PROCEEDINGS OF SCIENCE

Precise Standard Model tests

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I summarize the precision tests of the Standard Model in kaon physics which have been reported at this conference.

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1. Introduction

Particle physics experiments of the last three decades can always be viewed as either tests of the standard model (SM) or as searches for new physics (NP). The distinction between the two points of view is somewhat artificial, because testing a theory only makes sense if one admits the possibility that it may be wrong, and so that new phenomena may be discovered. On the other hand the search for new phenomena only makes sense if one can clearly define what is known and to what level of precision, which means, that one has tested the "old theory". Having made clear that this separation is somewhat artificial, I can also express my gratitude to the organizers for having decided to make this splitting, which made the impossible task of summarizing a conference half that difficult. I am sure that this gratitude is also shared by my colleague Yasuhiro Okada who had to summarize the part of the conference related to the search for NP [1].

Before discussing what precision SM tests have been reported at this conference, it is useful to define an ordering principle, which allows us to navigate among the many contributions and topics touched without losing the overview of this field. I will split the SM Lagrangian in three pieces which, at the energy scale of the kaon, can be described as the dominant, the small and the tiny one:

$$\begin{aligned} \mathscr{L}_{\rm SM} &= \mathscr{L}_{\rm QCD}^{(u=d)} + \mathscr{L}_{\not{iso}} + \mathscr{L}_{weak} \\ \mathscr{L}_{\not{iso}} &= \mathscr{L}_{\rm QED} + \mathscr{L}_{m_u-m_d} \end{aligned} \tag{1.1}$$

The dominant term is the QCD Lagrangian in the isospin limit, the small one is the isospin breaking part, which can itself be split into the QED Lagrangian and the isospin-breaking QCD mass term , and the tiny piece is due to weak interactions, which at this energy is best represented by a series of nonrenormalizable terms generated by the exchange of W and Z bosons.

2. Strong interactions

At energies of the order of the kaon mass and below, the SM Lagrangian is dominated by the QCD term. This part of the Lagrangian has only two free parameters (in the isospin limit only the masses of the light quarks, $m_u = m_d = \hat{m}$ and m_s – the heavy quark masses, which are also present, are fixed at their value). Once these two free parameters are fixed, for example to correctly reproduce the lowest part of the spectrum (the pion and kaon mass), QCD becomes a parameter-free theory and we must face the challenge to describe with it a very rich phenomenology. The non-perturbative nature of the problem makes this a highly nontrivial task.

At present there are three main approaches to tackle nonperturbative QCD phenomena: the lattice, the effective field theory and the dispersion-relation method. Lattice QCD is a fully first-principle approach; in the effective field theory one efficiently derives the consequences of symmetry in a quantum field theory framework (and so automatically respecting analyticity and unitarity, albeit perturbatively); in the dispersion theory approach one exactly implements analyticity and unitarity. The three methods are complementary and indeed they are sometimes used in combination in order to obtain a prediction which can be compared to experiments.

In the last few years lattice calculations with dynamical fermions and with realistically light quark masses have become available, so that a direct comparison with the phenomenology is now

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becoming possible. In some tests of the SM this is essential, as lattice calculations provide unique information (like in the case of $f_+(0)$ necessary for the extraction of V_{us}, see below).

For χPT and dispersion relations a direct comparison to the phenomenology is possible since much longer and what we have seen at this conference is the remarkable level of precision which has been reached.

2.1 Lattice results

There have been several talks about lattice results at this conference, which reflects the fact that nowadays lattice QCD provides invaluable information on hadronic matrix elements, necessary for the SM analysis of kaon phenomenology. Probably the best example today is the determination of V_{us}. Experiments on $K_{\ell 3}$ decays measure the product $f_+(0)V_{us}$, whereas those on $K_{\ell 2}$ and $\pi_{\ell 2}$ decays provide the ratio $(V_{us}F_K)/(V_{ud}F_\pi)$. The extraction of V_{us} or of the ratio V_{us}/V_{ud} requires input on the hadronic matrix elements $f_+(0)$ and F_K/F_π. Symmetry arguments (the Ademollo-Gatto theorem [2]) imply that the deviation of $f_+(0)$ from 1 is quadratic in the SU(3)-breaking – moreover the effective Lagrangian method allows one to express the SU(3)-breaking correction of order p^4 in terms of masses and decay constants only [3] and give an unambiguos numerical prediction. Unfortunately this is not enough at the level of precision reached today if one wants to test the unitarity of the CKM matrix, and in fact it was realized early on [4] that an estimate of the $O(p^6)$ correction was essential in order to have a precise extraction of V_{us}. Improving this estimate is difficult. Calculations of the chiral expansion of the form factor up to order p^6 allow one to identify unambiguously the unitarity contribution, but the final result depends on unknown $O(p^6)$ LECs. Estimating these with resonance saturation or other methods does not seem to lead to the necessary precision. The ratio F_K/F_π depends on LECs already at order p^4. A purely chiral prediction for these is impossible, because the relevant LEC combination is not known from other sources. Lattice calculations for both $f_+(0)$ and F_K/F_π are nowadays possible and have already reached the necessary precision to make a sensitive test of the unitarity of the CKM matrix. The current status of these has been reviewed in the talks of Boyle [5] and Mescia [6]. Moreover, if one uses the independent determination of V_{ud} from superallowed Fermi decay of nuclei together with F_K/F_π to extract V_{us} from the ratio $\Gamma(K \to \ell\nu)/\Gamma(\pi \to \ell\nu)$ one obtains two independent and compatible determinations of V_{us} [7, 8]. This nontrivial result confirms that the systematic uncertainties of lattice calculations are under good control.

Several lattice groups have studied pion physics in detail: the pion mass and decay constants, the scalar and vector form factors and the $I = 2$ scattering length have been calculated by different groups with different actions and at different values of the light quark masses. The description of the quark mass dependence of these quantities with χPT is quite satisfactory and has led to a determination of several low energy constants of the χPT Lagrangian both at order p^2 and p^4. Moreover results obtained with different actions and different computational techniques and setups show a good level of agreement. In view of this it appears to be useful to try and summarize lattice results in a unified way and where possible to offer averages or lattice-based estimates which any theorist or experimentalists could then directly use in his/her analysis of the phenomenology. Such an initiative is being carried forward by FLAG, a working group of the FLAVIAnet European network (the acronym stands for FLAVIAnet Lattice Averaging Group), for the moment concentrating on low-energy particle physics (*i.e.* mainly pions and kaons) – at this conference I have presented the status of this project [7]. The first paper of FLAG will appear soon.

At this conference there has been one talk dedicated to pion physics on the lattice, the one by J.-I. Noaki [9] on behalf of the JLQCD/TWQCD collaboration. This collaboration performs calculations with two (and more recently even with $2+1$) dynamical quarks with the overlap formulation. This formulation respects chiral symmetry exactly even at finite lattice spacing and is therefore particularly well suited to approach the chiral limit. These advantages come at a very high computer cost, unfortunately, which makes it even more remarkable that such calculations can be performed and produce competitive results. The results obtained by this collaboration show that χPT at NLO describes well the pion mass dependence only for pion masses below the kaon mass (similar conclusions have also been reached by other groups). To reach the kaon-mass region it is necessary to include NNLO terms. One must add, however, that even if one can describe data around M_K and beyond with NNLO χPT , these data can barely improve the precision of the LEC determination – one should rather aim at having more and more precise data at lower pion masses and determine the LECs only in the region where the chiral series converges well.

A different picture comes out of the lattice data on kaons, even if one considers only very simple quantities like masses and decay constants. Different collaborations have reported a failure of NLO χPT in describing the quark-mass dependence of the lattice data for m_s close to the physical value, but for $\hat{m}$ such that $M_\pi \sim 400$ MeV (cf. [5]). This is not that surprising, after all, in view of the fact that in this case the kaon mass is around 600 MeV and that the physical value of the kaon mass is already at the border of where one can hope to apply the chiral expansion successfully. From the practical point of view this difficulty has been overcome in two ways: either by doing an SU(2) chiral analysis of data on kaons (*i.e.* by considering the strange quark as heavy and expanding only around $\hat{m}=0$, an approach pioneered by Roessl [10]), or by doing polynomial fits of the m_s dependence of lattice data (*i.e.* based on a Taylor expansion around $m_s = m_s^{\rm phys}$). Since it is no problem for lattice calculations to work at $m_s = m_s^{\rm phys}$, it is clear that the chiral expansion around $m_s = 0$ is not needed to reach the physical point. On the other hand, one should not overlook the possibility to learn something interesting about QCD and investigate the dependence on m_s *per se*. This does require to invest considerable efforts in doing expensive simulations at $m_s \leq m_s^{\rm phys}$, but by doing this one may reliably determine the SU(3) LECs and shed light on the analysis of the phenomenology based on SU(3) χPT , which is nowadays done at NNLO.

This issue is even more urgent for the four-quark $K-\pi$ matrix elements discussed by Norman Christ in his talk [11], because for those quantities the failure of χPT is quite dramatic. Of course, the main motivation for carrying out such difficult calculations is to get the $K \to \pi\pi$ matrix elements which are relevant in the CP-violating decays and play an important role in the calculation of ε'/ε, and since it is possible to calculate these directly, this is the route that lattice collaborations interested in these problems will take (as announced in [11]). Nonetheless, I do not doubt that some time in the future lattice calculations of $K-\pi$ matrix elements with three light dynamical flavour will become *easy* – it will then be interesting to understand why the chiral expansion seems to fail here and at what values of the strange quark mass it breaks down.

The reader interested in learning more about the current status of lattice calculations relevant for kaon decays is referred to the contributions by Boyle [5], Christ [11], Mescia [6] and Noaki [9].

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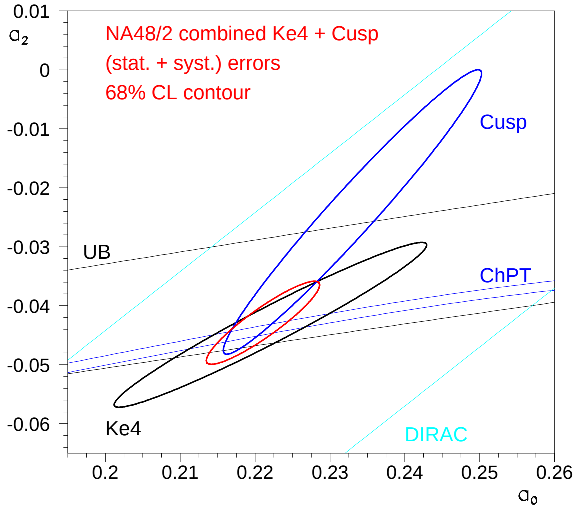

Figure 1: Ellipses representing the experimental measurements of a_0^0 and a_0^2 by NA48/2 and band for the DIRAC measurement. The small (red) one is obtained by combining the cusp and the K_{e4} measurement. Figure courtesy of Brigitte Bloch-Devaux.

2.2 $\pi\pi$ scattering lengths

The $\pi\pi$ *S*-wave scattering lengths represent one of the best examples of the power of the effective Lagrangian method, especially when combined with dispersion relations. The calculation of the *S*-wave scattering lengths based on this approach [12] yields a precision at the few-percent level (the superscript indicates the isospin, the subscript the angular momentum):

$$a_0^0 = 0.220 \pm 0.005 \qquad a_0^2 = -0.0444 \pm 0.0010 \ . \tag{2.1}$$

The first experimental measurements of the scattering lengths date back to the seventies, when a_0^0 was extracted from a measurement of $3 \cdot 10^4$ K_{e4} decays [13], but only in the last few years a precision close to the theoretical one has been reached. This not only thanks to a dramatic increase in the event statistics, but also thanks to the use of different methods: the two scattering lengths can now be measured in K_{e4} decays (E865 [14] and NA48/2 [15]), from the lifetime of pionium (DIRAC [16]) and from the cusp effect in $K^\pm \to \pi^\pm \pi^0 \pi^0$ (NA48/2 [17, 18]). At this conference preliminary results of analyses of the full data sample of K_{e4} decays of NA48/2 and of the cusp in $K \to 3\pi$ decays have been presented by Brigitte Bloch-Devaux [19] and Dmitry Madigozhin [20]. The most interesting outcome of these preliminary analyses is the combined result of the two measurements, as illustrated in Fig. 1, which gives [19]

$$a_0^0 = 0.2210 \pm 0.0047_{\text{stat}} \pm 0.0015_{\text{syst}} \qquad a_0^2 = -0.0429 \pm 0.0044_{\text{stat}} \pm 0.0016_{\text{syst}} \tag{2.2}$$

with an uncertainty in a_0^0 very close to the theoretical one and a remarkably good agreement.

A first attempt to measure $a_0^0 - a_0^2$ in a different channel has been discussed, on behalf of the KTeV collaboration, by Ed Blucher [21], who presented evidence for a cusp effect in $K_L \to 3\pi^0$

[22]. Making a two-parameter fit to the Dalitz-plot distribution they obtain $a_0^0 - a_0^2 = 0.215 \pm 0.031$, which is about 1.5σ away from the value obtained by the NA48/2 experiment. The discrepancy is not particularly worrying in view of the strong correlation with the Dalitz-plot parameter which is unknown otherwise. Indeed if they fix $a_0^0 - a_0^2$ at the measured NA48/2 value they can also obtain a good fit (albeit slightly less so), but get a very different value for the Dalitz-plot parameter. Moreover, the analysis is based on the Cabibbo-Isidori approach [23, 24] only and does not take into account the rediative corrections calculated in [25]. Preliminary results by the NA48/2 collaboration on the same measurement also point in the direction of some tension with their own cusp measurement in the K^+ decay [26] – this issue clearly calls for further investigations.

The experimental uncertainty in a_0^2, see (2.2), is not yet comparable to the theoretical one, but the result is nonetheless most remarkable as it represents a *world première*. Let me emphasize that pions in the final state of K_{e4} decays are never in an $I = 2$ state, and that a_0^2 plays a role here only because one describes the $\pi\pi$ phase shifts with solutions of the Roy equations (since these only embody analyticity, unitarity and crossing symmetry, no theory bias is introduced in this way). In this framework a_0^2 (more precisely the combination $2a_0^0 - 5a_0^2$) enters as a subtraction constant also in the $I = 0$ S-wave. The cusp in $K \to 3\pi$ decays, on the other hand, is mainly sensitive to the difference $a_0^0 - a_0^2$. Neither of the two experiments provides a reasonably precise measurement of a_0^2 alone, but since they are sensitive to two slightly different combinations of a_0^0 and a_0^2, the combined analysis yields a rather precise measurement of both scattering lengths, as illustrated in Fig. 1. This first measurement of a_0^2 is very interesting also because this quantity can be calculated on the lattice. At the moment there are two independent calculations which have been made with dynamical quarks and small enough quark masses to allow for a reliable extrapolation to the physical point (*i.e.* $M_\pi < 300$ MeV and the use of the χPT NLO formula). The results are [27, 28]

$$a_0^2(\text{latt.}) = \begin{cases} -0.04330(42) & \text{NPLQCD}, \ (N_f = 2+1) \\ -0.04385(28)(38) & \text{ETM}, \ (N_f = 2) \end{cases} \tag{2.3}$$

and nicely agree both with the chiral calculation (2.1) and the experimental measurement (2.2).

The NPLQCD collaboration has also calculated scattering lengths for other hadronic processes like, *e.g.* πK scattering [29]. The amplitude has been calculated at two loops [30]) in SU(3) χPT , although one may doubt whether this theory is still in its applicability domain at such high values of center of mass energy even. One can, however, consider the kaon as heavy and apply SU(2) χPT [10], just like one calculates πN scattering. In particular one may rely on a soft-pion theorem which states that the current algebra prediction for the isospin-odd S-wave scattering length is subject only to $O(M_\pi^2)$ corrections. The analysis in [31], however, has shown that with the values of the LECs estimated in [30], these corrections are of about 10% at $O(p^4)$ and an additional 10% at $O(p^6)$, despite their algebraic suppression, which is surprising. Alternatively, one can treat πK scattering with dispersion relations – the Roy-Steiner equations – [32] and translate high-energy data into constraints on the scattering lengths. This analysis also indicates large corrections to the soft-pion theorem. The lattice calculation [29], on the other hand is compatible with the current algebra prediction. This short summary clearly indicates that there is something to be better understood here. Direct measurements of this scattering process near threshold are unfortunately not available – the same methodology used by DIRAC to measure the $\pi\pi$ scattering lengths from

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the pionium lifetime could however also be used to measure the πK scattering lengths, and indeed at this conference Ewa Rondio [33] has discussed this exciting possibility as one of the future projects at CERN. Just before the conference the first evidence of the formation of such atoms has been announced by the DIRAC collaboration [34].

3. Isospin breaking

Isospin breaking effects, whether they are induced by electromagnetic interactions or by the up and down quark mass difference, are mostly a nuisance for theorists, who whenever possible try to ignore them – unless the precision reached forces them not to do so. After having discussed in detail the high level of precision of the scattering length measurement and of the theoretical calculations, it is appropriate to comment on the role of isospin breaking effects in those measurements. Indeed both the lattice and the χPT calculations are for the $\pi\pi$ scattering length in QCD in the isospin limit ($m_u = m_d$ tuned such that $M_\pi = 0.1396$ GeV). While such a quantity cannot be directly measured, one can correct the experimental data for isospin breaking effects and obtain the quantities in the isospin limit. This has in fact been done for all three measurements of the scattering lengths discussed above: for pionic atoms the experimental analysis is based on [35], for the cusp in $K \to 3\pi$ on [23, 24, 36, 37] (in fact also the calculation for the $K_L \to 3\pi^0$ decay is available, see [25]), and for K_{e4} decays on [38]. In particular for the K_{e4} analysis the isospin breaking corrections make a spectacular effect. For the decay of pionium and for the cusp effect, on the other hand, they are not corrections, but are responsible for the physical phenomenon – without them these measurements would not be at all possible.

At the level of precision reached by lattice calculations, isospin breaking effects start to play an important role. The QCD spectrum is one of the fundamental tests for the lattice approach and if the precision reaches the percent level, then one has to account for the fact that neutral and charged members of an isospin multiplet receive different QED radiative corrections to their mass. In addition, strong isospin-breaking contributions must also be taken into account. Most lattice calculations so far have been performed in the isospin limit, but the issue of isospin breaking is well known in the community and first efforts in calculating these effects have been performed already more than ten years ago [39], at the time still in the quenched approximation. More recently a method has been proposed to evaluate electromagnetic effects in dynamical simulations in a cost effective way [40]. In [41] a lattice calculation of the electromagnetic contributions to meson masses with two dynamical Domain Wall Fermions has been performed – in this calculation the coupling of the photon field to the sea quarks has been neglected. At this conference Taku Izubuchi has presented an update of this calculation [42] which now includes also strange quarks in the sea. For example the electromagnetic contribution to the kaon mass difference has been calculated to be ($M_{K^+} - M_{K^0}$ is split into a part due to the quark mass difference and the part due to e^2 effects):

$$[M_{K^+} - M_{K^0}]^{(e^2)} = \begin{cases} 1.443(55)\text{MeV} & [41] \\ 1.20(10)\text{MeV} & [42] \end{cases} \tag{3.1}$$

These results indicate rather small violations of Dashen's theorem (and the latter is even consistent with no violation at all), which is in contrast to what is typically obtained in analytical calculations performed in the framework of models [43, 44, 45, 46]. While it is probably too early to draw definitive conclusions on this issue, it is clear that a systematic improvement of the lattice calculations is possible, whereas on the analytical side this is rather unlikely. For other lattice calculations of these effects, see the literature cited in [42].

Isospin breaking effects play also an essential role in the study of weak decays of kaons, which is the subject of the next section. Whether in the determination of V_{us} and in the corresponding tests of unitarity of the CKM matrix, or in tests of lepton universality, or in analyses of rare decays, radiative corrections and strong isospin breaking effects have to be taken into account at the level of precision of today's experiments. The status of the theoretical calculations of these effects is summarized in the contributions of Emilie Passemar [47] and Christopher Smith [48].

4. Weak interactions

Weak interactions are the last tiny bit of the Lagrangian of the Standard Model at low energy and play a role in the analysis of the phenomenology only if one waits long enough to observe the decay of otherwise stable hadrons (like the pions, the kaons and the neutron). They break various symmetries which are otherwise conserved in QCD, like the P, the CP and the flavour symmetry, and make kaon physics especially interesting as they allow us to glimpse at the physics of an energy scale several orders of magnitude higher than the kaon mass. At this energy the weak interactions are represented by a series of nonrenormalizable terms in the Lagrangian, as illustrated in (1.1) – from a low-energy perspective the weak interactions are treated exactly like new physics, since at the kaon mass it does not make much of a difference whether the heavy particles exchanged in loops are the W and Z bosons, or the top quark, or their supersymmetric partners or a Z' and W' bosons which are maybe a factor two or three heavier. A technically important difference is of course that for the SM weak interactions one can explicitly integrate out the heavy degrees of freedom and derive the exact form of the nonrenormalizable operators which are so generated. Doing this one obtains the explicit form of the corresponding Wilson coefficients expressed in terms of CKM matrix elements, masses of the heavy degrees of freedom and of the gauge couplings. These steps have been illustrated with a historical perspective in the opening talk by Professor Lim [49], emphasizing in particular the question of whether the heavy top quark would decouple or not – the nondecoupling effects are very intimately related to the structure of the weak interactions in the SM.

In the long step from the electroweak scale down to the kaon mass the strong interactions play again a crucial role as they are responsible for the running of the nonrenormalizable operators. This is an effect which can be dealt with in perturbation theory, but because of the size of the strong coupling constant and the large ratio of the scales involved, to reach the necessary precision one has to go beyond the leading log approximation. Indeed the level of precision reached nowadays in evaluating QCD corrections, known at NNLO, and even electroweak corrections (at NLO) to the nonrenormalizable part of the SM Lagrangian at low energy is quite remarkable, as illustrated in the contribution by Martin Gorbahn [50] (for a very detailed introduction to the subject and a comprehensive review, see [51]).

4.1 Semileptonic decays, photon and Z penguin contributions

Among the various nonrenormalizable interactions those which involve fermions are partic-

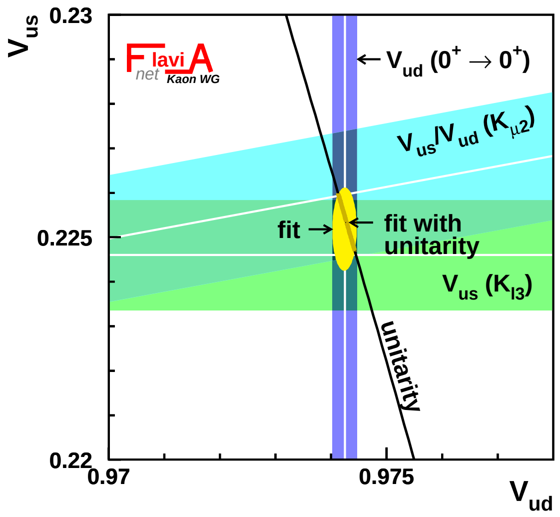

Figure 2: Summary of the analysis of V_{us} and V_{ud} as performed by the Flavianet Kaon Working group [8]. Figure courtesy of Matteo Palutan.

ularly easy to deal with: the quark currents do not renormalize and there is no running between the electroweak and the low-energy scale to worry about. Moreover the matrix elements of quark bilinears among kaon and pion states can be calculated reliably on the lattice (at present with no more than two hadrons in the external states) or in χPT . The calculation of the matrix elements $f_+(0)$ and the ratio F_K/F_π have been discussed in Sect. 2.1. These allow a precise determination of the CKM matrix elements V_{us} and of the ratio V_{us}/V_{ud}, as discussed in detail by Palutan [8] and illustrated in Figure 2: as seen there, the CKM passes perfectly the unitarity test, and with it an essential building block of the SM is experimentally confirmed.

Even safer against hadronic uncertainties are ratios of decay rates in which the hadronic part cancels out completely, like in

$$R_K \equiv \frac{\Gamma(K \to e\nu(\gamma))}{\Gamma(K \to \mu\nu(\gamma))} \quad . \tag{4.1}$$

In this ratio one tests the coupling of the leptons to the W's and the universality assumption (on which the SM is based). In the SM this ratio is tiny, of the order of 10^{-5} as it is proportional to the square of the ratio of the lepton masses. A precise SM prediction that can be tested against experiments requires an analysis of radiative corrections and in particular of those due to real photon emission. This has been provided recently in [52] and reads

$$R_K^{SM} = (2.477 \pm 0.001) \cdot 10^{-5} \tag{4.2}$$

A precise experimental measurement has been recently performed and presented at this conference both by KLOE [53] and NA62 [54], and their results read

$$R_K = \begin{cases} (2.500 \pm 0.016) \cdot 10^{-5} & \text{NA62 – prelim.} \\ (2.493 \pm 0.025_{\text{stat}} \pm 0.019_{\text{syst}}) \cdot 10^{-5} & \text{KLOE} \end{cases} \tag{4.3}$$

thus beautifully confirming the SM.

In the broad class of semileptonic decays belong also those due to flavour changing neutral currents originated by Z and photon penguins. Among these there are the “golden modes” $K^{\pm} \to \pi^{\pm} \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$, for which the standard model prediction is dominated by the uncertainty in the relevant CKM matrix elements. For these modes both the long-distance contributions, including isospin-breaking ones [48], as well as the short-distance contributions [50] have been evaluated to such an accuracy that the total theory error amounts to about 4%. There are several other modes where the electroweak penguins contribute, but none is as clean as the $K \to \pi \nu \bar{\nu}$ channel, because of much larger long-distance contributions. For a comprehensive analysis of the different modes and a detailed discussion of how essential it is to measure all these modes in order to fully exploit their potential in testing the SM, see [48].

4.2 Nonleptonic decays

For nonleptonic kaon decays the situation is unfortunately less satisfactory. NA48 and KTeV have measured these accurately and in particular both the direct and indirect *CP*-violating contributions in the decay $K \to \pi\pi$. These measurements are not new, but an update of the analysis by the KTeV experiment and their final number for ε'/ε has been presented for the first time at this conference [55]. The summary of the NA48 analysis can be found in [56]. These remarkable experimental results do not provide a significant test of the SM, unfortunately. On the theory side the calculation of the effective weak Hamiltonian and the running of the Wilson coefficients have been performed to NLO accuracy [57, 58]. The calculation of the hadronic matrix elements, on the other hand, cannot yet be performed reliably, as discussed in detail by Norman Christ [11], because the strategy to calculate the $K \to \pi$ and $K \to$ vacuum matrix elements and use χPT to obtain from these the $K \to \pi\pi$ matrix elements has shown to lead to results with uncontrolled uncertainties. This does not mean that they have given up the calculation, but rather that they will attack it from a different side: it has been announced that the RBC/UKQCD collaboration has started a new major project with the goal of calculating directly the $K \to \pi\pi$ matrix elements, following the method proposed by Lellouch and Lüscher [59] — a 10-20% accuracy for the $\Delta I = 3/2$ amplitudes will be reached in about two years. The $\Delta I = 1/2$ amplitudes are more difficult and will take longer but are also “within reach” [11].

There are several reasons that make these decay amplitudes difficult to calculate, in particular the fact that there are two light particles in the final state. Dealing with four-quark operators, on the other hand, is not anymore a major obstacle *per se*, as the successful calculations of B_K (see, [60, 61, 62], reviewed at this conference in [5]) show. The combination of these lattice calculations with the precise perturbative analysis of the weak Hamiltonian [50] leads to an accurate SM prediction for ε_K, with a theoretical error (excluding the parametric one coming from the uncertainties in the CKM matrix elements) of about 6%. I mention this quantity here not only to provide an example of a difficult matrix element which can be calculated accurately on the lattice, but also because this is one of the few quantities where a possible discrepancy with the SM may start becoming visible [63, 64]. A detailed analysis of this issue would require discussing the B-physics observables which provide an alternative measurement of the CKM matrix elements involved, which is outside the scope of this conference. Moreover, this would bring up the question of possible new physics signals in K physics, which is the topic of the parallel summary talk by Okada [1] — better for me to stop here.

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5. Conclusions

Since the very discovery of kaons the experimental studies of their decays have played an important role in shaping the Standard Model. Today, more than sixty years later, they still provide stringent tests of this theory, to an ever increasing level of precision. Summarizing a conference is an impossible task, even if it is split in two – the approach I have taken is to emphasize that at low energy, the SM is to a good approximation dominated by the strong interactions, with small corrections given by isospin–breaking contributions either of strong or electromagnetic origin. With enough precision in the data or if one looks at the proper observable (typically one which would be zero because of a symmetry of the strong interactions) can then be sensitive to yet smaller non-renormalizable terms in the Lagrangian — the low-energy remnant of the weak interactions, which become fully dinamical only at much higher energies. Experiments in kaon decays allow us to test all these aspects of the SM, even its high-energy scales, to a remarkable level of precision. No evident discrepancies have emerged so far other than in a few cases where the culprit could be either some experimental issue or our difficulty in calculating strong-interaction matrix elements.

Acknowledgments

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