eta^{(\prime)}$ decays.

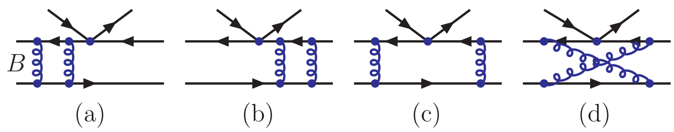

Figure 3: The typical vertex Feynman diagrams which contribute to the form factors at the NLO level.

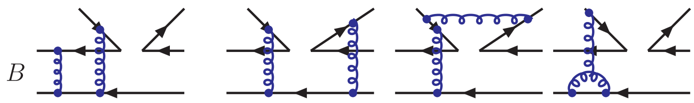

Figure 4: The typical hard spectator Feynman diagrams which contribute at the NLO level.

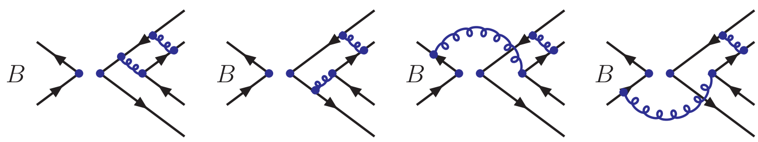

Figure 5: The typical annihilation Feynman diagrams which contribute at the NLO level.

the Feynman diagrams as shown in Figs. 3-5, however, the analytical calculations have not been completed yet.

The vertex corrections to the factorizable emission diagrams, as illustrated by Figs. 2a-2d, have been calculated years ago in the QCD factorization appeoach[16, 13, 14]. The difference of the calculations induced by considering or not considering the parton transverse momentum is rather small [19], say less than 10%, and therefore can be neglected. The vertex corrections can then be absorbed into the re-definition of the Wilson coefficients $a_i(\mu)$ by adding a vertex-function $V_i(M)$ to them[14, 19].

The contribution from the so-called "quark-loops" is a kind of penguin correction with the four quark operators insertion, as illustrated by Fig. 2e-2f. For the $b \to s$ transition, the contributions from the various quark loops are given by:

$$H_{eff}^{(ql)} = - \sum_{q=u,c,t} \sum_{q'} \frac{G_F}{\sqrt{2}} V_{qb} V_{qs}^* \frac{\alpha_s(\mu)}{2\pi} C^q(\mu, l^2) \left(\bar{s}\gamma_\rho \left(1-\gamma_5\right) T^a b\right) \left(\bar{q}' \gamma^\rho T^a q'\right), \tag{3.1}$$

It is straightforward to calculate the decay amplitude for Fig.2e and 2f. We found two kinds of topological decay amplitudes [22]: $M^{(q)}_{K\eta_s}$ for $B \to K$ transition and $M^{(q)}_{\eta_q K}$ for $B \to \eta$ transition. For $B \to K\eta'$ decays, we found the similar decay amplitudes. Finally, the total "quark-loop" contribution to the considered $B \to K\eta^{(\prime)}$ $(K = K^0, K^+)$ decays can be written as [22]

$$M^{(ql)}_{K\eta} = < K\eta|\mathscr{H}^{ql}_{eff}|B >= \frac{G_F}{\sqrt{2}} \sum_{q=u,c,t} \lambda_q \left[M^{(q)}_{K\eta_s} F_2(\phi) + M^{(q)}_{\eta_q K} F_1(\phi) \right], \tag{3.2}$$

$$M^{(ql)}_{K\eta'} = < K\eta'|\mathscr{H}^{(ql)}_{eff}|B >= \frac{G_F}{\sqrt{2}} \sum_{q=u,c,t} \lambda_q \left[M^{(q)}_{K\eta_s} F'_2(\phi) + M^{(q)}_{\eta_q K} F'_1(\phi) \right]. \tag{3.3}$$

For the magnetic penguins, the corresponding weak effective Hamiltonian contains the $b \to sg$ transition,

$$H^{cmp}_{eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} C^{eff}_{8g} O_{8g}, \tag{3.4}$$

with the chromo-magnetic penguin operator,

$$O_{8g} = \frac{g_s}{8\pi^2} m_b \, \bar{d}_i \, \sigma^{\mu\nu} \, (1+\gamma_5) \, T^a_{ij} \, G^a_{\mu\nu} \, b_j, \tag{3.5}$$

where i, j being the color indices of quarks. The corresponding effective Wilson coefficient $C^{eff}_{8g} = C_{8g} + C_5$.

The decay amplitudes, $M^{(g)}_{K\eta_s}$ and $M^{(g)}_{\eta_q K}$, have been obtained by evaluating the Feynman diagrams Figs.2g and 2h [22]. The total chromo-magnetic penguin contribution to the considered $B \to K\eta^{(\prime)}$ $(K = K^0, K^+)$ decays can be written as

$$M^{(cmp)}_{K\eta} = < K\eta|\mathscr{H}^{cmp}_{eff}|B >= -\frac{G_F}{\sqrt{2}} \lambda_t \left[M^{(g)}_{K\eta_s} F_2(\phi) + M^{(g)}_{\eta_q K} F_1(\phi) \right], \tag{3.6}$$

$$M^{(cmp)}_{K\eta'} = < K\eta'|\mathscr{H}^{cmp}_{eff}|B >= -\frac{G_F}{\sqrt{2}} \lambda_t \left[M^{(g)}_{K\eta_s} F'_2(\phi) + M^{(g)}_{\eta_q K} F'_1(\phi) \right]. \tag{3.7}$$

4. Numerical Results and Discussions

We use the following input parameters [2, 24] in the numerical calculations

$$\begin{aligned} f_B &= 0.21\text{GeV}, \quad f_K = 0.16\text{GeV}, \quad m_\eta = 547.5\text{MeV}, \quad m_{\eta'} = 957.8\text{MeV}, \\ m_K &= 0.49\text{GeV}, \quad m_{0K} = 1.7\text{GeV}, \quad M_B = 5.279\text{GeV}, m_b = 4.8\text{GeV}, \\ M_W &= 80.41\text{GeV}, \quad \tau_{B^0} = 1.527\text{ps}, \quad \tau_{B^+} = 1.643\text{ps}. \end{aligned} \tag{4.1}$$

For the CKM quark-mixing matrix elements, we use the values as given in Ref.[2, 24]:

$$\begin{aligned} V_{ud} &= 0.9745, \quad V_{us} = \lambda = 0.2200, \quad |V_{ub}| = 4.31 \times 10^{-3}, \quad V_{cd} = -0.224, \\ V_{cd} &= 0.996, \quad V_{cb} = 0.0413, \quad |V_{td}| = 7.4 \times 10^{-3}, \quad V_{ts} = -0.042, \quad V_{tb} = 0.9991, \end{aligned} \tag{4.2}$$

with the CKM angles $\beta = 21.6^\circ$, $\gamma = 60^\circ \pm 20^\circ$ and $\alpha = 100^\circ \pm 20^\circ$.

Using the wave functions and the input parameters as specified in previous sections, it is straightforward to calculate the CP-averaged branching ratios for the considered four $B \to K\eta^{(\prime)}$

Table 1: The pQCD predictions for the branching ratios (in unit of 10^{-6}). The label $\mathrm{LO_{NLOWC}}$ means the LO results with the NLO Wilson coefficients, and +VC, +QL, +MP, NLO means the inclusion of the vertex corrections, the quark loops, the magnetic penguin, and all the considered NLO corrections, respectively.

Mode	LO	$LO_{\rm NLOWC}$	+VC	+QL	+MP	NLO	Data	QCDF
$B^+ \to K^+\eta$	4.7	4.7	4.3	4.9	3.1	$3.2^{+3.2}_{-1.8}$	2.6 ± 0.6	$1.9^{+3.0}_{-1.9}$
$B^+ \to K^+\eta'$	30.2	46.8	74.6	48.1	30.2	$51.0^{+18.0}_{-10.9}$	70.5 ± 3.5	$49.1^{+45.2}_{-23.6}$
$B^0 \to K^0\eta$	3.2	3.4	3.1	3.8	2.3	$2.1^{+2.6}_{-1.5}$	< 2.0	$1.1^{+2.4}_{-1.5}$
$B^0 \to K^0\eta'$	31.3	46.5	69.7	48.5	20.7	$50.3^{+16.8}_{-10.6}$	68 ± 4	$46.5^{+41.9}_{-22.0}$

decays, which are listed in Table 1. For comparison, we also list the corresponding updated experimental results [2] and numerical results evaluated in the framework of the QCDF approach [14].

It is worth stressing that the theoretical predictions in the pQCD approach have relatively large theoretical errors induced by the still large uncertainties of many input parameters, such as quark masses($m_{u,d}, m_s$), chiral scales (m_{0K}, m_0^q, m_0^s), Gegenbauer coefficients ($a_i^{(K,\eta)}, \cdots$), ω_b and the CKM angles (α, γ), etc. From the numerical results about the branching ratios, one can see that

- The LO pQCD predictions for branching ratios are much smaller (larger) than the measured values for $B \to K\eta'$ ($B \to K\eta$) decays, show the same tendency as found in Ref. [15].
- The NLO contributions can interfere constructively (destructively) with the corresponding LO parst for $B \to K\eta'$ ($B \to K\eta$) decays. For $B^0 \to K^0\eta'$ and $B^+ \to K^+\eta'$ decays, the NLO contributions provide a 70% enhancement to their branching ratios . For $B^0 \to K^0\eta$ and $B^+ \to K^+\eta$ decays, on the other hand, the NLO contributions give rise to a 30% reduction to their branching ratios and result in the good agreement between the pQCD predictions and the data.
- The NLO pQCD predictions for branching ratios $Br(B \to K\eta^{(\prime)})$ agree very well with the measured values within one standard deviation. The NLO contributions play an important role in understanding the observed pattern of branching ratios of the four $B \to K\eta^{(\prime)}$ decays.

It is easy to calculate the direct CP-violating asymmetries for the considered decays, which are listed in Table 2. As a comparison, we also list currently available data [2] and the corresponding QCDF predictions [14]. As to the CP-violating asymmetries for the neutral decays $B^0 \to K^0\eta^{(\prime)}$, one can see the numerical results and discussions as given in Ref.[22].

In short, we calculated the branching ratios and CP-violating asymmetries of $B^+ \to K^+\eta^{(\prime)}$ and $B^0 \to K^0\eta^{(\prime)}$ decays in the pQCD approach. The partial NLO contributions considered here include: QCD vertex corrections, the quark-loops and the chromo-magnetic penguins.

From our calculations and phenomenological analysis, we found the following results:

(a) The NLO contributions in the pQCD approach can provide a 70% enhancement to $Br(B \to K\eta')$, but a 30% reduction to $Br(B \to K\eta)$. The large branching ratio of $B \to K\eta'$ decays, as well as the large disparity $Br(B \to K\eta') \gg Br(B \to K\eta)$ can therefore be understood naturally.

Table 2: The pQCD predictions for the direct CP asymmetries in the NDR scheme (in units of 10^{-2}), the QCDF predictions [14] and the world average as given by HFAG [2].

Mode	LO	+VC	+QL	+MP	NLO	Data	QCDF
$\mathscr{A}_{CP}^{dir}(B^\pm \to K^\pm\eta)$	9.3	31.1	7.8	7.6	−11.7	-27 ± 9	$-18.9^{+29.0}_{-30.0}$
$\mathscr{A}_{CP}^{dir}(B^\pm \to K^\pm\eta')$	−10.1	−10.6	−5.9	−10.4	−6.2	1.6 ± 1.9	$-9.0^{+10.6}_{-16.2}$

(b) The pQCD predictions for the CP asymmetries of $B \to K\eta^{(\prime)}$ decays are consistent with currently available data.

(c) One should note that Only the partial NLO contributions in the pQCD approach have been taken into account here. To achieve a complete NLO calculations in the pQCD approach, of course, the still missing pieces should be evaluated as soon as possible.

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