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The Golden Modes: $B_d \to J/\psi K_s$ and $B_s \to J/\psi \phi$

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Motivated by the precision of the data for the "golden mode" $B \to J/\psi K_s$ the theoretical prediction of the golden modes $B_d \to J/\psi K_s$ and $B_s \to J/\psi \phi$ is re-investigated. The major question with respect to theoretical uncertainties is, how to reliably estimate the effect of doubly Cabibbo suppressed penguin contributions. Perturbative approaches are considered as well as methods based on flavour symmetries. The overall situation is not conclusive; penguin contributions large enough to shift β by a few degrees cannot be excluded.

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1. Introduction

It has been pointed out already quite some time ago that a measurement of the time dependent CP asymmetry in the decay $B \to J/\psi K_s$ will allow a precise determination of the CKM angle β [1]. The theoretical uncertainty of this method is due to penguin contributions which induce amplitudes with a different weak phase. These will modify the naive relation between β and the measured time dependent CP asymmetry and, in addition, can induce direct CP violation. However, in the so called "golden modes" these penguin contributions appear only strongly CKM suppressed, and hence only small corrections to the naive formulae are expected.

Explicitly, the typical amplitude for a non-leptonic two-body decay can be written as

$$A(B^0 \to f) = \mathscr{A}\left[1 + r_f\, e^{i\delta_f}\, e^{i\theta_f}\right] \tag{1.1}$$

where usually $\mathscr{A}$ is the tree amplitude and r_f denotes the modulus of the penguin-over-tree ratio, which has a strong phase θ_f. The weak phase δ_f is in the cases at hand the CKM angle γ, while the modulus of the CKM factors is absorbed into r_f.

The key observable is the time dependent CP asymmetry, which is given by

$$A_{\mathrm{CP}}(t;f) \equiv \frac{\Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)}{\Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f)} = \frac{A_{\mathrm{D}}^f \cos(\Delta M_q t) + A_{\mathrm{M}}^f \sin(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2) - \mathscr{A}_{\Delta\Gamma}^f \sinh(\Delta\Gamma_q t/2)} \tag{1.2}$$

where $q = d, s$ and M_q and $\Delta\Gamma_q$ denotes the mass and width differences in the B_d-$\bar{B}_d$ system.

We my express the observables A_{D}^f and A_{M} in terms of the parameters of the amplitude given in (1.1)

$$A_{\mathrm{D}}^f = -2 r_f \sin\theta_f \sin\delta_f \tag{1.3}$$
$$A_{\mathrm{M}}^f = \left[\sin\phi_s + 2 r_f \cos\theta_f \sin(\phi_s + \delta_f) + r_f^2 \sin(\phi_s + 2\delta_f)\right] \tag{1.4}$$

where ϕ_s is the mixing phase stemming from the $\Delta B = 2$ interaction. We omit the expression for $\mathscr{A}_{\Delta\Gamma}^f$, since we shall not discuss effects originating from finite lifetime differences.

The reason why the decay $B \to J/\psi K_s$ is called "gold plated" is that the ratio r_f in this case is suppressed by small CKM angles

$$r_f \propto \varepsilon = \left|\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*}\right| \sim 5\%\,, \tag{1.5}$$

where the constant of proportionality is a ratio of the hadronic matrix elements of penguin and tree operators, which in general is believed to be less than unity. Hence to a very good approximation we get

$$A_{\mathrm{D}}^f = 0 \qquad \text{and} \qquad A_{\mathrm{M}}^f = \sin\phi_s = \sin 2\beta \tag{1.6}$$

As far as data is concerned, the time-dependent CP asymmetry in $B \to J/\psi K_s$ is the flagship measurement of the B factories. The current precision is already at a level, where small effects become important. In particular, if there is a non-standard contribution in the $\Delta B = 2$ interaction, we would have an additional piece due to "new physics" in the mixing phase

$$\phi_d = 2\beta + \phi_d^{\mathrm{NP}}\,. \tag{1.7}$$

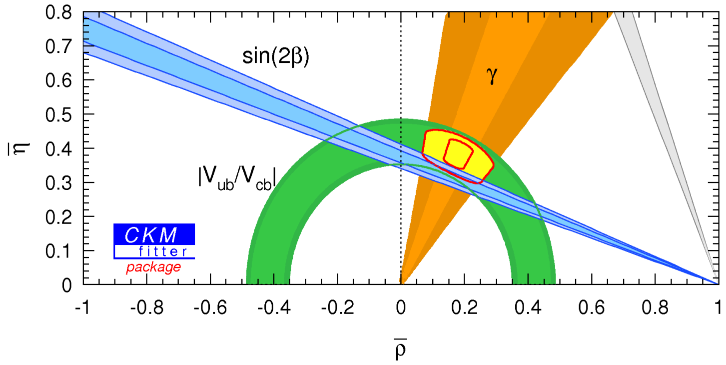

Figure 1: Illustration of the current tension in the CKM triangle

However, to become sensitive to a small $\phi_d^{\rm NP}$ a good control over the standard model contribution to the CP asymmetries coming from r_f is mandatory.

The current data has an indication for a tension in the unitarity-triangle fit. Extracting a value for β from $|V_{ub}/V_{cb}| = 0.0958^{+0.003}_{-0,005} \pm 0.007$ and $\gamma = (65 \pm 10)^\circ$ alone, one obtains a "true" value

$$(\sin 2\beta)_{\rm true} = 0.76^{+0.02+0.04}_{-0.04-0.05} \tag{1.8}$$

which may be compared with the measurement of the CP asymmetry in $B \to J/\psi K_s$. One obtains

$$(\phi_d)_{J/\psi K^0} - 2\beta_{\rm true} = -(8.7^{+2.6}_{-3.6} \pm 3.8)^\circ \tag{1.9}$$

indicating a tension between these two quantities, which is illustrated in fig. 1. A similar tension can be constructed from the ratio of the oscillation frequencies $\Delta M_d/\Delta M_s$ and the parameter ε from Kaon CP violation, which makes the input of the somewhat controversial (due to the tension between its exclusive and inclusive value) parameter V_{ub} obsolete [2, 3].

Assuming that this becomes evidence in the near future, it is necessary to re-investigate the standard-model contributions to r_f in order to obtain the correct interpretation of this tension, namely either in terms of sizable penguin contributions or in term of new physics. In the next section we investigate the different theoretical approaches to a calculation (or better to an estimate) for r_f.

2. Theoretical Approaches

We shall first look at the purely theoretical approaches which try to estimate the hadronic matrix elements by more or less QCD based models. The partonic calculation is based on the QCD diagrams which are evaluated in perturbation theory. However, the scales are quite low, rendering perturbation theory unreliable. Alternatively, one may try to relate the hadronic matrix elements to other observables, which, however, requires some additional assumptions.

The second way to estimate r_f is to make use of flavour symmetries and data of similar decay modes. The drawback here is that the flavour symmetry relations needed are affected by flavour symmetry breaking and hence again sizable uncertainties remain.

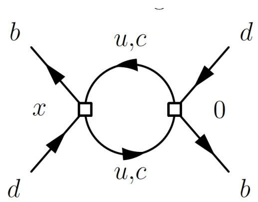

Figure 2: Diagrams for the contribution to the $\Delta B = 2$ interaction from internal up and charm quarks

2.1 Partonic Calculation

The suggestion to estimate r_f is in fact quite old; it has been proposed originally by Bander, Silverman and Soni [4] and has been applied more recently to $B \to J/\psi K_s$ in [5].

Aside from the corrections to r_f one has also a contribution to the $\Delta B = 2$ interaction coming from internal up and charm quarks shown in fig. 2. Due to the GIM mechanism these contributions are suppressed by a factor m_c^2/M_W^2, but contain large logarithm $\ln\left(m_c^2/M_W^2\right)$. The total contribution from this source can be calculated reliably and turns out to be very small

$$\Delta\phi_d \approx -2\frac{m_c^2}{m_t^2}\ln\left(\frac{m_c^2}{M_W^2}\right) \approx -4\times 10^{-4}. \tag{2.1}$$

A more severe problem is the estimate of the penguin contribution r_f. According to [4] we calculate the diagrams shown in fig. 3 which yields a contribution to the "effective interaction" of the form

$$\begin{aligned}\mathscr{H}_{\text{eff}}^{\text{Peng.}}(b\to c\bar{c}s) = -\frac{G_F}{\sqrt{2}}\Bigg\{&\frac{\alpha}{3\pi}(\bar{s}b)_{V-A}(\bar{c}c)_V\cdot\left[1+\mathscr{O}\left(\frac{M_\Psi^2}{M_Z^2}\right)\right]\\ &+\frac{\alpha_s(k^2)}{3\pi}(\bar{s}T^a b)_{V-A}(\bar{c}T^a c)_V\Bigg\}\cdot\left(\frac{5}{3}-\ln\left(\frac{k^2}{\mu^2}\right)+i\pi\right)\end{aligned} \tag{2.2}$$

The matrix elements that appear in this expression can be estimated from the rate for $B \to J/\psi K_s$; in particular, the color-octet matrix element can be obtained from the non-factorizable contribution to this decay.

The problem with expression (2.2) is that two parameters appear which have to be fixed. On one hand there is the scale k^2 of α_s the natural value of which is $k^2 = M_{J/\psi}^2$ on the other hand

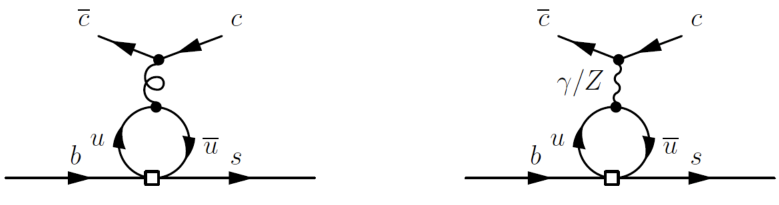

Figure 3: Diagrams for the perturbative estimate of r_f.

there is a dependence on the renormalization point μ, which we set to $\mu = m_b$. These choices are ambiguous, reflecting the fact that there are substantial uncertainties in this approach.

Inserting the numbers and taking into account the date from the rate of $B \to J/\psi K_s$, the authors of [5] obtain

$$S(J/\psi K_S) = (\sin 2\beta)_0 - (2.16 \pm 2.23) \times 10^{-4} \tag{2.3}$$

$$C(J/\psi K_S) = (5.0 \pm 3.8) \times 10^{-4} \tag{2.4}$$

where the uncertainties are only the ones due to the extraction of the matrix elements from the rate of $B \to J/\psi K_s$.

Although this estimate suffers from substantial uncertainties, the conclusion form this calculation is that the standard contribution to r_f in case of $B \to J/\psi K_s$ is too small to matter at the current level of precision. However, in many other circumstances (such as e.g. perturbative calculations of form factors) it is observed that the perturbative approach tends to underestimate the true effects, sometimes even dramatically.

2.2 Hadronic Calculation

A different ansatz to estimate r_f has been proposed by Gronau and Rosner [6]. Here the backscattering of the quarks $\bar{u}u \to \bar{c}c$ is estimated by inserting hadronic intermediate states. Denoting the T-matrix for he weak interactions with T and the strong scattering with S_0, one obtains to leading order in the weak interactions

$$T = S_0 T S_0 \tag{2.5}$$

This yields for the $\bar{u}u \to \bar{c}c$ backscattering

$$\langle J/\psi K^0 | T^u | B \rangle = \sum_f \langle J/\psi K^0 | S_0 | f \rangle \langle f | T^u | B \rangle \tag{2.6}$$

by inserting a complete set of states. Here T^u is the $b \to s\bar{u}u$ penguin contribution, and we now need an estimate of $\langle f | T^u | B \rangle$ for $f = K^* \pi, K^{**}\pi, \ldots$.

This estimate is obtained by applying the same reasoning to the T^c piece of T, which corresponds to the tree contribution $b \to s\bar{c}c$.

$$\langle f | T^c | B \rangle = \sum_k \langle f | S_0 | k \rangle \langle k | T^c | B \rangle \tag{2.7}$$

where we inserted again a complete set of states.

If we saturate the sum by a single state, one may set up the inequality

$$|\langle f | T | B \rangle| \geq |\langle f | S_0 | D^* D_s \rangle| \, |\langle D^* D_s | T | B \rangle| \tag{2.8}$$

which leads to [6]

$$\xi_f \equiv \frac{|\langle J/\psi K^0 | S_0 | f \rangle \langle f | T^u | B \rangle|}{|\langle J/\psi K^0 | T | B \rangle|} \leq \frac{1}{3} \frac{|\langle f | T^u | B \rangle|}{|\langle f | T^c | B \rangle|} \left(\frac{|\langle f | T | B \rangle|}{\langle J/\psi K^0 | T | B \rangle|} \right)^2 \tag{2.9}$$

which is the contribution to the penguin-over-tree ratio from the intermediate state f This relation may thus serve to bound the $\bar{u}u \to \bar{c}c$ backscattering contributions from each individual intermediate state f.

The right-hand side of this inequality can be estimated by using data on charmless two-body decays into a pseudoscalar and a vector meson. The authors of [6] estimate the contributions of the channels $K^*\pi$, ρK, ωK and $K\eta$ to be less than $\xi_f \le 8\times 10^{-4}\cdots 8\times 10^{-5}$, depending on the channel. Since the individual contributions are all of order 10^{-3} or less, the authors of [6] end up with the conclusion that the full re-scattering, obtained from summing over all intermediate states, is less than 10^{-2}.

However, it is known e.g. from D-$\bar{D}$ mixing and from re-scattering estimates that a sum over a finite number of intermediate states does not represent the full answer very well. In particular, in cases where a sizable energy release is present, a partonic calculation based on the assumption of duality seems to work better. In the case at hand, the estimates are based on the two-body modes for which measurements exist, and it is no clear, which fraction of the total charmless rate mediated by T^u is going into these particular decays.

3. Approaches based on Data

As an alternative, one may try to obtain information on the penguin contributions by relating the matrix elements appearing in $B \to J/\psi K_s$ to the matrix elements of other decays, which are e.g related by flavour symmetries. This idea has been exploited extensively, for an application to the golden modes see e.g. [7].

Here we shall discuss the results of a recent fit from [8]. Here the golden mode $B \to J/\psi K$ is compared to the situation in the decay $B \to J/\psi\pi$. For the charged modes, these tow decays are U-spin partners, while this is not the case for the neutral B mesons, see e.g.. [9]. Nevertheless we shall use the size of the r_f ratio obtained from $B_d \to J/\psi\pi^0$ as an estimate of the size of the corresponding ratio in $B_d \to J/\psi K_s$.

We shall express our results in terms of the shift $\Delta\phi_d$ of the observed mixing angle induced by the penguin contribution. We obtain [8]

$$\tan\Delta\phi_d = \frac{2r_f\cos\theta\sin\gamma + r_f^2\sin 2\gamma}{1+2r_f\cos\theta\cos\gamma + r_f^2\cos 2\gamma} \tag{3.1}$$

where γ is the weak (CKM) phase and θ is the strong phase.

We may determine a value for r_f from the data of $B_d \to J/\psi\pi^0$, where the CKM suppression of the penguin contributions is much less severe than in the golden mode. The time-dependent CP asymmetry $A_{\rm CP}(t;J/\psi\pi^0)$ was recently measured by the BaBar (SLAC) [10] and Belle (KEK) [11] collaborations, yielding the following averages [12]:

$$C(J/\psi\pi^0) = -0.10\pm 0.13, \tag{3.2}$$

$$S(J/\psi\pi^0) = -0.93\pm 0.15\,. \tag{3.3}$$

Note that the error of $S(J/\psi\pi^0)$ is that of the HFAG, which is not inflated due to the inconsistency of the data.

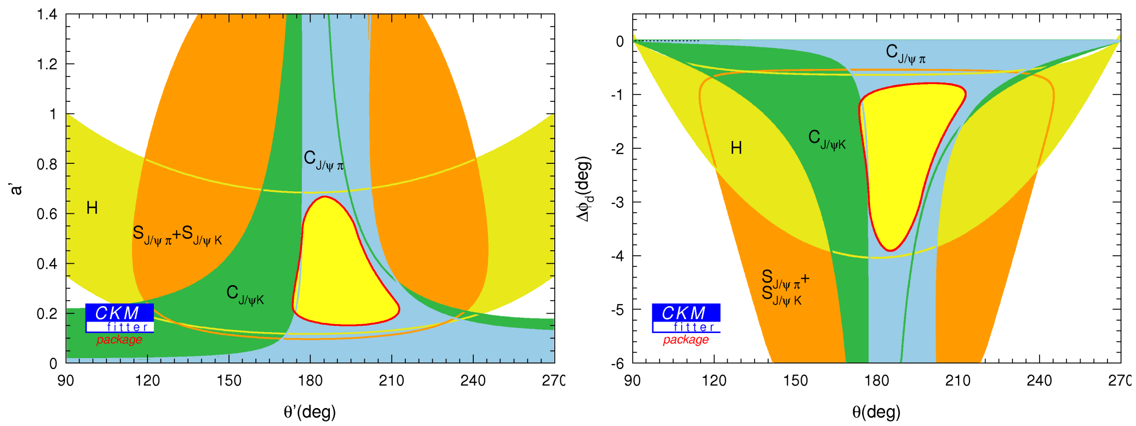

Figure 4: Left: The $1\,\sigma$ ranges in the θ'–a' plane with current data ($a' = r_\pi$). Right: $\Delta\phi_d$ for the constraints shown in the left plot.

In addition to the CP asymmetries we will also use the data on the CP averaged branching ratios. We introduce

$$H \equiv \frac{2}{\varepsilon}\left[\frac{\mathrm{BR}(B_d \to J/\psi\pi^0)}{\mathrm{BR}(B_d \to J/\psi K^0)}\right]\left|\frac{\mathscr{A}}{\mathscr{A}'}\right|^2 \frac{\Phi_{J/\psi K^0}}{\Phi_{J/\psi\pi^0}} = \frac{1 - 2r_\pi \cos\theta' \cos\gamma + r_\pi^2}{1 + 2r_K \cos\theta \cos\gamma + r_K^2} \tag{3.4}$$

where $\varepsilon = \lambda^2/(1-\lambda^2) = 0.053$ is a CKM factor and Φ_f are phases space factors. The ratio of the amplitudes $|\mathscr{A}/\mathscr{A}'|$ takes into account flavour-symmetry breaking corrections. Furthermore, have expressed H in terms of the weak phase γ, the strong phase θ' and the ratio r_π for the penguin in $B_d \to J/\psi\pi^0$ and the strong phase θ and the ratio r_K for the penguin in $B_d \to J/\psi K_s$.

As discussed above, we shall make the assumption that the hadronic matrix elements and the strong phases are roughly the same in the to decays $B_d \to J/\psi K_s$ and $B_d \to J/\psi\pi^0$. To this end we shall assume

$$\theta = \theta' \qquad r_K = \varepsilon r_\pi \tag{3.5}$$

and leave generous margin for a possible violation of these relations.

In the flavour symmetry limit, we would also have $|\mathscr{A}/\mathscr{A}'| = 1$. However, we shall include some of the breaking effects by assuming that the breaking behaves like the form factor ratio of the $B \to \pi$ and $B \to K$ form factors, the values of which are taken from a QCD sum rule calculation. Clearly this is debatable assumption, since non-factorizable contributions are known to be dominant in $B_d \to J/\psi K_s$.

Fig. 4 shows the resulting fit. In the left plot we show the constraints coming from the data on the decay $B \to J/\psi\pi^0$. From this we conclude that the preferred values, indicated by the yellow region, of $r_\pi \in [0.15, 0.67]$ and $\theta' \in [174, 213]^\circ$ at the one sigma level. Making use of relation (3.5), we end up with the right plot, which gives the shift $\Delta\phi_d$ and the value of the strong phase θ. We see that a negative value of $\Delta\phi_d$ emerges; the global fit to all observables yields $\Delta\phi_d \in [-3.9, -0.8]^\circ$, mainly due to the constraints from H and $C(J/\psi\pi^0)$.

The impact of violations of (3.5) is rather mild. Varying $r_K = \xi r_\pi$ and with $\xi \in [0.5, 1.5]$ and leaving θ and θ' completely uncorrelated in the region $\theta, \theta' \in [90, 270]^\circ$ the fit still prefers negative values of $\Delta\phi_d$; the global fit yields $\Delta\phi_d \in [-6.7, 0.0]^\circ$.

It is remarkable that the data seem to prefer sizable penguin contributions. The ratio r_f which are obtained by this fits is of the order of $r_f \sim 0.1...0.7$ which is considerably larger than what is obtained by the purely theoretical approaches discussed above.

4. Conclusion

The situation which emerges from these investigations remains inconclusive. While the theoretical approaches tend to yield small r_f ratios, the data seem to prefer larger values. Evidently, the partonic calculation will result in an r_f value of the order of $1/(16\pi^2)$, since the penguins are induced through a loop diagram.

Likewise, saturating the $\bar{u}u \to \bar{c}c$ scattering with hadronic intermediate states, the contributions of each individual state is much smaller that what is obtained from the fit to the data. However, this smallness could be compensated by a large number of possible intermediate states.

4.1 Discussion of $B_d \to J/\psi K_s$

The relatively large shift $\Delta\phi_d$ that is obtained from the fit to the data could as well originate from a "new physics" contribution. This has been investigated also in [8]. However, it is interesting to note that this shift softens the tensions observed in the standard-model fit of the unitarity triangle. In particular, the tension with the (inclusive) value of V_{ub} becomes milder once a negative shift in ϕ_d is taken into account. Furthermore, also the tensions in the other fits such as the ones considered in [2, 3] are softened.

In summary, even if the central values for CKM parameters remain as they are and the uncertainties get smaller, the situation can still be explained by the standard model; however, the penguin-over-tree ratios are larger than the current theoretical prejudices indicate.

4.2 Outlook: What to expect for $B_s \to J/\psi\phi$?

Once sizable r_f ratios are assumed, this will also have an impact on the interpretation of the data of $B_s \to J/\psi\phi$. As it has been discussed in [13], one may also use data to constrain the r_f ratio in $B_s \to J/\psi\phi$ using the control channel $B_s \to J/\psi K^*$. Since there is no data yet on this channel, nothing detailed can be said.

However, if ratios of the order $r_f \sim 0.1...0.7$ turn out to be typical numbers, it will have a sizable impact on the CP asymmetries in $B_s \to J/\psi\phi$. In the standard model, the B_s-$\bar{B}_s$ mixing phase is small, $\phi_s = -2\lambda^2\eta \sim -1.5^\circ$, and hence only a small mixing-induced CP asymmetry is expected.

However, if $r_f \sim 0.1...0.7$, the penguin contributions could even dominate the mixing induced CP asymmetry, since the shift $\Delta\phi_s$ can be larger than the value oft the mixing phase. In particular, the sign would be negative, thereby increasing the absolute value of ϕ_s. From this we would conclude that a measurement of a time-dependent CP asymmetry, which would correspond to a value of 5° for ϕ_s, could still have a standard-model explanation. However, if the central value of the current measurements of ϕ_s [12]

$$\phi_s = \left(-43^{+15}_{-22}\right)^\circ \vee \left(-136^{+22}_{-15}\right)^\circ . \qquad (4.1)$$

stabilizes unchanged, this will be a clear indication of a new effect.

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