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Standard(-like) Model from an SO(12) Grand Unified Theory in six-dimensions with S^2 extra space

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We analyze a gauge-Higgs unification model based on a gauge theory on a six-dimensional space-time which has an S^2 extra-space. We impose a symmetry condition for a gauge field and non-trivial boundary conditions on the S^2 for each fields. We briefly review the scheme for constructing a four-dimensional theory from the six-dimensional gauge theory under these conditions. We then construct a specific model based on an SO(12) gauge theory with fermions which lie in a 32 representation of SO(12), under the scheme. We find that this model leads a Standard-Model(-like) gauge theory which has gauge symmetry SU(3) $\times$ SU(2)$_L$ $\times$ U(1)$_Y$($\times$ U(1)^2) and one generation of SM fermions, in four-dimensions. The Higgs sector of the model is also analyzed, and it is shown that the electroweak symmetry breaking and the prediction of W-boson and Higgs-boson masses are obtained.

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1. introduction

The gauge-Higgs unification is one of the attractive approaches to the physics beyond the SM [1, 2, 3]. In this approach, the Higgs particles originate from the extra-dimensional components of the gauge field of a gauge theory defined on spacetime with dimensions larger than four. Thus the Higgs sector is embraced into the gauge interactions in the higher-dimensional spacetime and part of the fundamental properties of Higgs scalar is determined from the gauge interactions. We consider, in this paper, gauge-Higgs unification model on six-dimensional spacetime which has S^2 extra-space with non-trivial boundary conditions of fields on S^2.

2. Model

We consider a gauge-Higgs unification model based on a gauge theory as defined on the six-dimensional spacetime with the extra-space which has the structure of two-sphere S^2 [4]. We can impose on the fields of this gauge theory the symmetry condition which identifies the gauge transformation as the isometry transformation of S^2 as in the coset space dimensional reduction(CSDR) scheme, since the S^2 has the coset space structure such as S^2=SU(2)/U(1). We then impose on the gauge field the symmetry in order to carry out the dimensional reduction of the gauge sector.

The action of this theory is given by

$$S = \int dx^4 \sin\theta d\theta d\phi \left(\bar{\psi} i \Gamma^\mu D_\mu \psi + \bar{\psi} i \Gamma^a e_a^\alpha D_\alpha \psi - \frac{1}{4g^2} g^{MN} g^{KL} Tr[F_{MK} F_{NL}] \right), \tag{2.1}$$

where $F_{MN} = \partial_M A_N(X) - \partial_N A_M(X) - [A_M(X), A_N(X)]$ is the field strength, D_M is the covariant derivative including spin connection, and Γ_A represents the 6-dimensional Clifford algebra. We impose on the gauge field $A_M(X)$ the symmetry which connects SU(2)$_I$ isometry transformation on S^2 and the gauge transformation on the fields in order to carry out dimensional reduction, and the non-trivial boundary conditions of S^2 to restrict four-dimensional theory. The symmetry requires that the SU(2)$_I$ coordinate transformation should be compensated by a gauge transformation. The symmetry further leads to the following set of the symmetry condition on the fields [1, 5, 6]:

$$\xi_i^\beta \partial_\beta A_\mu = \partial_\alpha W_i + [W_i, A_\mu], \tag{2.2}$$
$$\xi_i^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_i^\beta A_\beta = \partial_\alpha W_i + [W_i, A_\alpha], \tag{2.3}$$

where ξ_i^α is the Killing vectors generating SU(2)$_I$ symmetry and W_i are some fields which generate an infitesimal gauge transformation of G. Here index $i = 1,2,3$ corresponds to that of SU(2) generators. The LHSs of Eq (2.2,2.3) are infintesimal isometry SU(2)$_I$ transformation and the RHSs of those are infintesimal gauge transformation. The non-trivial boundary conditions are defined so as to remain the action Eq (2.1) invariant, and are written as

$$\psi(x,\pi-\theta,-\phi) = \gamma_5 P\psi(x,\theta,\phi), \tag{2.4}$$
$$A_\mu(x,\pi-\theta,-\phi) = PA_\mu(x,\theta,\phi)P, \tag{2.5}$$
$$A_\theta(x,\pi-\theta,-\phi) = -PA_\theta(x,\theta,\phi)P, \tag{2.6}$$
$$A_\phi(x,\pi-\theta,-\phi) = -PA_\phi(x,\theta,\phi)P, \tag{2.7}$$
$$\psi(x,\theta,\phi+2\pi) = P'\psi(x,\theta,\phi), \tag{2.8}$$
$$A_\mu(x,\theta,\phi+2\pi) = P'A_\mu(x,\theta,\phi)P', \tag{2.9}$$
$$A_\theta(x,\theta,\phi+2\pi) = P'A_\theta(x,\theta,\phi)P', \tag{2.10}$$
$$A_\phi(x,\theta,\phi+2\pi) = P'A_\phi(x,\theta,\phi)P', \tag{2.11}$$

where $P(P')$s act on the representation space of gauge group G and satisfy $P^2 = 1((P')^2 = 1)$; we can take element of $P(P')$ as ± 1. The fermion sector of four-dimensional action is obtained by expanding fermions in normal modes of S^2 and then integrating S^2 coordinate in six-dimensional action. Thus, the fermions have massive KK modes which would be a candidate of dark matter. Generally, the KK modes do not have massless mode because of the positive curvature of S^2. The existence of the positive curvature is expressed as spin connection term of covariant derivative in six-dimensional Lagrangian. We, however, can show that the fermion components satisfying the following condition have massless mode:

$$-i\Phi_3\psi = \frac{\Sigma_3}{2}\psi, \tag{2.12}$$

since spin connection term in Eq. (Dphi) is canceled by this condition.

We then construct a model based on a gauge group G=SO(12) and a representation F=32 of SO(12) for fermions. Our set up is as follows.

1. We assume that U(1)$_I$ is embedded into SO(12) such as

$$SO(12) \supset SO(10) \times U(1)_I. \tag{2.13}$$

2. The parity assignment is written in 32 dimensional spinor basis of SO(12) such as

$$\begin{aligned} SO(12) &\supset SU(3)\times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_I \\ 32 = &\ (3,2)^{(+-)}(1,-1,1) + (\bar{3},2)^{(+-)}(-1,1,-1) \\ &+(3,1)^{(--)}(4,1,-1) + (\bar{3},1)^{(--)}(-4,-1,1) \\ &+(3,1)^{(-+)}(-2,-3,-1) + (\bar{3},1)^{(-+)}(2,3,1) \\ &+(1,2)^{(++)}(3,-3,-1) + (1,2)^{(++)}(-3,3,1) \\ &+(1,1)^{(--)}(6,-1,1) + (1,1)^{(--)}(-6,1,-1) \\ &+(1,1)^{(-+)}(0,-5,1) + (1,1)^{(-+)}(0,5,-1), \end{aligned} \tag{2.14}$$

where e.g. $(+,-)$ means that the parities (P,P') of the associated components are (even, odd).

3. We introduce two types of left-handed Weyl fermions that belong to 32 representation of SO(12), which have parity assignments such as $\psi^{(+P')} \to \gamma_5 P\psi^{(+P')}(P'\gamma_5\psi^{(+P')})$ and $\psi^{(-P')} \to \gamma_5 P\psi^{(-P')}(-P'\gamma_5\psi^{(-P')})$ respectively.

3. The consequences of the model

As a result of this set up, we obtain gauge symmetry breaking by symmetry condition and boundary condition as SO(12) $\supset$ SO(10) $\times$ U(1)$_I$ $\supset$ SU(5)$\times$ U(1)$_X$ $\times$ U(1)$_I$ $\supset$ SU(3) $\times$ SU(2)$_L$ $\times$ U(1)$_Y$ $\times$ U(1)$_X$ $\times$ U(1)$_I$, SM Higgs doublet (1,2)(3,2,-2) and (1,2)(-3,-2,2), and one generation of SM fermions $\{$(3,2)(1,-1,1)$_L$,(3,1)(4,1,-1)$_R$,(3,1)(-2,-3,-1)$_R$,(1,2)(-3,3,1)$_L$,(1,1)(-6,1,-1)$_R$,(1,1)(0,5,-1)$_R$ $\}$.

We also analyzed Higgs potential and obtain vaccume expectation value of Higgs doublet as

$$< \phi >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{4}{3}} \frac{1}{gR}, \tag{3.1}$$

and W boson mass m_W and Higgs mass m_H are given in terms of radius R

$$m_W = g_2 \frac{v}{2} = \sqrt{\frac{2}{3}} \frac{1}{R}, \quad m_H = \sqrt{3} g v = \sqrt{4} \frac{1}{R}. \tag{3.2}$$

The ratio between m_W and m_H is predicted

$$\frac{m_H}{m_W} = \sqrt{6}. \tag{3.3}$$

The electroweak symmetry breaking is then realized and the Higgs mass value is predicted.

4. Summary

We analyzed a gauge theory defined on the six-dimensional spacetime which has an S^2 extra-space, with the symmetry condition and non-trivial boundary conditions and constructed the model based on SO(12) gauge theory. We found that this model leads Standard Model like particle contents in four-dimensional spacetime and prediction for the Higgs sector.

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