

## Quasiregular singularities taken seriously

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I discuss a special class of singularities obtained as a natural 4-dimensional generalization of the conical singularity. Such singularities (called quasiregular) are ruinous for the predictive force of general relativity, so one often assumes (implicitly as a rule) that they can be somehow excluded from the theory. In fact, however, attempts to do so (without forbidding the singularities by fiat) have failed so far. It is advisable therefore to explore the possibility that their existence is not prohibited after all. I argue that quasiregular singularities, if allowed, may appear either in situations where causality is endangered or in the early Universe. In the latter case objects might appear strongly (though not quite) resembling cosmic strings. Those objects would be observable and, moreover, it is not impossible that we already do observe one.

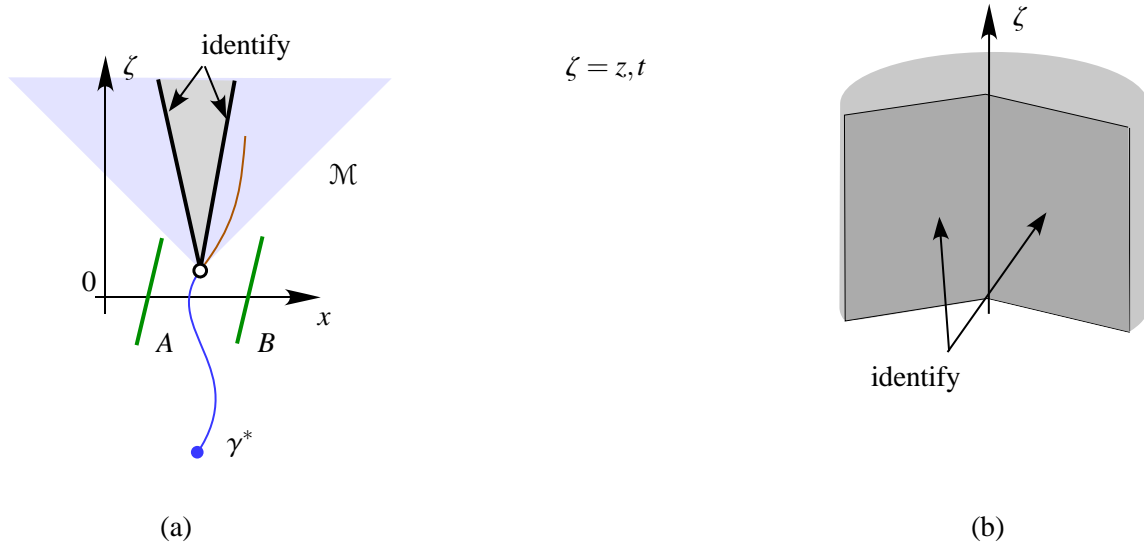
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### 1. Introduction

This lecture is devoted to a special kind of singularity. Instead of giving a precise definition let me start with a simple example. Take a two-dimensional space, either Euclidean, or Minkowskian.



**Figure 1:**  $\zeta$  in this and subsequent figures may be  $z$  or  $t$ . a) In the Minkowski case both sides of the angle should be timelike. b) Initially the space is  $\mathbb{R}^3$  with the flat metric.

Cut an angle out of it and glue the rays bounding the angle together (the vertex is not regarded a part of a ray). The resulting cone  $\mathcal{M}$  is still a smooth connected paracompact flat manifold (so, in particular, in the Lorentzian case it is a nice spacetime)<sup>1</sup>, but it is *singular* — the vertex cannot be glued back into the space (without sacrificing either the smoothness or the non-degeneracy of the metric), so the geodesics terminating at the hole are endless though incomplete. In Fig. 1b a similar construction is shown in the two plus one case. We again remove a wedge and glue together its boundaries (this time they are half-planes). Again the intersection of the faces — this time it is the straight line — cannot be glued back into the space and we again have a singularity, this time “in the form” of a straight line.

It is the singularities of this type that are our subject. The reason why they deserve most serious consideration is that they, in fact, deprive general relativity of its predictive power. Indeed, in contrast to the “usual”, curvature singularities, these “topological” ones are absolutely “sudden”: *nothing* would tell an observer approaching such a singularity that his world line will terminate in a moment. As we shall see soon, in spacetimes with such singularities everything (the geometry of the universe, its topology, causal structure, etc.) may change whimsically and (apparently) causelessly. For example, observers (like  $A$  and  $B$  in the figure) may think — up to some moment — that they live in the Minkowski space. But after that moment they will discover that without experiencing any acceleration they acquired some speed towards each other. Likewise, in the otherwise Minkowskian space time machines or wormholes may appear with no visible cause if their appearance is accompanied by singularities of this kind. As Geroch [2] stated in this connection:

<sup>1</sup>A proof of this, hopefully obvious, assertion requires giving a precise meaning to the term “gluing”; see, e. g., [1].

“Thus general relativity, which seemed at first as though it would admit a natural and powerful statement at prediction, apparently does not”. Two hard questions that immediately arise are:

1. How to predict the evolution of the Universe? We see that anything can happen any time.
2. Why don't we encounter that problem in our everyday life?

A temptingly simple answer would be this: “Such singularities are unphysical. They are excluded by . . .” In place of the dots a strong argument must stand, or a new (physically motivated) postulate.

## 2. Inevitability of quasiregular singularities

In this section my goal is to explain why, contrary to what one might expect, it is hard (if possible at all) to find that appropriate argument or postulate.

### 2.1 Identification

Before discussing possible candidates we, of course, have to specify clearly what are “such singularities”. This can be done in many ways, but I shall restrict myself to three most known variants. References to some others can be found in [3].

#### 2.1.1 Quasiregular singularities

Consider a curve in a spacetime  $M$

$$\gamma(s): [0, 1) \rightarrow M$$

and let  $\{e_{(i)}\}$  be an orthonormal frame parallel transported along  $\gamma$ . In this frame we find the components of  $\gamma$ 's velocity and define the following integral:

$$b[\gamma] \equiv \int_0^1 \sqrt{(\dot{\gamma}^1)^2 + (\dot{\gamma}^2)^2 + \dots} d\xi.$$

called the *b-length* of  $\gamma$ . In the Riemannian case it is simply the length of the curve. On the other hand, in the Minkowski space the *b-length* of a curve is merely its coordinate length in the standard (Cartesian) coordinates.

Clearly, the *value* of  $b$  may depend on the choice of  $\{e_{(i)}\}$ , but its *finiteness* does not.

**1. Definition.** An inextendible spacetime is said to be singular (*b-incomplete*) if there is an *endless* curve  $\gamma^*$  with a finite *b-length*.

Obviously, our exemplary spacetime  $\mathcal{M}$  is singular: the blue curve is endless (to be more precise, it is *future* endless) even though it has a finite *b-length*.

**2. Definition.** A singularity is *quasiregular* if in the basis  $\{e_{(i)}\}$  the components of the Riemann tensor and all its derivatives remain bounded on  $\gamma^*$ .

Evidently, singularities in flat spacetimes (including  $\mathcal{M}$ ) are quasiregular. A *general* quasiregular singularity, however, is a much more complex object than simply a punctured plane.

**3. Example. (Misner space).** Take the Minkowski plane

$$ds^2 = -dt^2 + dx^2 = -d\alpha d\beta \quad \alpha \equiv t - x, \quad \beta \equiv t + x, \quad (2.1)$$

and consider the isometry  $\zeta$  (it is a Lorentzian boost, in fact)

$$\zeta: \quad \alpha \mapsto \kappa\alpha, \quad \beta \mapsto \kappa^{-1}\beta$$

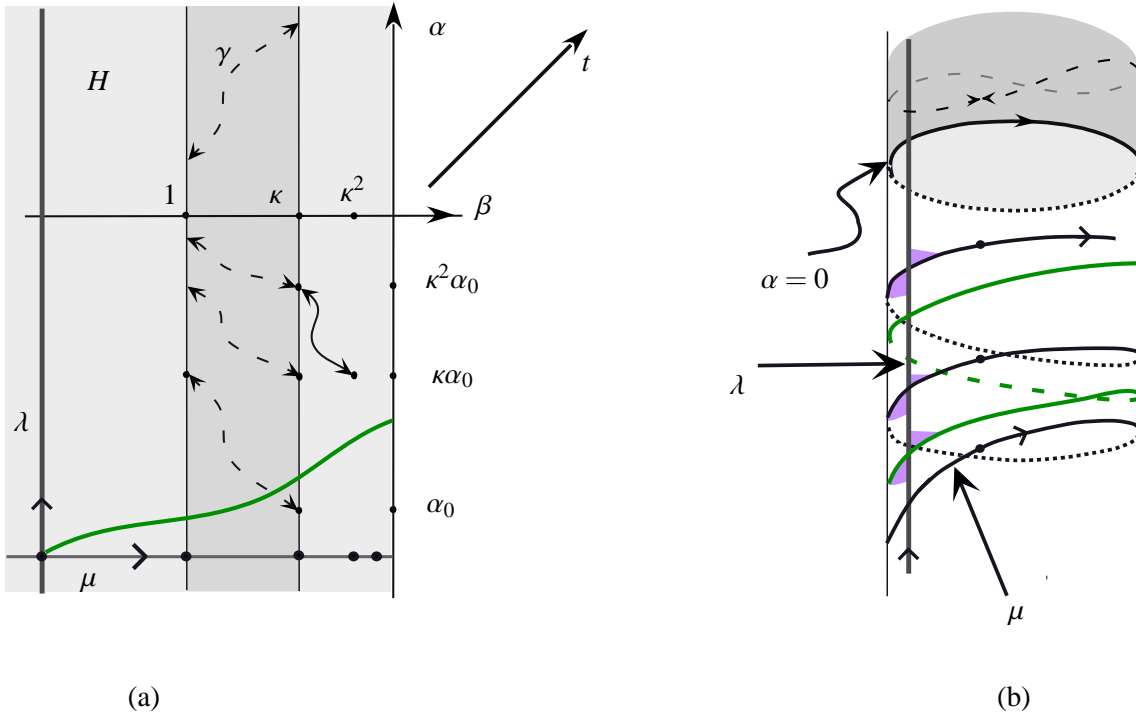
This isometry induces an equivalence relation: a point  $p$  is equivalent to any point  $q$  related to it by the isometry:

$$p \approx q \iff p = \zeta^k(q) \quad \forall k \in \mathbb{Z}$$

The Misner space  $M_M$  is defined, see, e.g., [4], as the quotient of the left half-plane  $H$  over this equivalence

$$M_M = H / \approx .$$

It follows right from the definition (I drop the proof) that the Misner space is a nice legitimate spacetime, though with a lot of surprising properties. To see some of them it is instructive to



**Figure 2:** (a) The double-headed lines connect (some of) identified points. (b) By purple some of the null cones are shown.

isolate the strip  $\beta \in (1, \kappa)$  as a fundamental region. Then the Misner space is obtained simply by identifying the left border of the strip with its right border according to the rule

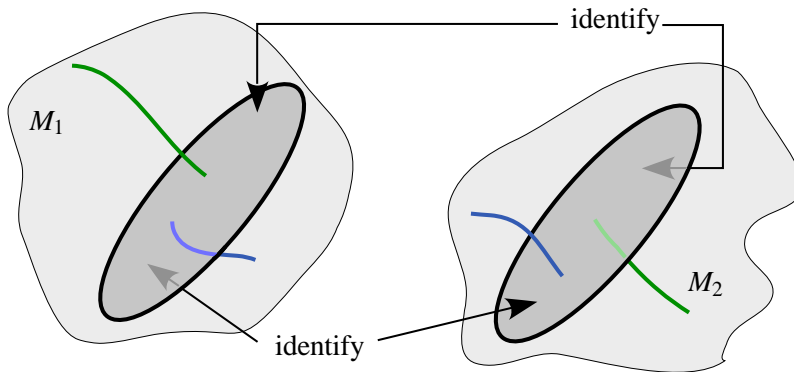
$$(\alpha, 1) = (\kappa^{-1}\alpha, \kappa)$$

as shown by dashed lines in Fig. 2a. So,  $M_M$  is a cylinder. The metric on the cylinder is flat, but the causal structure, is nevertheless quite bizarre. The lower part of the cylinder is obtained by

identifying causally disconnected pairs of points and, therefore, causality holds here. But the upper half originates from the quadrant  $\alpha > 0, \beta < 0$ , where we identify causally related points, so there are closed causal curves in this part of  $M_M$  (the blue one, for example, originating from  $\gamma$ ). The boundary between these regions is formed by the circle obtained from the geodesic  $\alpha = 0$ . The fate of the other null geodesics of  $H$  differ: the vertical ones map to generators of the cylinder (as, for example,  $\lambda$  does). And the horizontal null geodesics, like  $\mu$ , say, turn into spirals which infinitely wind themselves approaching the horizon and never crossing it. The timelike curves in  $H$  which do not cross the ray  $\alpha = 0$  behave the same way, see the green curve in Fig. 2. Their images in the Misner space being sandwiched between two null spirals also infinitely approach the horizon and, correspondingly, have no end points. So, we have a lot of endless curves with finite  $b$ -lengths. Thus the Misner space is *singular*. And as the metric is flat the singularity is quasiregular (even though it is so different from that considered above).

**2.1.2 Local extendability**

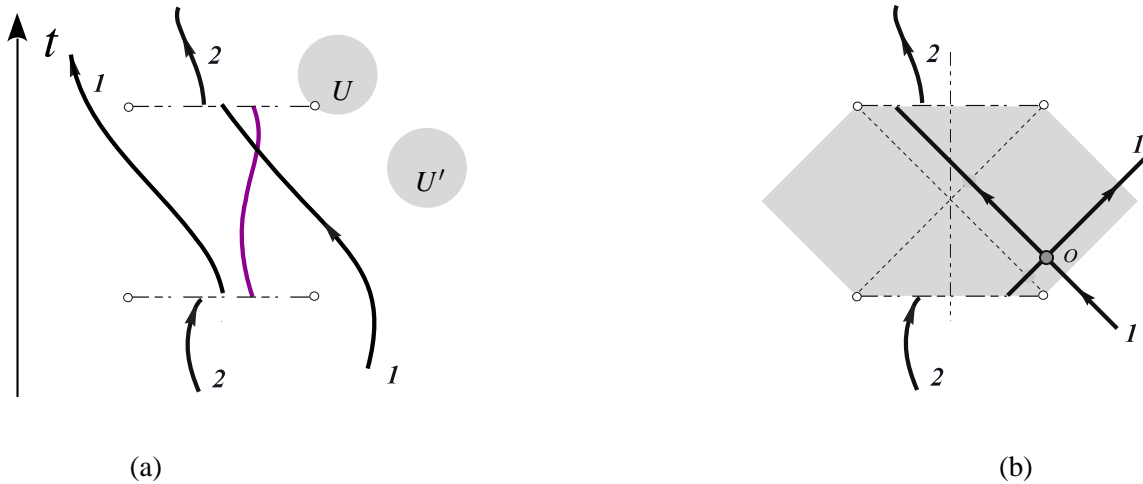
Let us consider one more singularity. Pick two isometric regions (of, perhaps, different space-



**Figure 3:** The banks of the slits are identified (which implies that they lie in isometric domains) so that the green and the blue lines become continuous. If the isometry can be extended from the mentioned domains to the entire  $M_{1,2}$ , the result of the surgery is the double covering of  $M_1$  – (a sphere of co-dimension 2).

times). Remove a closed disk from one of them, the corresponding disk from the other, and identify the banks of the slits as is shown in Fig. 3: the upper bank of either slit is glued to the lower bank of the other. The edges of the disks [in the (2+1)-dimensional case these will be circles  $S^1$ ] cannot be glued back in the space, so the spacetime is singular.

**4. Example.** Two equal spacelike slits separated by time are made in the Minkowski plane. The banks of the slits are identified as shown in Fig. 4a. The resulting spacetime [5] contains closed causal curves — the purple one, for example — and is referred to as the Deutsch-Politzer (DP) time machine. Later we shall use a slightly different spacetime, see Fig. 3b, called the twisted Deutsch-Politzer (TDP) space. It is obtained the same way as the DP space, but one of the banks before being glued to the other is mirror inverted. The TDP space is non-orientable, but this doesn't matter much, because its 4-dimensional analogue can be made orientable. What does matter is that a typical null geodesic entering the domain of causality violation has a self-intersection.



**Figure 4:** Black curves 1 and 2 are continuous. (a) The purple curve is timelike and closed. (b) Shaded is the region where causality does not hold.

Both spacetimes in Example 4 have singularities. However, if we had glued the banks in the “normal” way we would have been able to remove the singularities arriving simply at the Minkowski space. To capture this idea Hawking and Ellis introduced [4] the concept of local extendability:

**5. Definition.** A spacetime  $M$  is said to be *locally extendible* if it contains an open set  $U$  such that (i) the closure of  $U$  is non-compact, but (ii)  $U$  is isometric to a subset  $U'$  of a spacetime  $M'$  in which  $CU'$  is compact.

Loosely speaking, locally extendible spaces are those which lack points that “might have” existed. The DP space is obviously locally extendible (for example, the closure of the ball  $U$ , see Fig. 3a, is evidently non-compact, while the closure of  $U'$ , which is isometric to  $U$ , is compact) and so is  $\mathcal{M}$ .

### 2.1.3 Holes

The concept of local extendability operates with a spacetime as a whole, with no reference to anything like evolution. There is an alternative concept, however, formulated in causal rather than in topological terms.

First, for a given set  $\mathcal{S}$  we define the domain of dependence  $D^+(\mathcal{S})$  to be the collection of all points  $p$  such that every past endless curve through  $p$  meets  $\mathcal{S}$ . Let, for example,  $\mathcal{S}$  be a line of constant time in the Minkowski plane. Its domain of dependence is the whole upper half-plane. At the same time in the cone  $\mathcal{M}$  the domain of dependence of the “same” line does not include the pale purple region, see Fig. 1a. Indeed, through any point of this region there is a past endless curve (like that drawn in brown) which does not meet  $\mathcal{S}$ . Such curves appear, of course, due to the “missing” points and hence the definition<sup>2</sup>.

<sup>2</sup>On the (minor) deviations of this definition from the original one [2], see [3].

**6. Definition.** A space-time  $(M, g)$  is called *hole-free* if it has the following property: given any achronal<sup>3</sup> hypersurface  $\mathcal{S}$  in  $M$  and any metric preserving embedding  $\pi$  of an open neighbourhood of  $D^+(\mathcal{S})$  into some other spacetime  $(M', g')$ , then  $\pi(D^+(\mathcal{S})) = D^+(\pi(\mathcal{S}))$ .

In other words, a hole-free space is that where the domain of dependence of any surface is “as big as possible”. Clearly, the spacetime  $\mathcal{M}$  is not hole-free.

Summing up, the singularity in  $\mathcal{M}$  is a quasiregular singularity. At the same time  $\mathcal{M}$  is a locally extendible spacetime. And, finally,  $\mathcal{M}$  is not hole-free. Which of these properties should be forbidden?

## 2.2 Impasses

### 2.2.1 The “unphysical nature” of quasiregular singularities

In their pioneering paper on quasiregular singularities [6] Ellis and Schmidt speaking through Salviati say: “We know lots of examples of quasiregular singularities, all constructed by cutting and gluing together decent space-times; and because of this construction, we know that these examples are not physically relevant.” But this argument — in spite of its popularity — is emphatically untenable: the cuttings and gluings that we used are not *a property* of the relevant spacetimes, they only are a means of description. *Any* spacetime can be constructed by cutting and gluing together some other decent spacetimes and *any* of them can be constructed otherwise. The spacetimes with the singularities in discussion are absolutely no different in this respect from the others.

Yet another idea was to take into consideration the Einstein equations and to show that quasiregular singularities appear only in spacetimes with the stress-energy tensor of a very special type. Then matter of that type could have been declared unphysical and the whole problem would have been solved. This program, however, does not work. At least, not in the general case. Indeed, pick an arbitrary spacetime  $M_1$  and define  $M$  to be the double covering of  $M_1$  — (a sphere of co-dimension 2).  $M$  can be visualized as the result of the surgery described in the beginning of section 2.1.2. The missing sphere gives rise to a quasiregular singularity in  $M$ , but since the spacetime  $M_1$  was chosen arbitrarily,  $M$  can be built obeying the Einstein equations with an arbitrarily nice right-hand side. Moreover, the same would be true for *any* local condition which one could impose on spacetimes. So, we conclude that generally

No local condition can exclude quasiregular singularities.

### 2.2.2 The local extendability postulate

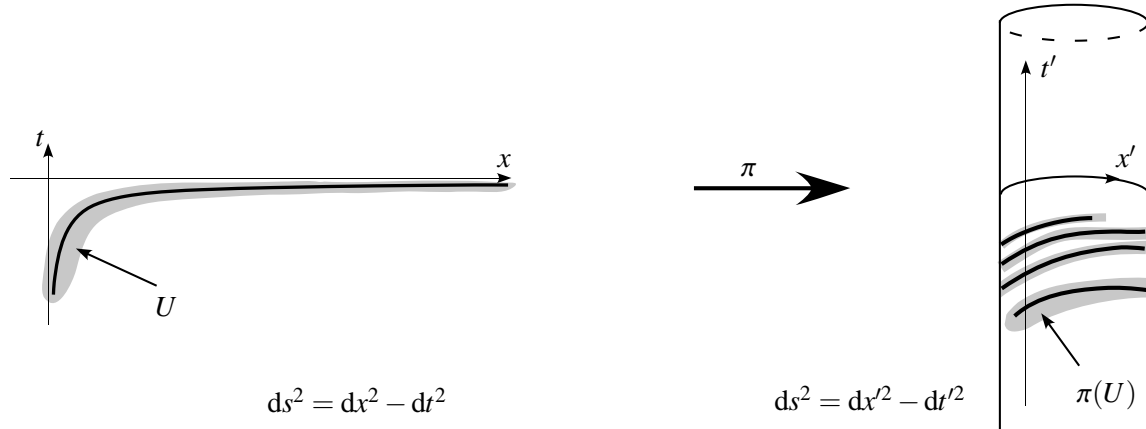
Maybe then we should forbid a realistic spacetime to be locally extendible<sup>4</sup> [4]? Indeed, the singularities considered in Section 2.1.2 appeared, as Hawking and Ellis put it, only because we were perverse enough to extend the top and bottom sides of the slits “wrong way”. So, if we consider a four-dimensional spacetime as a result of some “evolution” (the evolution, say, of a three-dimensional space with time), such a postulate seems self-suggesting. What it says is, loosely speaking, the following. In its evolution a spacetime at every moment of time has to choose

<sup>3</sup>A set is *achronal* if no its points can be connected with a timelike curve.

<sup>4</sup>In spite of its name, this property is not local.

between developing a quasiregular singularity and avoiding it, and the spacetime always choose the latter. However, as Beem and Ehrlich showed [7], this approach does not work either.

Consider a flat cylinder with the period  $l$  and a map  $\pi$  sending each point of the Minkowski



**Figure 5:** The Minkowski plane is locally extendible.

plane to a point of the cylinder according to the rule

$$t' = t, \quad x' = x \pmod{l}$$

(we wrap the plane around the cylinder, see Fig. 5). The part of the hyperbolae shown in Fig. 5 is mapped to a spiral: a curve which infinitely approaches the circle  $t' = 0$  and has no self-intersections. A neighbourhood  $U$  of the hyperbolae, if chosen appropriately (i. e., to be narrow enough), will map to its image one-to-one and, obviously, isometrically. So, the restriction of  $\pi$  to  $U$  is a metric preserving embedding. And, nevertheless, the closure of  $\pi(U)$  is compact, while the closure of  $U$  is not. Thus we conclude that

the Minkowski space is locally extendible.

But a postulate forbidding even the Minkowski space is definitely *too* strong.

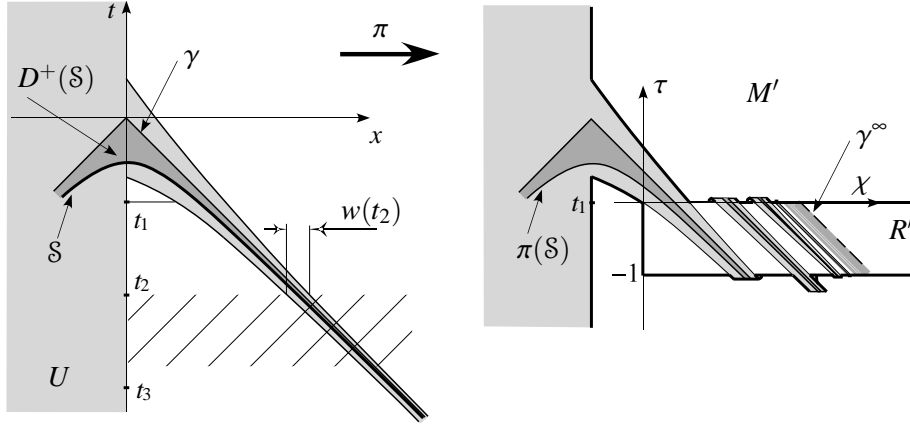
### 2.2.3 Hole-freeness

The last possibility<sup>5</sup> is to use “hole-freeness” as a criterion and, following Geroch’s proposal, to “modify general relativity as follows: the new theory is to be general relativity, but with the additional condition that only hole-free spacetimes are permitted” [2]. The proposal seems to be physically well motivated. Indeed, what happens in the domain of dependence of  $\mathcal{S}$  is causally determined by the data on  $\mathcal{S}$ , so one can easily imagine that the existence itself of a point of  $D^+(\mathcal{S})$  is also determined by them and not by the remainder of the spacetime.

However, this approach fails by exactly the same reason as the previous one: even the Minkowski plane is not hole-free.

<sup>5</sup>Of those considered in this lecture.





**Figure 6:** The dark-gray regions are  $D^+(\mathcal{S})$  and its image. By light-gray  $U$  and  $\pi(U)$  are shown.

*Proof.* Let  $\mathcal{S}$  be a hyperbolae in the Minkowski plane. Its domain of dependence  $D^+(\mathcal{S})$  is the closed set shown in Fig. 6; it is bounded (in particular) by a past directed null geodesic  $\gamma$ . The neighbourhood  $U$  of  $D^+(\mathcal{S})$  is defined to be the left half-plane plus a “beak”, see Fig. 6. The beak is characterized by width  $w$  and is chosen so that

$$w \text{ is monotone and } w(t) \rightarrow 0 \text{ at } t \rightarrow -\infty.$$

The idea is to find for  $U$  an alternative extension (not the Minkowski plane, but some  $M'$  instead) and to check that in that extension the domain of dependence of the image of  $\mathcal{S}$  is larger than simply the image of  $D^+(\mathcal{S})$ :

$$D^+(\pi(\mathcal{S})) \supsetneq \pi(D^+(\mathcal{S})). \quad (*)$$

The extension  $M'$  is built by gluing some portions  $O_k, k = 1, \dots$  of the beak to a rectangular strip  $R$ :

$$ds^2 = -d\tau^2 + d\chi^2, \quad \tau \in (-1, 0), \quad \chi > 0.$$

Specifically, pick a sequence  $\{t_k\}$  of negative numbers such that  $t_{k+1} < t_k - 1$  and define  $O_k$  as follows:

$$O_k \equiv \{p \in U: \quad x(p) > 0, \quad t_k > t(p) > t_k - 1\}$$

(for example, the hatched strip in Fig. 6 cuts  $O_2$  out of the beak). Further, let  $\Psi$  be the isometry which sends, for every  $k$ , each point  $p \in O_k$  to the point  $q \in R$  according to the rule

$$\tau(q) = t(p) - t_k, \quad \chi(q) = x(p) - \chi_k$$

Now  $M'$  is defined as the quotient:

$$M' \equiv U \cup_{\Psi} R$$

(i. e., as the result of gluing together  $U$  and  $R$  by  $O$ ) and  $\pi$  as the natural projection of  $U$  to  $M'$ .

I have not specified  $\chi_k$  (see [3] for details). but, in fact, all required of them is that  $\pi(O_k)$  would not overlap and at the same time the series  $\sum w_k$  would converge, where  $w_k$  is the maximal width of  $O_k$  (for  $\{t_k\}$  falling fast enough such a choice of  $\chi_k$  is obviously possible). The above-mentioned

convergence implies that there is a null geodesic segment  $\gamma^\infty$  — the dashed line in Fig. 6 — to which the null geodesics  $\pi(\gamma \cap O_k)$  converge.

It is the existence of  $\gamma^\infty$  that proves (\*) and hence the proposition. Indeed,  $\gamma^\infty$  is disjoint with  $\pi(D^+(\mathcal{S}))$ . But, on the other hand, a timelike curve through any its point meets inevitably  $\pi(\mathcal{S})$ . So,  $\gamma^\infty \subset D^+(\pi(\mathcal{S}))$ . □

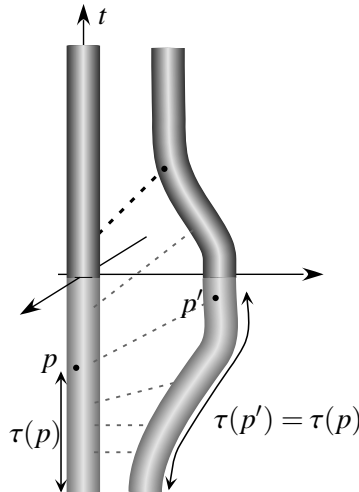
### 3. Formation of quasiregular singularities

As we have just seen there is no obvious way to expel quasiregular singularities from relativity. Now let us formulate the question the other way round: Are there any indications that quasiregular singularities do exist, or may appear under favorable conditions? At first glance the property which makes such singularities interesting — their “suddenness” — makes at the same time their appearance unlikely. Indeed, if everything prior to the singularity looks *as if* the spacetime is singularity-free, why not assume that it *is* — and will remain — singularity-free. Actually, however, the situation is somehow less trivial. I think, two possibilities are of interest here. Let us start with the more subtle and academic one.

#### 3.1 How to artificially create a quasiregular singularity

##### 3.1.1 Time machines

In our consideration we have already met two spacetimes — the DP space and the Misner space — which evolve nicely up to some moment, but then lose causality; in other words closed causal curves appear to the future of some globally hyperbolic domain. The regions where causality breaks down and the entire spacetimes containing them are called *time machines*. The most known



**Figure 7:** Wormhole-based time machine. The lengths are measured from a surface  $t = const$ . The thick dashed line is the earliest closed causal (null) curve.

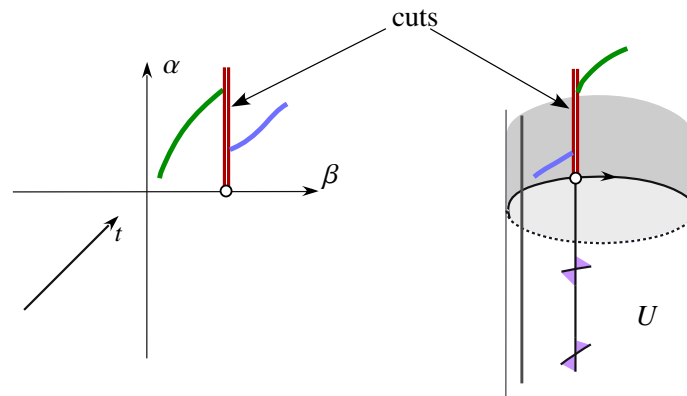
time machine is probably that based on a wormhole [8]. The relevant spacetime, see Fig. 7, is

obtained, for example, by removing<sup>6</sup> two timelike tubes from the Minkowski space and identifying the boundaries of the holes (the vicinity of the junction is understood to be smoothed out appropriately, so that the junction is seamless). The identification is done as follows: every generator of the left cylinder is glued to the corresponding generator of the right cylinder so that the length of the corresponding segments are equal. In other words, the clocks traveling with one mouth of the wormhole are synchronized (when seen through the throat) with those left at rest near the other mouth. Clearly, if the right tube is tortuous enough, then the points which are to be identified become causally separated beyond some surface. So, closed causal curves appear to the future of a globally hyperbolic region and the spacetime becomes a time machine.

At first glance the mentioned time machines form two — fundamentally different — classes. The DP time machine, as we discussed, may appear or may not. One can neither predict its appearance, nor cause it. And it seems to be completely different with the wormhole-based time machine or the Misner space, where one prepares suitable initial conditions and waits until the spacetime governed by the Einstein equations gives birth to a closed causal curve. So, one can even think that time machines are divided into spontaneously appearing and artificially manufactured. In fact, however, this difference is a sheer illusion. Any spacetime in its evolution can avoid transformation into a time machine. The following theorem is proved

**Theorem [9].** Any spacetime  $U$  has a maximal extension  $M^{\max}$  such that all closed causal curves in  $M^{\max}$  (if they exist there) are confined to the chronological past of  $U$ .

The proof is quite lengthy, so I shall only illustrate the theorem by the example of the Misner space. Denote by  $U$  its initial, globally hyperbolic part. An inhabitant of  $U$  sees that in the course of time the null cones open more and more, see Fig. 8. So, he anticipates the unavoidable (as he might



**Figure 8:**  $U$  is the lower (causality-respecting) half of the Misner space. The circles designate a ‘new’ quasiregular singularity.

think, knowing from the Einstein equations that the spacetime will have to remain flat) appearance of a time machine (when the inclined generator becomes horizontal). What the theorem says is that his hopes may be vain: besides the Misner space,  $U$  also has another, *causality-respecting* (and also flat) maximal extension. To verify this assertion recall that in the Misner space the vertical

<sup>6</sup>Again, this surgery is nothing more than a description. No real cutting of a spacetime is meant.

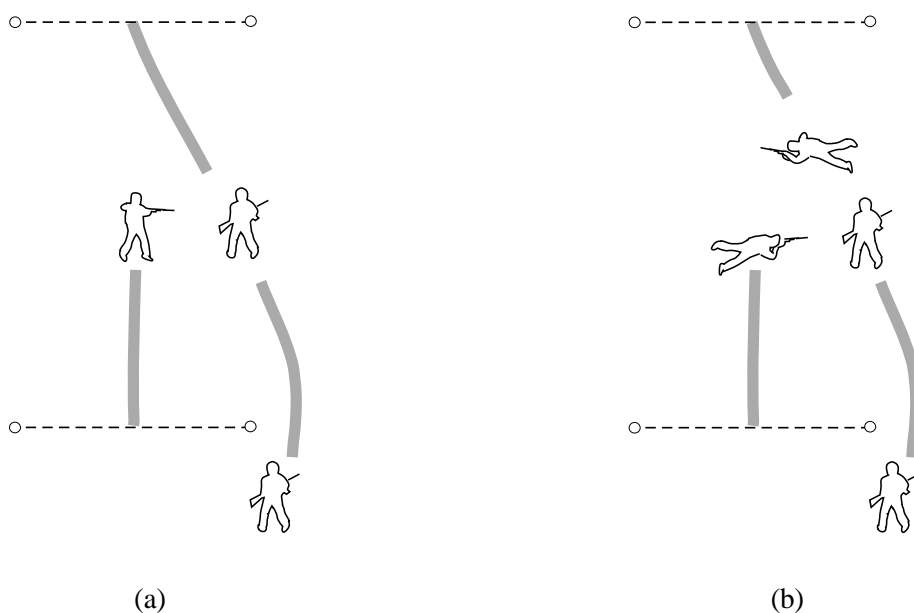
lines generating the cylinder are null geodesics. So, there is a neighbourhood of the brown, say, ray isometric to a neighbourhood of a null ray of the Minkowski plane. The desired extension now can be described as the result of removing these two rays and gluing the right bank of either cut to the left bank of the other (so that the green and blue curves in Fig. 8 are continuous). That causality holds in the thus constructed extension is obvious. Note, however, that this is achieved at the cost of allowing a new quasiregular singularity to appear.

It's important that by "spacetime" in the theorem not just a smooth connected Hausdorff pseudo-Euclidean manifold is understood. One can impose *any* additional condition and, as long as it is local (like the Einstein equations), the theorem remains true [9].

Summing up, whatever we do within general relativity we cannot force the universe to give birth to a time machine. What we *can* do, however, is force the universe to *choose* between creating a time machine, or a quasiregular singularity. And there are indications as I shall show in a moment that the universe might prefer the latter.

### 3.1.2 Time travel paradox

Consider the following situation. An experimenter learns how to build a time machine (out of a wormhole, say). He makes all necessary preparations to ensure that it will appear in 5 minutes. Then he loads his rifle and makes a firm decision to enter the time machine as soon as it appears,



**Figure 9:** (a) An apparent time travel paradox. (b) One of its possible resolutions.

and to return with its help "back in time", where to waylay his younger self, see Fig. 9a, and shoot the latter dead. It is clear that his plan cannot be realized. What is *not* obvious is what actually will happen.

Let us reformulate the situation in more general terms. We have a system in some initial state (an armed person in a room where a time machine is to appear). The system is governed by some quite plausible laws of motion (the person must wait for 5 minutes then make a few steps, raise the rifle, and pull the trigger). There is nothing pathological either in the initial state, or in the

laws of motion. In a globally hyperbolic spacetime they would uniquely determine the evolution of the system. But, because the spacetime contains a time machine, we are facing a paradox<sup>7</sup>: no evolution corresponds to the specified combination of the initial state and laws of motion. How should this be interpreted?

A possible solution is this. The system in consideration is highly complex and actually we did not specify its laws of motions accurately enough. So, it might happen that we are just overlooking the solution. For example, one can advocate the evolution depicted in Fig. 9b: the experimenter is wounded (not killed!) and it is the wound that in due time prevents him from shooting accurately and thus leads to wounding the target instead of killing it.

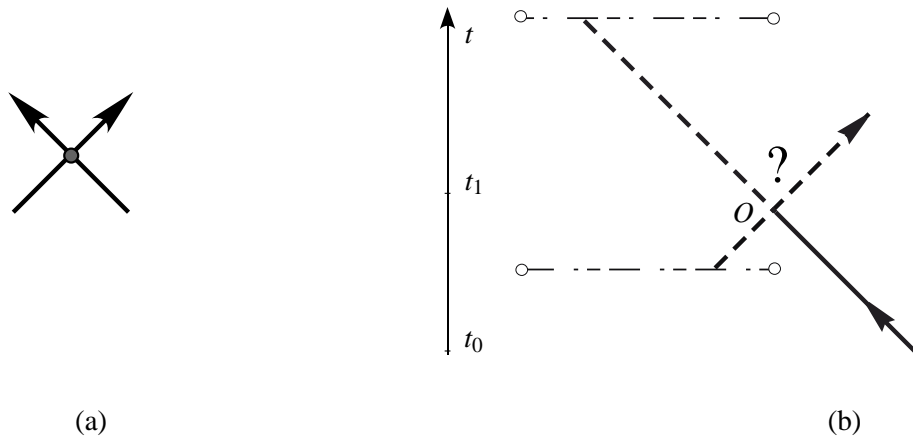
Such reasonings may lead one to the following

**7. Conjecture.** Any reasonable laws of motion being combined with any reasonable initial state must correspond to some evolution whether or not a time machine appears.

To cast doubt on this conjecture and to construct a time travel paradox [10] it is instructive to analyze a toy model of this situation. Consider the twisted DP universe, see example 4, populated only by pointlike particles, whose evolution is determined by the following simple laws.

- (a) Particles move along null geodesics;
- (b) They cannot appear from nothing or disappear (local conservation);
- (c) Their interaction reduces to the vertex shown in Fig. 10a.

At first glance it might seem that already the initial data shown in Fig. 10b — a single particle ready to fly into the time machine — constitute the desired paradox. Indeed, in the TDP space all



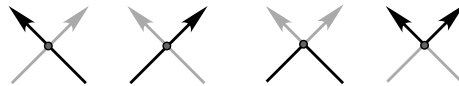
**Figure 10:** (a) The vertex defining dynamics in our toy model. (b) The ‘paradox’ appearing if the number of particles assumed to be conserved.

null geodesics entering the time machine have self-intersections. So, if the particle enters the time machine it must hit its younger self in the point  $O$  thus preventing the latter from getting into the time machine. But if this younger particle does change its trajectory and fly away, then where the older particle (the second participant of the collision) came from? A paradox, apparently.

<sup>7</sup>Not to be confused with the ‘grandfather paradox’, which is, strictly speaking, not a paradox at all [10].

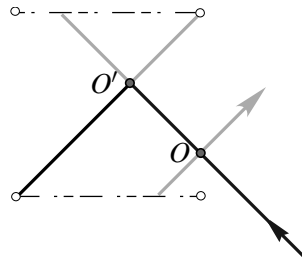
The entire reasoning, however, is based on the false implicit assumption that the number of particles is a conserved quantity. So, if there was a single particle on the surface  $t = t_0$ , then there must be only one particle on the surface  $t = t_1$  too. But this global conservation doesn't follow from our laws (a-c) and no reasons at all are seen to impose it as a separate additional law. And as soon as we abandon it, a nice solution appears satisfying all those laws: it contains *two* different particles. One of them has a closed world line. The other collides with that former one in  $O$  and bounces away from the time machine.

To exclude such a solution we can sophisticate the model. Let now the particles be of two different kinds: dark and light and the interaction is defined by this table



The laws look quite realistic in the sense that they are local and respect all the symmetries of the problem. And again at first glance it seems that there is no evolution from the initial data of Fig. 10b: one cannot assign in a consistent way a tint to the closed world line in the figure. Indeed, it is a single particle, so it must have the same tint all along, but according to the laws that we have adopted the incoming left and the outgoing left particles always have different tints.

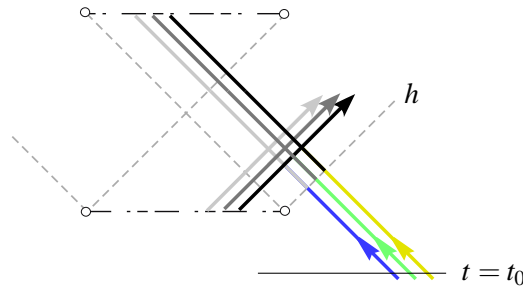
And still this is not a paradox yet. In Fig. 11 we see one of the admissible solutions. It



**Figure 11:** There are three light particles in this world: one is emitted in  $O$  and escapes to infinity and two others are born in the collision at  $O'$ . Of them one is absorbed in  $O$  and the other disappears in the singularity.

contains three dark and three light particles. The former include the particle coming from infinity to the point  $O$ , another one, emitted in  $O$  and absorbed in  $O'$ , and, finally, the particle emerging from the singularity to be absorbed in  $O'$ . This is a legitimate evolution satisfying all the laws formulated above.

Now we have discovered all pitfalls and the next sophistication brings us the desired paradox [10]. Namely, suppose that every particle in the world under discussion has one more characteristic, let us call it color. There are three different colors and the particles of different colors do not interact. Then the initial state shown in Fig. 12 does not correspond to any evolution. Indeed, any solution must have three self-intersecting world lines to the future of  $h$ . Only two of them may be affected by collisions with particles of the same color (the possible world lines of such particles are shown by dashed lines). So, the tint of at least one particle with the self-intersecting world line remains constant, which, as discussed above, contradicts the laws of interaction.



**Figure 12:** Any evolution must include the solid lines as world lines of particles. It also may include some of the dashed lines, but nothing else.

Of course, this model is *very* simple. Nevertheless, it strongly suggests that Conjecture 7 is false and in some cases the configuration of the matter fields may exclude the appearance of a time machine. If so, we come to a rather unexpected conclusion:

By arranging matter particles one can make some of geometrically admissible extensions of a given spacetime impossible.

In other words, the matter content of our world determines its geometry *not only* via Einstein’s equation [10].

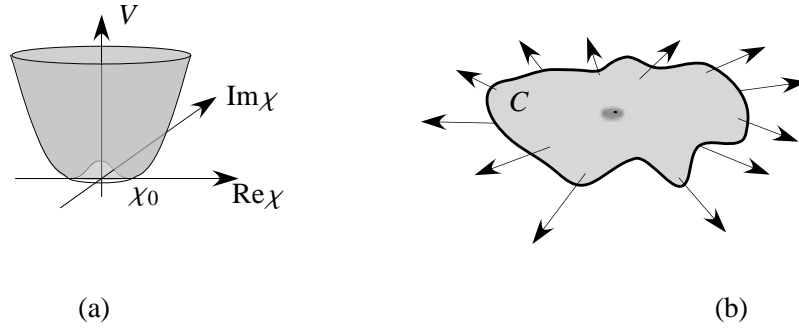
A pertinent consequence is that all one needs to create a quasiregular singularity is a wormhole and a rifle. Then forcing the universe to choose between the singularity and the time machine one can make at the same time the appearance of the latter impossible.

### 3.2 Primordial quasiregular singularities

Now let us turn to another relevant environment, which is the early Universe. In this section I briefly remind the reader the arguments suggesting that there are cosmic strings in nature (all details can be found in [11]) and then reproduce them in a purely gravitational case to show that the existence of string-like singularities (a variety of quasiregular singularities) is equally realistic.

#### 3.2.1 Cosmic strings

Suppose, after the universe had cooled below some critical temperature, a complex scalar field  $\chi$  appeared with the potential shown in Fig. 13a. One expects the field to take the value  $\chi_0$  corresponding to the minimal energy. But the evolutions of the field in different regions are *uncorrelated* and this can make the process of taking this value by the field energetically prohibited *globally* even though *locally* it is energetically favorable. Indeed, suppose on some loop  $C$  the field happened to take values like those depicted in Fig. 13a. Then, evidently, on a surface enclosed by  $C$  there will be a point at which the field vanishes. The energy density in that point will be, on the contrary, non-zero (because  $V(0) \neq 0$ ). Now we can deform the disk. The same reasoning will show that there is a non-vacuum spot on the new surface, too. So, what we have is actually not just a single spot of non-zero energy density but an endless curve (either infinite, or closed), or rather an endless thin tube, because they must be of finite — though small — thickness. The tubes are stable:



**Figure 13:** (a) The potential has the minimum at  $\chi_0 = |\chi_0|e^{i\sigma}$ , where  $\sigma$  is real, but otherwise arbitrary. (b) The arrows are complex numbers of the same modulus  $\chi_0$  (the values of  $\chi$ ) and not three-vectors.

even though they are surrounded by vacuum, they cannot dissolve for the topological reasons just discussed. It is such tubes that are called cosmic strings.

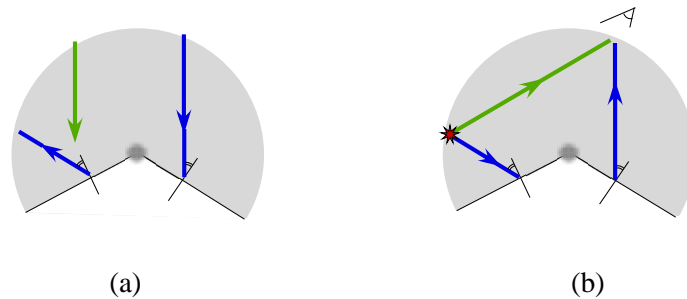
An important thing about cosmic strings is their gravitational fields. In particular, the universe with a straight cosmic string is believed to be described — *at large  $\rho$*  — by the spacetime depicted in Fig. 1b, or rather, by its four-dimensional analog

$$ds^2 = -dt^2 + dz^2 + d\rho^2 + \rho^2 d\varphi^2, \tag{3.1}$$

$$t, z \in \mathbb{R}, \quad \rho > 0, \quad \varphi = \varphi + 2\pi - d, \quad d \in (0, 2\pi).$$

In other words, at large  $\rho$  the straight cosmic string produces the same gravitational field as the singularity considered in the Introduction.

When a string moves through the cosmological fluid it leaves a wake behind it: as is seen from Fig. 14a two *parallelly* moving galaxies may nevertheless collide after a string has passed



**Figure 14:** Identify the thin line to obtain the section  $t = const, z = const$  of the spacetime with a cosmic string along the  $z$ -axis. In fact, such a section is just a flat cone with a smooth cap.

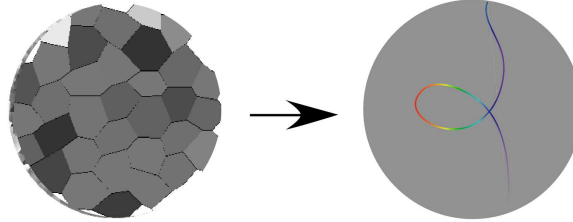
between them — a phenomenon of obvious importance to cosmology. On the other hand, two light rays emitted from the same source may, by exactly the same reasons, come to an observer from different directions, see Fig. 14b. Thus, a string acts as a gravitational lens producing multiple images of a single object. Note that both rays propagate in flat spacetime, so the images are neither distorted, nor fuzzy. They may differ, however, because we see the source from different angles.

### 3.2.2 String-like singularities [12]

Now let us apply all the above to the purely gravitational case. In the end of the Planck era the classical spacetime had emerged and started to expand obeying the Einstein equations.



By the time it could be confidently called classical it was practically flat (by Planck’s standards, anyway), so we can speak of the emergence (whatever it means) of a flat spacetime. One can think, however, that remote regions evolved uncorrelatedly and again the locally favorable process (of



**Figure 15:** Each patch tends to develop into the Minkowski space. But as they do so independently, global obstruction may appear resulting in formation of “topological defects”—string-like singularities.

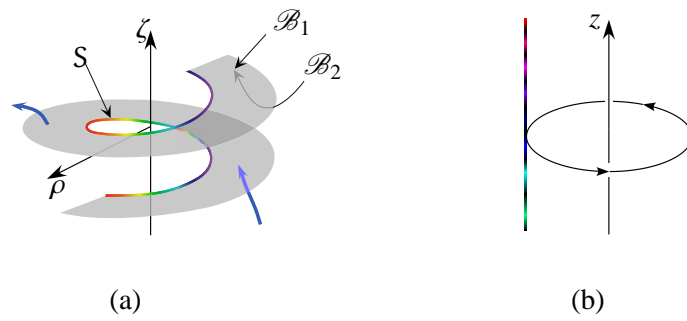
becoming Minkowskian) might be impeded by some *global* obstructions. Exactly as with matter fields, such obstructions must have given rise to singularities. This time, however, these would be true *geometrical* singularities and exactly of the type we are considering — singularities in the otherwise Minkowski spacetime.

Of course, this scenario contains some handwaving as we know nothing about how the classical space emerges, but the beauty of it is that we almost do not need to. Suppose, for example, that a circle lying in a newborn (non-simply connected) flat region happened to be too short (or too long) for its radius of curvature. Then we do not, in fact, need to know anything else to conclude that when the spacetime eventually becomes entirely classical and flat it will contain a quasiregular singularity (presumably, it will be similar to the straight-line one considered in the Introduction and will be represented by an endless — though not necessarily straight — line). This scenario seems so natural and convincing that I think the appearance of quasiregular singularities in the Early universe is at least as realistic as appearance of cosmic strings.

#### 4. Observations

Now suppose quasiregular singularities did appear in the Early Universe and survived to the present day. What do they look like? How do we detect them?

Unfortunately, there is still neither an exhaustive list of such singularities, nor even a detailed classification. Almost all we have is a number of examples, see [1] and references therein. One — often referred to as “straight string” — is the spacetime (3.1). A few more [13] are obtained by changing the way in which the two half-planes mentioned in the Introduction (see, Fig. 1b) are identified. If before gluing them together we shift (or boost) one of them with respect to the other, the properties of the resulting singularity will change significantly. Suffice it to say that if the shift is in the  $t$ -direction the spacetime will contain closed causal curves. The property shared by all these singularities is that they all in a sense are straight lines at rest. Recently, however, a number of singularities in flat spacetime were found [1] which are represented, in the same sense, by curved or moving lines. All such singularities including those “straight” ones I shall collectively call *string-like*.



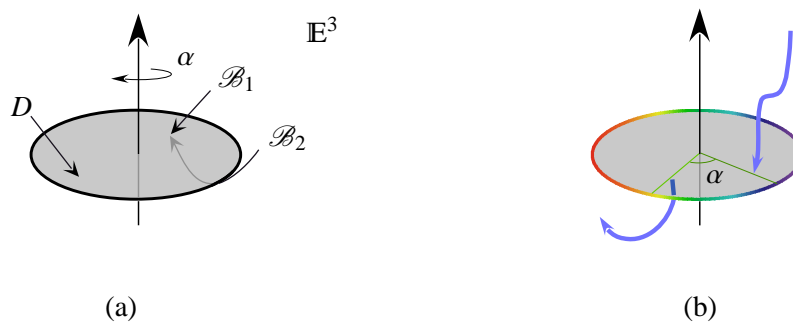
**Figure 16:** (a) The cut is made along the gray surface (which actually spreads to infinity). (b) The spiral singularity in the case  $\xi = t$ .

**8. Examples.** A) In the flat space  $\mathbb{R}^3$  consider the surface  $H$  given, in the cylinder coordinates, by the equation  $\phi = b\xi \pmod{2\pi}$ , where  $\rho > \rho_0 > 0$  and  $b \neq 0$ .  $H$  is (a half of) a helicoid without the core, see Fig 16a, and is bounded by the spiral

$$S: \quad \phi = b\xi \pmod{2\pi}, \quad \rho = \rho_0.$$

Make a cut in  $\mathbb{R}^3$  along  $CH$  (note that  $S$  is removed, too), rotate the lower bank of the slit — it is denoted by  $\mathcal{B}_2$  in the figure — anti-clockwise shifting it at the same time upward so that  $\mathcal{B}_1$  slides over  $\mathcal{B}_2$ , and paste the banks together again into a single surface. The resulting space  $\mathbb{M}^3$  is smooth, flat, etc., but it lacks the points that formed  $S$  (these points cannot be returned back insofar as the metric is required to be smooth). Thus,  $\mathbb{M}^3$  has a quasiregular singularity and this singularity, when  $\xi = z$ , has the form of a spiral (and is called, accordingly, *spiral* [1]). The structure of this singularity becomes more transparent when  $\mathbb{M}^3$  is depicted in the original coordinates  $z, \rho, \phi$  as in Fig. 16a. These coordinates are invalid, of course, on  $\mathcal{B}_{1,2}$ , that is why a smooth curve looks discontinuous in the picture. Another curious singularity is obtained if in the previous procedure one sets  $\xi = t$  (and the full spacetime is obtained by multiplying  $\mathbb{M}^3$  by the  $z$ -axis), see Fig. 16b. It is easy to see that the singularity in this case is represented by a straight line moving in quite a bizarre manner: it *circles around nothing*.

B) From the ordinary 3-dimensional Euclidean space  $\mathbb{E}^3$  remove a closed circle  $D$ . Then rotate one



**Figure 17:** The *loop* singularity can be view as a special case of the spiral one.

of the banks ( $\mathcal{B}_1$  in Fig. 17a) of the thus obtained slit w. r. t. the other ( $\mathcal{B}_2$ , correspondingly) by some  $\alpha$ , and, finally, glue the banks together. The resulting space is shown in Fig. 17b, where as

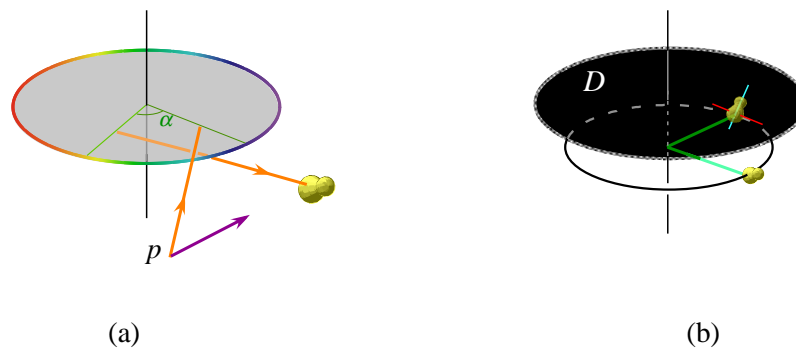
usual we use the old coordinates (so, the blue curve is actually continuous). The missing circumference  $S$  (as before, it cannot be glued back into the space) is a closed string-like singularity called *loop*. It is convenient to think of the spacetime as the Euclidean space minus the circumference plus the rule that a curve meeting the disk in some point  $p$  is continued from the point obtained from  $p$  by rotation by  $\alpha$ .

The relation between the string-like singularities and strings is summarized in the following table. In contrast to the strings, the singularities are in the *empty* space. So, their evolution is

	Cosmic strings	String-like singularities
Source	Matter	Empty space
Evolution	Governed by the field equations	?
Grav. field	Determined by (field eqs + Einstein eqs)	Rigid
Origin	Topological obstructions	
Cosmological rôle	Wakes	
Manifestations	Multiple images of a single source	

not connected to properties of any field and, in particular, is not described by the Nambu action. The cosmic strings, unlike the singularities, can bend, curving the spacetime around (in particular, they can emit gravitational waves). On the other hand, both entities have similar mechanisms of formation and stability, both produce wakes and, finally both may give rise to multiple images of a single source.

To appreciate the latter property consider the loop singularity from Example 8 and an observer

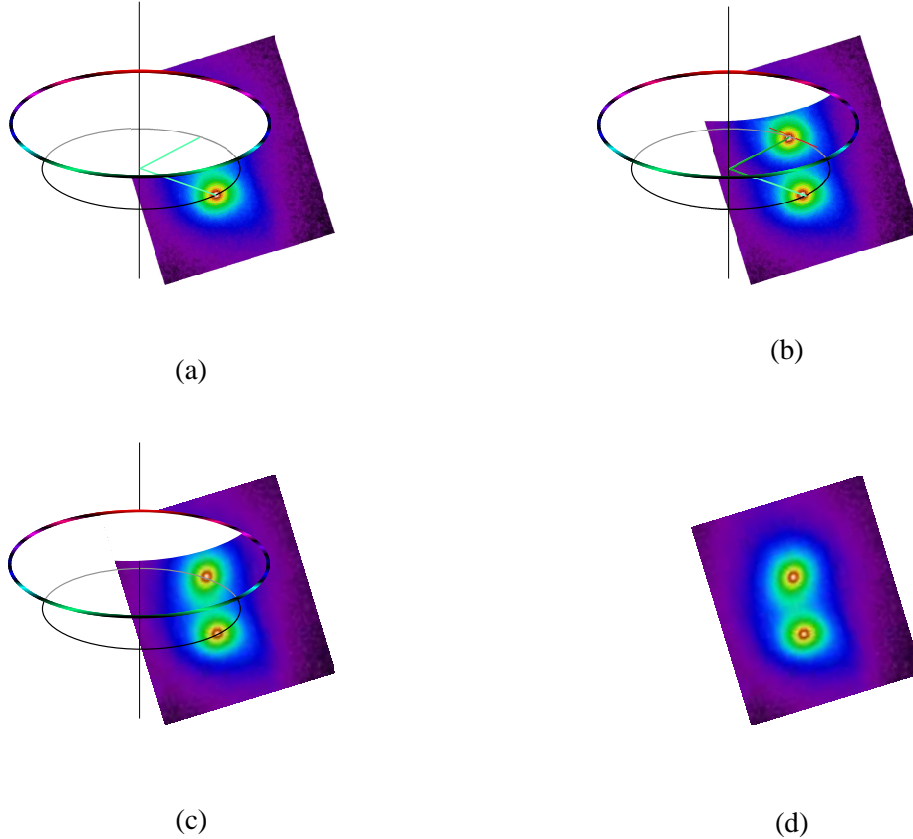


**Figure 18:** (a) The line of sight (the orange line) of the observer located in  $p$  ends up at the object which he also sees when looks in the direction shown by the purple arrow. (b) The object seen through the disk looks rotated (w. r. t. all three axes —blue, green, and red) in comparison with the same object observed directly.

who looks in the direction of the singularity, see Fig 18a. His line of sight after reaching the disk jumps, changes its direction and may end on an object which he could see somewhere aside. Thus

he will see *two* images of the same source — one in the direction shown by the orange ray and another in the direction of the purple ray. Equivalently, one can pick a source seen out of the disk and rotate it in one’s mind by  $\alpha$ , see Fig 18b. If it gets behind the disk one will see both images. Note that generally the images differ, because one sees the object from different angles.

Thus, suppose there is a loop string-like singularity somewhere. And suppose there is a galaxy not far from it, see Fig. 19a. Then following the prescription given above we rotate the galaxy by  $\alpha$



**Figure 19:** (c) is what we must see if there is a galaxy near a loop string-like singularity as in (a). (d) The image of the extragalactic object CSL-1.

w. r. t. the axis of the disk  $D$  corresponding to the singularity, and find that (for a suitable size of the loop and value of  $\alpha$ ) a terrestrial observer would see two images of the galaxy, see Fig. 19b. The second image must be rotated w. r. t. to the other one to make allowance for the effect mentioned above, so, it will look as in Fig. 19c. Thus, if there are galactic size loop string-like singularities, one may expect to observe in the sky something like that depicted in Fig. 19c. And Fig. 19d is the *real image* of the extragalactic object CSL-1 obtained with the Hubble Space Telescope, see [14] and references therein. Apparently the object is a pair of giant galaxies with the same velocities, with the same (at the 98% c.l.) spectra and with no explanation<sup>8</sup> for this similarity other than sheer coincidence.

<sup>8</sup>As is argued in [14] this cannot be a result of lensing by a compact object (because the isophotes are undistorted) or by a straight string (because no other pairs are found nearby).

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