PROCEEDINGS OF SCIENCE

$g_{B*B\pi}$ coupling in the static heavy quark limit

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By means of QCD simulations on the lattice, we compute the coupling of the heavy-light mesons to a soft pion in the static heavy quark limit. The gauge field configurations used in this calculations include the effect of $N_f = 2$ dynamical Wilson quarks, while for the static quark propagator we use its improved form (so called HYP). On the basis of our results we obtain that the coupling $\hat{g} = 0.44 \pm 0.03^{+0.07}_{-0.00}$, where the second error is flat (not gaussian).

European Physical Society Europhysics Conference on High Energy Physics
July 16-22, 2009
Krakow, Poland

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PoS(EPS-HEP 2009)060

1. Introduction

Heavy Quark Effective Theory (HQET) is an effective theory which offers in its static limit (i.e. with just the first term of the expansion in $1/m_Q$ of the QCD Lagrangian) a simplified framework to solve the non-perturbative dynamics of light degrees of freedom in the heavy-light systems. That dynamics is constrained by heavy quark symmetry (HQS): it is blind to the heavy quark flavor and its spin. As a result the total angular momentum of the light degrees of freedom becomes a good quantum number (j_ℓ^P) and therefore the physical heavy-light mesons come in mass-degenerate doublets.

In phenomenological applications the most interesting information involves the lowest lying doublet, the one with $j_\ell^P = (1/2)^-$, consisting of a pseudoscalar and a vector meson, such as (B_q, B_q^*) or (D_q, D_q^*) states, where $q \in \{u,d,s\}$. When studying any phenomenologically interesting quantity from the QCD simulations on the lattice that includes heavy-light mesons (decay constants, various form factors, bag parameters and so on), one of the major sources of systematic uncertainty is related to the necessity to make chiral extrapolations. Indeed simulations at the physical point are out of reach, despite a lot of recent improvements [1]: the lightest masses used in fully controlled simulations are in the range $m_q \sim 5-10\,m_{u/d}$. Since the QCD dynamics with very light quarks is bound to be strongly affected by the effects of spontaneous chiral symmetry breaking, a more suitable (theoretically more controllable) way to guide such extrapolations is by using the expressions derived in heavy meson chiral perturbation theory (HMChPT), which is an effective theory built on the combination of HQS and the spontaneous chiral symmetry breaking $[SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V]$. Its Lagrangian is given by [2]

$$\mathscr{L}_{\rm heavy} = -{\rm tr}_a \mathscr{T}\nabla[\overline{H}_a i v\cdot D_{ba} H_b] + \hat{g}\ {\rm tr}_a \mathscr{T}\nabla[\overline{H}_a H_b \gamma_\mu \mathbf{A}^\mu_{ba}\gamma_5]\,, \tag{1.1}$$

$$D^\mu_{ba}H_b = \partial^\mu H_a - H_b \frac{1}{2}[\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger]_{ba}\,, \quad \mathbf{A}^{ab}_\mu = \frac{i}{2}[\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger]_{ab}\,, \tag{1.2}$$

where

$$H_a(v) = \frac{1+\not{v}}{2}\left[P^{*\,a}_\mu(v)\gamma_\mu - P^a(v)\gamma_5\right]\,, \tag{1.3}$$

is the heavy meson doublet field containing the pseudoscalar, $P^a(v)$, and the vector meson field, $P^{*\,a}(v)$. In the above formulae, the indices a,b run over the light quark flavors, $\xi = \exp(i\Phi/f)$, with Φ being the matrix of (N_f^2-1) pseudo-Goldstone bosons, and "f" is the pion decay constant in the chiral limit. We see that the term connecting the Goldstone boson ($\mathbf{A}_\mu$) with the heavy-meson doublet $[H(v)]$ is proportional to the coupling $\hat{g}$, which will therefore enter into every expression related to physics of heavy-light mesons with $j_\ell^P = (1/2)^-$ when the chiral loop corrections are included. In the rest of the proceedings we will summarise what has have been reported in [3] concerning the computation of $\hat{g}$.

2. Extraction of $\hat{g}$ by numerical simulations

Since the charm quark is not very heavy, the use of the experimentally known [4] value of $g_{D^*D\pi} \equiv \frac{2\hat{g}_c\sqrt{m_D m_{D^*}}}{f_\pi}$ to fix the value of $\hat{g}$-coupling and its use in chiral extrapolations of the quantities

Figure 1: Sketch of the 2-pts and 3-pts static light correlation functions. Single lines refer to light quark propagators, double line to static quark propagators while grey ovals refer to smeared interpolating fields.

relevant to B-physics phenomenology may be dangerous mainly because of the potentially large $\mathscr{O}(1/m_c^n)$-corrections. Unfortunately the decay $B^* \to B\pi$ is kinematically forbidden and therefore, to determine the size of $\hat{g}$, we have to resort to a non-perturbative approach to QCD. Unlike for the computation of the heavy-to-light form factors, QCD sum rules proved to be inadequate when computing $g_{D^*D\pi}$, most likely because of the use of double dispersion relations when the radial excitations should be explicitly included in the analysis, as claimed in [5]. Therefore we have estimated $\hat{g}$ from lattice simulations with $N_f = 2$ flavours of dynamical quarks. From the definition of the coupling $g_{H^*H\pi}$

$$\langle H(p)\pi(q)|H^*(p',\varepsilon_\lambda)\rangle = g_{H^*H\pi}\, q\cdot\varepsilon_\lambda, \quad q = p'-p, \tag{2.1}$$

and the expression of the matrix element $\langle H|A_\mu|H^*\rangle$, with $A^\mu = \bar{q}\gamma^\mu\gamma^5 q$, in terms of the form factors $A_i, i = 1,2,3$

$$\begin{aligned}\langle H(p)|A^\mu|H^*(p',\varepsilon_\lambda)\rangle \;=\; & 2m_V A_0(q^2)\frac{q\cdot\varepsilon_\lambda}{q^2}q^\mu + (m_H+m_H^*)A_1(q^2)\left(\varepsilon_\lambda^\mu - \frac{q\cdot\varepsilon_\lambda}{q^2}q^\mu\right)\\ &+ A_2(q^2)\frac{q\cdot\varepsilon_\lambda}{m_H+m_H^*}\left(p^\mu + p'^\mu - \frac{m_{H^*}^2 - M_H^2}{q^2}q^\mu\right),\end{aligned} \tag{2.2}$$

one can write that in the soft pion limit

$$\langle H(p)|q_\mu A^\mu|H^*(p',\varepsilon_\lambda)\rangle = g_{H^*H\pi}\frac{q\cdot\varepsilon^\lambda}{m_\pi^2 - q^2} f_\pi m_\pi^2 + \cdots \tag{2.3}$$

Finally, with $\vec{q} = \vec{p} = \vec{p}\,' = 0$, $\langle H|A^i|H^*(\varepsilon_\lambda)\rangle = (m_H^* + m_H)A_1(0)\varepsilon_\lambda^i$. This kinematical situation is physically meaningful in static limit of HQET (H and H^* are degenerate in mass) and we conclude that $\hat{g}$ is given by $A_1(0)$.

3 lattice spacings have been considered with several sea quark masses to make chiral extrapolation. We have performed our computation on publicaly available ensembles [6] -[8] whose main characteristics are that the lattice spacing is smaller than 0.1 fm, $m_q \in [m_s/4, 1.5m_s]$ and the volume is between 1.5 and 2.5 fm. We have computed 2-pts and 3-pts correlation functions $C^{(2)}(t_x)$ and $C^{(3)}(t_x,t_y)$, schematically drawn in Figure 1, from which one extracts the effective energy $\mathscr{E}_q = \ln\left(\frac{C^{(2)}(t+1)}{C^{(2)}(t)}\right)\Big|_{t\gg 0}$, the coupling $\mathscr{Z} = \langle 0|P|H\rangle$ and $\hat{g} = \frac{C^{(3)}(t_y,t_x)}{\mathscr{Z}^2 e^{-\mathscr{E}_q t_x}}\Big|_{t_y\gg 0, t_x - t_y \gg 0} \equiv R(t_x,t_y)|_{t_y\gg 0, t_x-t_y\gg 0}$. In order to suppress more strongly the coupling with radial excitations with have used smeared interpolating fields $P(V_i)(\vec{x},t) = \sum_{\vec{y}} \overline{h}(\vec{x},t)\Phi(\vec{x}-\vec{y})\gamma^{5(i)}q(\vec{y},t)$.

All our fits to extract $\hat{g}$ from R are made on the common interval, $5 \le t_y \le 8$. The final ingredient necessary to relate the results of our calculation to the continuum limit is the appropriate axial current renormalization. We prefer to apply the same procedure to all our data sets and determine non-perturbatively the axial renormalization constant from hadronic Ward identities [9].

The last step to reach the coupling $\hat{g}$, which is our final goal, is to make the extrapolation to the chiral limit. To that end we attempt either a simple linear fit or a fit guided by the expression derived in HMChPT [10], i.e.,

$$\hat{g}^q = \hat{g}_{\rm lin}\left(1+c_{\rm lin}m_\pi^2\right), \quad \hat{g}^q = \hat{g}_0\left[1-\frac{4\hat{g}_0^2}{(4\pi f)^2}m_\pi^2\log(m_\pi^2)+c_0 m_\pi^2\right], \tag{2.4}$$

where $\hat{g}_0$ is then the soft pion coupling that is to be used in applying the HMChPT formulae when extrapolating the phenomenologically interesting quantities computed on the lattice to the physical light quark mass limit. From Figure 2 it is obvious that this task is quite difficult if one is doing it separately for each β. Thus, performing a global fit of all our data, without introducing $\mathcal{O}(a^2)$ terms in the formula because of rather large statistical uncertainties, we obtain

$$\hat{g}_{\rm lin} = 0.51\pm 0.04, \quad c_{\rm lin} = (0.21\pm 0.12)\ {\rm GeV}^{-1}, \tag{2.5}$$

while with the HMChPT formula we have

$$\hat{g}_0 = 0.44\pm 0.03, \quad c_0 = (0.40\pm 0.12)\ {\rm GeV}^{-1}. \tag{2.6}$$

Another possibility is to exclude the data with $m_\pi^2 \geq 0.6\ {\rm GeV}^2$, which gives $\hat{g}_0 = 0.46\pm 0.04$. We also checked that our resulting $\hat{g}_0$ is insensitive to the variation of $f\in(120,132)$ MeV, latter being $f_\pi^{\rm phys.}$. Our result is in good agreement with a first unquenched calculation performed on coarser lattices ($a > 0.15$ fm) [11] where all to all propagator techniques have been used in order to reduce statistical fluctuations. It is also in very good agreement with $\hat{g}$ extracted from a quite different approach based on the measurement of B meson axial charge distribution [12].

3. Conclusions

We have reported on the results of our calculations of the soft pion coupling to the lowest lying doublet of static heavy-light mesons. From our computations, in which we use the fully

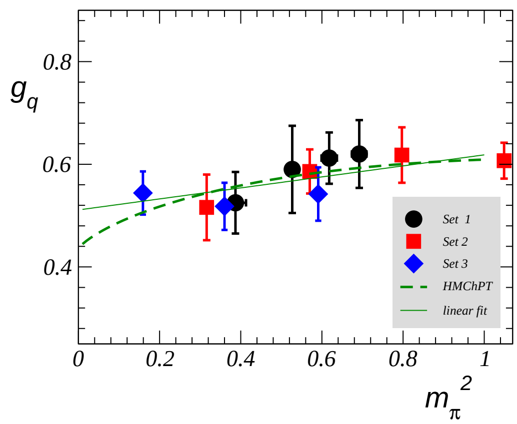

Figure 2: *$\hat{g}_q$ computed from the ratio R for all of our lattice data sets, after accounting for the axial current renormalization constants computed on the same ensembles of gauge field configurations. They are plotted as a function of the light pseudoscalar meson ("pion") mass squared (in* ${\rm GeV}^2$*).*

unquenched set-up and three different sets of gauge field configurations, all produced with Wilson gauge and fermion actions, we obtain that $\hat{g}_0 = 0.44 \pm 0.03^{+0.07}_{-0.00}$. The second error reflects the uncertainty due to chiral extrapolation and it is the difference between the results of linear fit and the fit in which HMChPT is used. On the more qualitative level, our results show/confirm that this coupling is smaller in the static limit than what is obtained when the heavy quark is propagating and is of the mass equal to that of the physical charm quark, $\hat{g}_{\rm charm} = 0.68 \pm 0.07$ [13]. It is intriguing that the $\mathscr{O}(1/m_c^n)$ corrections are quite large for the quantity in which the heavy quark contributes only as a spectator. An obvious perspective concerning the determination of $\hat{g}_0$ is to further reduce the errors, both statistical (by using the “all-to-all" propagator technique, like in ref. [11]), and the systematic ones, in particular those associated with chiral extrapolations and the contribution of excited states to 3pts Green functions: for the latter one might solve a Generalised Eigenvalue Problem, as recently discussed for f_B in [14].

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