PROCEEDINGS OF SCIENCE

Precision Measurement of $\pi\pi$ Scattering Lengths in K_{e4} Decays by NA48/2

Brigitte Bloch-Devaux[*] [†]
IRFU/SPP, CEA-Saclay
E-mail: bbloch@hep.saclay.cea.fr

Very large statistics of charged kaon decays have been accumulated in 2003-2004 by the NA48/2 experiment at the CERN SPS. The analysis of $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ (K_{e4}) decays allows a model-independent approach to the study of low energy $\pi\pi$ scattering. From a data sample of more than 1 million K_{e4} decays, scattering lengths can be extracted with an improved experimental precision of few percents, providing an accurate test of Chiral Perturbation Theory predictions. This result can be combined with the independent but complementary NA48/2 result obtained in the analysis of 60 millions $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$ ($K_{3\pi}$), leading to an experimental measurement of a_0 and a_2, the isospin 0 and 2 S-wave $\pi\pi$ scattering lengths, of unprecedented precision.

2009 KAON International Conference KAON09,
June 09 - 12 2009
Tsukuba, Japan

[*]Speaker.
[†]On behalf of the NA48/2 collaboration: Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Florence, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Turin, Vienna

 http://pos.sissa.it/

PoS(KAON09)033

1. Introduction

In the past years, $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ decays (K_{e4}) were traditionally the cleanest laboratory to study $\pi\pi$ scattering close to threshold and extract the values of the S-wave scattering lengths. Two experiments [1] collected sizable samples of K_{e4} decays: the Geneva-Saclay collaboration (S118) at the CERN/PS analyzed 30000 such K^{+} decays and the E865 collaboration at BNL about 400000 K^{+} decays. The NA48/2 collaboration at the CERN/SPS has collected more than one million decays in both charge modes in 2003-2004 and has already published results based on part of the data sample [2].

The analysis of the full data sample (1.13 million K_{e4} decays) will be presented here. The form factors and $\pi\pi$ phase shift which characterize the K_{e4} decay are measured simultaneously using a model independent method. S-wave $\pi\pi$ scattering lengths are then extracted with an improved precision using recent theoretical work [3] including detailed isospin symmetry breaking effects.

2. Experimental setup

Two simultaneous K^{+} and K^{-} beams were produced by 400 GeV$/c$ primary CERN/SPS protons, impinging on a beryllium target. The beams were then deflected in a front-end achromat to select momenta in the range (60 ± 3) GeV$/c$ and focused at ~ 200 m downstream in front of the first spectrometer chamber. A schematic view of the beam line and detector can be found in Ref. [4]. The NA48 detector and its performances are described elsewhere [5]. The main components used in the K_{e4} analysis are:

- a magnetic spectrometer consisting of a dipole magnet surrounded by two sets of drift chambers achieving a momentum resolution $\sigma(p)/p = (1.02 \oplus 0.044\ p)\%$ (p in GeV/c).
- a 27 radiation length liquid krypton calorimeter used to measure electromagnetic deposits and identify electrons through their E/p ratio. The transverse segmentation into 13248 projective cells gives an energy resolution $\sigma(E)/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV) and a space resolution for isolated showers $\sigma_x = \sigma_y = (0.42/\sqrt{E} \oplus 0.06)$ cm.
- a two plane segmented scintillator hodoscope allowing to trigger the detector readout on charged track topologies with a time resolution $\sim$150 ps.

3. The K_{e4} decay analysis

The data were selected for three well reconstructed charged track topologies, requiring two opposite sign pions and one $e^{+}(e^{-})$ identified from its E/p ratio. The reconstructed 3-track invariant mass (assigning a pion mass to each track) and p_t relative to the beam axis had to be outside an ellipse centered on the kaon mass and zero p_t, with semi-axes ± 20 MeV$/c^2$ and ± 35 MeV$/c$, allowing missing energy and p_t for the neutrino. No more than 3 GeV energy deposits in the calorimeter, non associated to tracks but in-time with the considered track combination, were allowed, keeping only a limited amount of soft radiative decays. The background sources are $K^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$ decays with subsequent $\pi \to e\nu$ decay or a pion misidentified as an electron and $K^{\pm} \to \pi^{\pm}\pi^{0}(\pi^{0})$ decays with subsequent Dalitz decay of a π^{0} to $e^{+}e^{-}\gamma$ with an electron misidentified as a pion and photon(s) undetected. The fraction of Wrong Sign events (WS), $K^{\pm} \rightarrow \pi^{+}\pi^{+}e^{\mp}\nu$, which would violate the ΔS=ΔQ rule, brings a direct control of the background level. The relative level of background/signal is $\sim 0.5\%$ and has been cross-checked using Monte Carlo simulation of the contributing background processes. This is illustrated in Figure 1-left by the distribution of the reconstructed beam momentum under the assumption of a missing neutrino .

3.1 Kinematics and Form Factor analysis procedure

The K_{e4} decay is fully described by the five kinematic Cabibbo-Maksymowicz variables [6]: two invariant masses $M_{\pi\pi}$ and $M_{e\nu}$ and three angles θ_π, θ_e and Φ as shown in Figure 1-right. Three axial (F, G, R) and one vector (H) complex form factors contribute to the transition amplitude expressed in terms of four form factor combinations $(F1, F2, F3, F4)$ which can be then developed in a partial wave expansion of S, P-waves, identified with the phases of the $\pi\pi$ scattering [7] under the assumption of T invariance:

$$F = F_s\, e^{i\delta_s} + F_p\, e^{i\delta_p} \cos\theta_\pi, \quad G = G_p\, e^{i\delta_g}, \quad H = H_p\, e^{i\delta_h}.$$

The third axial form factor R is suppressed by a factor m_e^2/S_e. Consequently, there is no way to measure it in K_{e4} decays. Assuming the same phase for F_p, G_p, H_p, only one phase difference and four real form factors are left ($\delta = \delta_s - \delta_p$, and F_s, F_p, G_p, H_p).

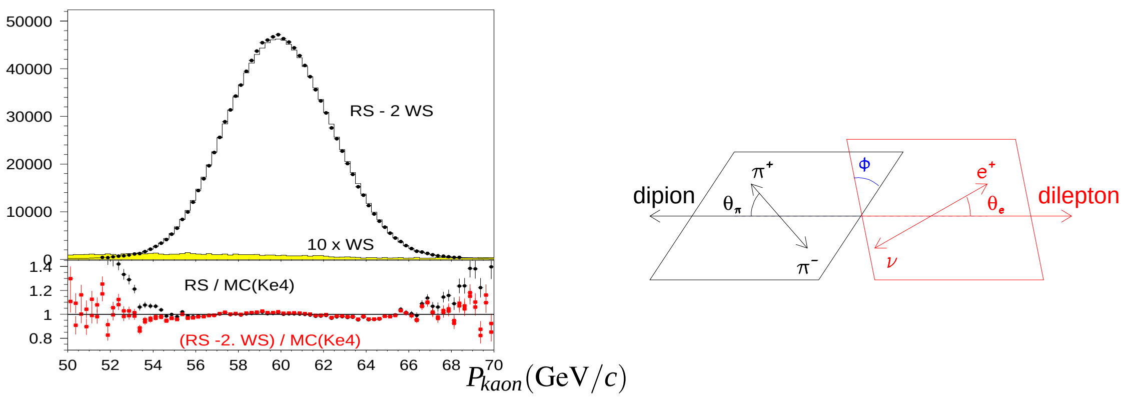

Figure 1: Left: Distribution of the reconstructed beam momentum (GeV/c). Data (background subtracted) are shown as symbols with error bars, simulation as open histogram and background (increased by a factor of 10 to be visible) from wrong sign events as shaded area. The insert shows the ratio of Data and simulated distributions before and after background subtraction. Right: Topology of the charged K_{e4} decay showing the angle definitions.

The event sample is then distributed over a grid of equal population boxes in the five-variable space. The chosen grid has ten bins in $M_{\pi\pi}$, five bins in $M_{e\nu}$, five bins in $\cos\theta_\pi$, five bins in $\cos\theta_e$ and twelve bins in ϕ, i.e. a total of 15000 five-dimensional boxes. Using the total sample of 726000 K^+ (404000 K^-) decays, there are 48 K^+ (27 K^-) per box. The Monte-Carlo simulation sample, 25 times larger than the data sample, takes into account acceptance, resolution effects as well as time-dependence of the beam geometry and experimental conditions. It also includes some of the electromagnetic effects, namely the classical Coulomb attraction between the two charged pions and the emission of real photon(s) as described by the PHOTOS generator [8] interfaced to the simulation. In this analysis, the branching fraction is not measured, so only relative form factors are accessible: F_p/F_s, G_p/F_s, H_p/F_s and the phase shift δ. Without prior assumption on the shape of their variation with $M_{\pi\pi}$, the form factors and phase shift are measured in the independent $M_{\pi\pi}$ bins and do not depend upon any particular model. At a second stage of the analysis, the observed variations of the form factors and phase shift with $M_{\pi\pi}$ are used to determine other parameter values through specific models. Under the assumption of isospin symmetry, the form factors can be developed in a Taylor expansion of dimensionless invariants q^2 ($q^2 = (S_\pi/4m_\pi^2) - 1$) and $S_e/4m_\pi^2$. The F_s form factor variation is described with three terms:

$$F_s = f_s\,(1 + f'_s/f_s\,q^2 + f''_s/f_s\,q^4 + f'_e/f_s\,S_e/4m_\pi^2)$$

while a q^2 term is enough to describe the G_p form factor:

$$G_p/f_s = g_p/f_s + g'_p/f_s\,q^2$$

and constants terms to describe the F_p and H_p form factors. The numerical results for all terms are given below:

$$\begin{aligned}
f'_s/f_s &= 0.152 \pm 0.007_{\text{stat}} \pm 0.005_{\text{syst}}\\
f''_s/f_s &= -0.073 \pm 0.007_{\text{stat}} \pm 0.006_{\text{syst}}\\
f'_e/f_s &= 0.068 \pm 0.006_{\text{stat}} \pm 0.007_{\text{syst}}\\
f_p/f_s &= -0.048 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}}\\
g_p/f_s &= 0.868 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}}\\
g'_p/f_s &= 0.089 \pm 0.017_{\text{stat}} \pm 0.013_{\text{syst}}\\
h_p/f_s &= -0.398 \pm 0.015_{\text{stat}} \pm 0.008_{\text{syst}}
\end{aligned}$$

The overall agreement between data and simulated distributions is excellent for each of the five kinematic variable projections. The systematic errors quoted are mainly due to background and acceptance uncertainties. They have been revisited compared to [2], including a restricted range [54.,66.] GeV/c of the reconstructed kaon momentum in the selection to minimize the background contamination (see Figure 1-left).

3.2 Determination of the scattering lengths

To extract scattering length values from the phase shift measurements of ($\delta = \delta_s - \delta_p$), more theoretical ingredients are needed. The Roy equations [9], based on analyticity, unitary and crossing symmetries, allow to predict the $\pi\pi$ phase values close to threshold using experimental measurements above the matching point ($E = 0.8$ GeV) and two subtraction constants, a_0 and a_2, the isospin 0 and 2 S-wave scattering lengths (in units of m_{π^+}). In a reverse way, from measurements of the phases and using the Roy equations, one can find the corresponding values of the subtraction constants. Numerical solutions of the Roy equations have been developed by several groups [10] and can be used for that purpose. Recent theoretical work, triggered by NA48/2 early results, has shown that isospin symmetry breaking may also alter the phases measured in the K_{e4} decays when all mass effects ($m_{\pi^+} \neq m_{\pi^0}, m_u \neq m_d$), neglected so far in the analyses, are considered [3]. Figure 2-left shows all experimental phase measurements from K_{e4} data. The simultaneous determination of both scattering lengths a_0 and a_2 translates into a 68%CL contour in the (a_0, a_2) plane as shown in Figure 2 right. The wide Universal Band corresponds to the range of allowed solutions from the experimental inputs at high energy, while the narrow band corresponds to an additional constraint from Chiral Perturbation Theory (ChPT) [11]. The NA48/2 experimental phase measurements translate in a 2-parameter fit as:

$$a_0 = 0.2220 \pm 0.0128_{\rm stat} \pm 0.0050_{\rm syst} \pm 0.0037_{\rm theo}\ ,$$
$$a_2 = -0.0432 \pm 0.0086_{\rm stat} \pm 0.0034_{\rm syst} \pm 0.0028_{\rm theo}\ .$$

with a 97% correlation coefficient. Using the ChPT constraint ($a_2 + 0.0444 = 0.236(a_0 - 0.22) - 0.61(a_0 - 0.22)^2 - 9.9(a_0 - 0.22)^3 \pm 0.0008$), the value from the 1-parameter fit is:

$$a_0 = 0.2206 \pm 0.0049_{\rm stat} \pm 0.0018_{\rm syst} \pm 0.0064_{\rm theo}$$

The theoretical error has been estimated in the 2-parameter fit following the prescription of the authors of [3] and is dominated by the experimental precision of the inputs to the Roy equation and the neglected Higher Order terms when introducing the mass effects. This result can be compared to the most precise prediction of ChPT which uses additional inputs [11]:

$$a_0 = 0.220 \pm 0.005_{\rm theo},\ \ a_2 = -0.0444 \pm 0.0010_{\rm theo}$$

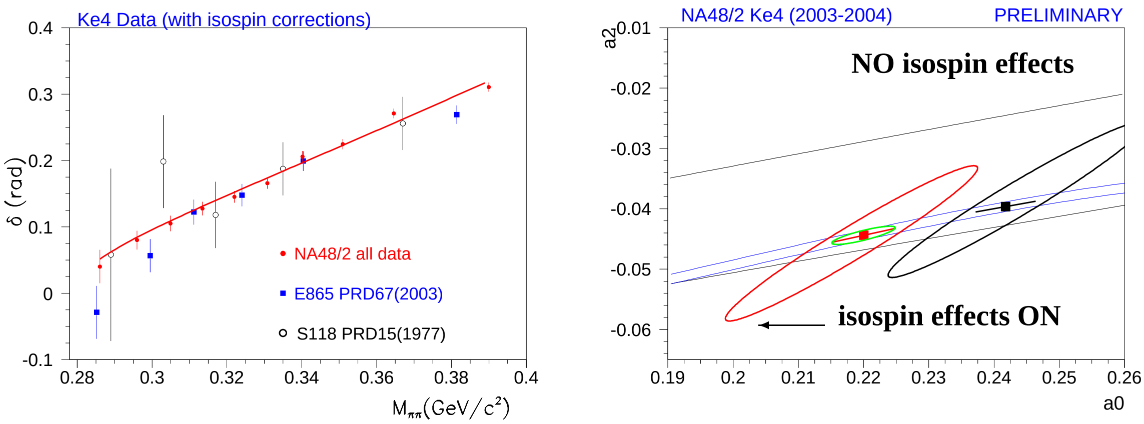

Figure 2: Left: Phase shift (δ) measurements with mass effects included for all available results. The line corresponds to the 2-parameter fit of the NA48/2 data alone. Right: Fits of the NA48/2 K_{e4} data in the (a_0,a_2) plane without (black) and with (red) isospin mass effects. The symbols are the result of the one-parameter fit imposing the ChPT constraint. The small (green) ellipse corresponds to the prediction from ChPT.

4. Combination of NA48/2 results and summary

The scattering length results from the analysis of the cusp effect in $K^\pm \to \pi^0\pi^0\pi^\pm$ decays have been reported at this conference [12]. The cusp and K_{e4} results are statistically independent, have very different systematic uncertainties (control of calorimetry and neutral trigger in one case, background and particle identification in the other) and show different correlations in the (a_0, a_2) plane. It is therefore interesting to combine the two analyses to get a more precise result:

$$a_0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0015_{\rm syst},\ \ a_2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0016_{\rm syst}$$
$$a_0 - a_2 = 0.2639 \pm 0.0020_{\rm stat} \pm 0.0004_{\rm syst}.$$

Figure 3 shows the two results and their combination in the planes (a_0, a_2) and $(a_0 - a_2, a_2)$, including statistical and systematic errors. The result $|a_0 - a_2|$ obtained from the pionium life time measurement by the DIRAC collaboration [13] is also shown.

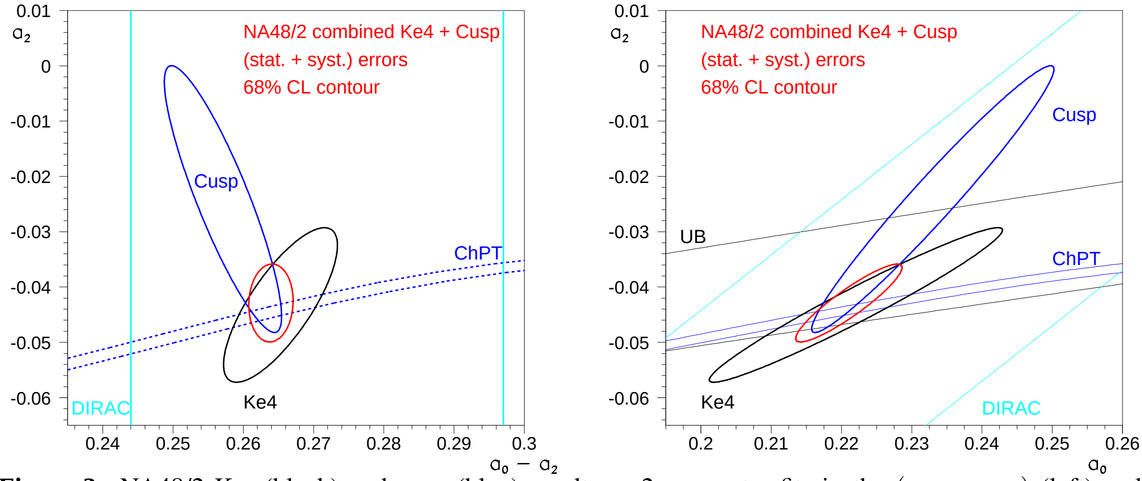

Figure 3: NA48/2 K_{e4} (black) and cusp (blue) results as 2-parameter fits in the $(a_0 - a_2, a_2)$ (left) and (a_0, a_2) (right) planes. In each plane the red contour corresponds to the combination of the results. The correlation coefficient is then 0.28 (resp. 0.91). The light blue lines correspond to the DIRAC result band.

Precise results have been obtained for the form factor and $\pi\pi$ phase shift values in the analysis of 1.13 million K_{e4} decays. The observed variation of the $\pi\pi$ phase shift with $M_{\pi\pi}$ allows to determine the scattering lengths a_0 and a_2, found in beautiful agreement with the predictions of ChPT once all isospin symmetry breaking effects are taken into account. The combination of two independent measurements of the S-wave scattering lengths by NA48/2 give precise values of both a_0 and a_2 with similar ± 0.005 (statistical + systematic) uncertainties. One should note that this is the first experimental determination ever obtained of a_2 and with a 10% relative precision.

References

[1] S118: L. Rosselet et al., Phys. Rev. **D15** (1977) 574.
E865: S. Pislak et al., Phys. Rev. Lett.**87** (2001) 221801, Phys. Rev. **D67** (2003) 072004.

[2] NA48/2: J. Batley et al., Eur. Phys. J. **C54** (2008) 411.

[3] G. Colangelo, J. Gasser and A. Rusetsky, Eur. Phys. J. **C59** (2009) 777.

[4] NA48/2: J. Batley et al., Phys. Lett. **B649** (2007) 349.

[5] NA48: V. Fanti et al., *Nucl. Inst. Methods* **A574** (2007) 433.

[6] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137** (1965) B438; Phys. Rev. **168** (1968) 1926.

[7] A. Pais and S. Treiman, Phys. Rev. **168** (1968) 1858.

[8] E. Barberio and Z. Was, PHOTOS, Comp. Phys. Comm. **79** (1994) 291,

[9] S. Roy, Phys. Lett. **B36** (1971) 353.

[10] B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rept. **353** (2001) 207,
S. Descotes, N. Fuchs, L. Girlanda, J. Stern, Eur. Phys. J. **C24** (2002) 469.

[11] G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. **B603** (2001) 125, Phys. Rev. Lett.**86** (2001) 5008

[12] NA48/2: D.Madigozhin, this conference, J. Batley et al., Phys. Lett. **B633** (2006) 173.

[13] DIRAC: B. Adeva et al., Phys. Lett. **B619** (2005) 50.