MaVaN's in the generalized Chaplygin gas scenario - A perturbative approach

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A perturbative approach for arbitrary choices of the equation of state of the universe is introduced in order to treat scenarios for mass varying neutrinos (MaVaN's) coupled to the dark sector. It allows for considering viable varying mass neutrino models coupled to any quintessence-type field. The Generalized Chaplygin model is considered as an example. Upon certain conditions, the usual *stationary condition* found in the context of MaVaN models together with the perturbative contribution can be employed to predict the dynamical evolution of the neutrino mass. Results clearly indicate that the positiveness of the squared speed of sound of the coupled fluid and the model stability are not conditioned by the *stationary condition*.

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The existence of a cosmological neutrino background is a firm prediction of the cosmological standard model, hence hints about their contribution to the energy density are quite relevant.

If from one hand, the most natural candidate to couple a SM singlet quintessence-like scalar field is the Higgs sector [1, 2], an exciting alternative is the coupling to neutrinos as in the context of the so-called mass varying neutrino (MaVaN) models [3, 4, 5, 6, 7]. This last possibility is particularly interesting since the coupling of neutrinos to the dark energy scalar field component may lead to a number of significant phenomenological consequences. Actually, this coupling renders the MaVaN mechanisms fairly natural. Indeed, if the neutrino mass m_v is generated by the dynamical value of a cosmologically active scalar field ϕ instead of through a vacuum expectation value (VEV) it would be an evolving quantity.

The neutrino energy density and pressure are expressed through a Fermi-Dirac distribution function without a chemical potential, $f(q)$, where $q \equiv \frac{|p|}{T_{v0}}$, and sub-index 0 denotes present-day values. Assuming a flat Friedman-Robertson-Walker cosmology with $a_0 = 1$, one thus have

$$\rho_v(a, \phi) = \frac{T_{v0}^4}{\pi^2 a^4} \int_0^\infty dq \, q^2 \left(q^2 + \frac{m^2(\phi) a^2}{T_{v0}^2} \right)^{1/2} f(q), \tag{1}$$

$$p_v(a, \phi) = \frac{T_{v0}^4}{3\pi^2 a^4} \int_0^\infty dq \, q^4 \left(q^2 + \frac{m^2(\phi) a^2}{T_{v0}^2} \right)^{1/2} f(q).$$

By observing that

$$m_v(\phi) \frac{\partial \rho_v(a, \phi)}{\partial m_v(\phi)} = (\rho_v(a, \phi) - 3 p_v(a, \phi)), \tag{2}$$

and from Eq. (2), one can obtain the energy-momentum conservation for the neutrino fluid

$$\dot{\rho}_v(a, \phi) + 3H(\rho_v(a, \phi) + p_v(a, \phi)) = \dot{\phi} \frac{dm_v(\phi)}{d\phi} \frac{\partial \rho_v(a, \phi)}{\partial m_v(\phi)}, \tag{3}$$

where $H = \dot{a}/a$ is the expansion rate of the universe and the *dot* denotes differentiation with respect to cosmic time.

The coupling between cosmological neutrinos and the scalar field as specified in Eq. (2) is restricted to times when neutrinos are non-relativistic (NR), i. e. $\frac{\partial \rho_v(a, \phi)}{\partial m_v(\phi)} \simeq n_v(a) \propto a^{-3}$ [4, 6, 8]. On the other hand, as long as neutrinos are relativistic ($T_v(a) = T_{v0}/a >> m_v(\phi(a))$), the decoupled fluids should evolve adiabatically since the strength of the coupling is suppressed by the relativistic increase of pressure ($\rho_v \sim 3 p_v$).

Treating the system of NR neutrinos and the scalar field as a unified fluid (UF) which adiabatically expands with energy density $\rho_{UF} = \rho_v + \rho_\phi$ and pressure $p_{UF} = p_v + p_\phi$ leads to

$$\dot{\rho}_{UF} + 3H(\rho_{UF} + p_{UF}) = 0 \implies \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\dot{\phi} \frac{dm_v}{d\phi} \frac{\partial \rho_v}{\partial m_v}, \tag{4}$$

where the last step is derived from the substitution of Eq. (3) into (4).

It is well known that the relative contribution of the energy densities components of the universe with respect to the one of the dark energy sector is on its own a problem. The assumptions proposed in Ref. [4] and subsequently developed elsewhere [8, 9, 6, 10] introduce a stationary

condition (SC) which allows circumventing the coincidence problem for cosmological neutrinos, by imposing that the dark energy is always diluted at the same rate as the neutrino fluid, that is,

$$\frac{dV(\phi)}{d\phi} = -\frac{dm_v}{d\phi}\frac{\partial \rho_v}{\partial m_v}.$$

(5)

This condition introduces a constraint on the neutrino mass since it promotes it into a dynamical quantity, as indicated in Eq. (4). In this context, the main feature of the scenario of Ref.[4] is that, in what concerns to dark sector, it is equivalent to a cosmological constant-like equation of state and an energy density that is as a function of the neutrino mass [7].

At our approach, the effect of the coupling of the neutrino fluid to the scalar field fluid is quantified by a linear perturbation $\varepsilon\phi$ ($|\varepsilon| << 1$) such that $\phi \to \varphi \approx (1+\varepsilon)\phi$. It then follows a novel equation for energy conservation

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV(\varphi)}{d\varphi} = -\frac{dm_v}{d\varphi}\frac{\partial \rho_v}{\partial m_v}.$$

(6)

After some straightforward manipulation one obtains for the value of the coefficient of the perturbation [7]

$$\varepsilon \simeq \frac{-\frac{dm_v}{d\phi}\frac{\partial \rho_v}{\partial m_v}}{\left[\phi^2\frac{d}{d\phi}\left(\frac{1}{\phi}\frac{dV(\phi)}{d\phi}\right)\right]},$$

(7)

for which the condition $|\varepsilon| << 1$ is required. Upon fulfilling all known phenomenological requirements, the above result allows for addressing a wide class of scalar field potentials and related equations of state for various candidates for the dark sector (dark energy and dark matter), which through the SC would be incompatible with realistic neutrino mass generation models.

In order to verify under which conditions Eq. (5) agrees with our perturbative approach for a given background equation of state, the coefficient of the linear perturbation should be given by

$$\varepsilon \simeq \frac{\frac{dV(\phi)}{d\phi}}{\left[\phi^2\frac{d}{d\phi}\left(\frac{1}{\phi}\frac{dV(\phi)}{d\phi}\right)\right]} \quad (|\varepsilon| << 1).$$

(8)

This means that one must search for a neutrino mass dependence on the scale factor for which the above condition is satisfied. Thus, once one sets the equation of state for the dark sector, there will be a period at late times for which the SC and the perturbative approach match. In particular, this feature can be reproduced by the generalized Chaplygin gas (GCG) equation of state [11, 12].

Given a potential, the explicit dependence of m_ϕ on the scale factor can be immediately obtained from Eq. (5). Furthermore, it is necessary to determine for which values of the scale factor the neutrino-scalar field coupling becomes important. For convenience one sets the value of $a = a_{NR}$ for which $\rho_{v,NR} = \rho_{v,UR}$ holds, usually established by the condition of $m_v \gtrsim T_v$, that parameterizes the transition between the NR and the ultra-relativistic (UR) regime. In fact, this takes place when

$$m_v(a) = m_0(\phi_0/\phi(a))^n = \chi\frac{T_{v,0}}{a}$$
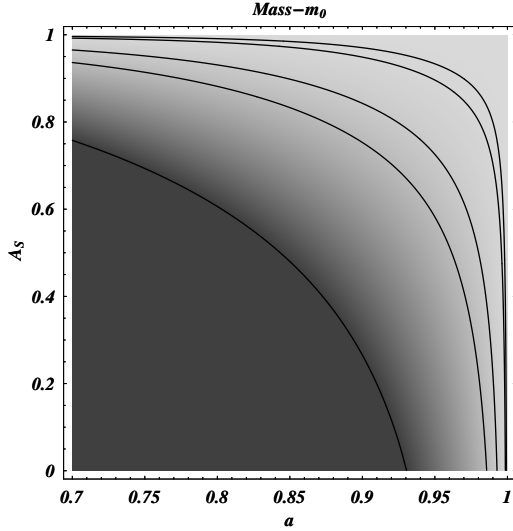
(9)

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Figure 1: Present-day values of the neutrino mass m_0 and the corresponding values of a_{NR} for which the transition between the NR and UR regimes takes place in the GCG phenomenological scenario with $\alpha = 1/2$ and variable A_s. The increasing *graylevel* corresponds to increasing values of m_0, for the boundary values $m_0 = 0.05\,eV, 0.1\,eV, 0.5\,eV, 1\,eV, 5\,eV$, respectively.

where χ is estimated to be about $\chi \simeq 94$ considering that $\rho_\nu/\rho_{\mathrm{Crit}} = m_0\,[eV]/(94\,h^2\,[eV])$, where h is the value of the Hubble constant in terms of $100\,km\,s^{-1}\,Mpc^{-1}$. Such a correspondence between a_{NR} and m_0 is illustrated in the Fig. 1 for the case of $\alpha = 1/2$. Considering the whole set of parameters that characterize the background fluid, one notices that it is rather difficult to see that the maximal value assumed by the ε parameter corresponds to its present-day value.

One observes that the interval of parameters A_s, m_0 and eventually α, for which our approximation can be applied ($\varepsilon < 1$), is valid for $a > a_{NR}$ and severally constrained by the imposition $a_{NR} \lesssim 1$. For values of A_s ($0.7 \lesssim A_s \lesssim 1$) [13, 14] one finds that $0.1 < \varepsilon \lesssim 1$. Just under quite special circumstances the usual SC and the perturbative contribution of MaVaN's match. In the original MaVaN scenario [4], the SC corresponds to the adiabatic solution ($H^2 \ll \mathrm{d}^2V/\mathrm{d}\phi^2$) of the scalar field equation of motion. In this case, the kinetic energy terms of the scalar field can be safely neglected. The consistency of our perturbative scenario with the stationary condition can be achieved only when the kinetic energy contribution is not relevant at late times.

For $A_s = 0$, the GCG behaves always as matter, whereas for $A_s = 1$, it behaves always as a
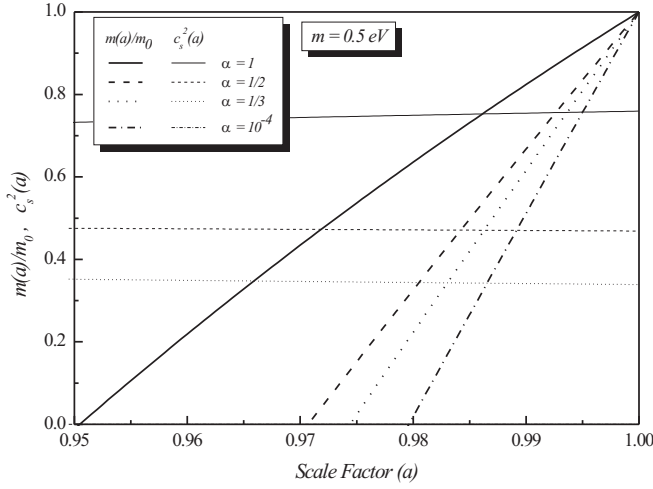
Figure 2: Model independent perturbative modification on squared speed of sound c_s^2 as a function of the scale factor for the neutrino-GCG coupled fluid in comparison with the adiabatic GCG fluid for $A_s = 0.7$ with $\alpha = 1, 1/2, 1/3, 10^{-4}$, for a present-day value of neutrino mass, $m_0 = 0.5\,eV$.

cosmological constant. Consequently, it is natural that the relevance of the kinetic energy term at present times is suppressed when the parameter A_s gets close to unity, which further ensures the agreement between the perturbative approach and the SC analysis.

Fig. 2 illustrates the results for an increasing neutrino mass with the scale factor for a set of phenomenologically consistent parameters in the context of the GCG model. Interestingly, for $m_0 = 0.5\,eV$, a fairly typical value, one can see that stable MaVaN perturbations correspond to a well defined effective squared speed of sound,

$$c_s^2 \simeq \frac{dp_\phi}{d(\rho_\phi + \rho_\nu)} > 0. \tag{10}$$

The greater the m_0 values, the more important are the corrections to the squared speed of sound, up to the limit where the perturbative approach breaks down. However, one finds that as far as the perturbative approach is concerned, our model does not run into stability problems in the NR neutrino regime. In opposition, in the SC treatment, where neutrinos are just coupled to dark energy, cosmic expansion in combination with gravitational drag due to cold dark matter have a major impact on the stability of MaVaN models. Usually, for a general fluid for which the equation of state is known, the dominant behaviour on c_s^2 arises from the dark sector component and not by the neutrino component. For the models where the SC (cf. Eq. (5)) implies in a cosmological constant-type equation of state, $p_\phi = -\rho_\phi$, one inevitably obtains $c_s^2 = -1$.

Thus, the perturbative approach is in agreement with the assumption that the coupling between neutrinos and dark energy (and/or dark matter) is weak. It is found that the stability condition related to the squared speed of sound of the coupled fluid is predominantly governed by dark energy equation of state. It also reproduces a dynamics similar to that one of the weakly coupled *cosmon* fields [15]. Such a troublesome behaviour should have already been observed as the SC implies

that $c_s^2 = -1$ from the very start, and the role of recovering the stability is relegated to the neutrino contribution [10]. The loosening of the stationary constraint Eq. (5) emerges from the dynamical dependence on φ, more concretely due to a kinetic energy component [7]. The knowledge of the background fluid equation of state for the dark sector (the GCG in our example), and the criterion for the applicability of the perturbative approach, do allow to overcome the c_s^2 negative problem, independently from the neutrino mass dependence set by the SC.

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