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A generating functional for equal-time correlation functions

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Our aim is to put the partially successful analytic noncovariant approaches to Coulomb gauge QCD on a firm and systematic basis. To this end, we develop a generating functional approach to the equal-time correlation functions. In fact, such a functional is given in terms of the vacuum wave functional, however, in a perturbative expansion of the equal-time correlation functions, the vacuum wave functional has to be known to the corresponding order. As a consequence, there are many contributions that correspond to one and the same Feynman diagram in the covariant theory. A remarkable simplification occurs on summing up these different contributions. We comment on the possibility of formulating new diagrammatic rules directly for the sum of all contributions that correspond to the same Feynman diagram.

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Recently, there has been much interest in the formulation of QCD in Coulomb gauge [1]. For the terms that are expected to be dominant in the deep IR limit, an intriguing relation between Landau and Coulomb gauge QCD has been found [2]: equal-time correlation functions in Coulomb gauge appear as the strict three-dimensional counterpart of the covariant correlation functions in (four-dimensional) Landau gauge. The latest numerical evaluation of equal-time correlation functions in the Coulomb gauge [3] seems to confirm this scenario.

In this contribution, we will look, in a more general setting, into the representation of equal-time correlation functions that is analogous to covariant correlation functions. In fact, such a representation can be written down immediately: given that the equal-time correlation functions are nothing but the true vacuum expectation values of products of the field operators, the Schrödinger representation of the field theory directly yields a generating functional for these correlation functions where the (absolute) square of the true vacuum wave functional plays the rôle of the exponential of the negative of the Euclidean classical action in the corresponding generating functional of the covariant correlation functions (in Euclidean space).

In order to write down the generating functional for the equal-time correlation functions explicitly, we hence need an explicit expression for the vacuum wave functional. To make progress, we will consider a definite theory, $\lambda\phi^4$ theory in this contribution. We make an exponential ansatz for the vacuum wave functional which is suggested by the covariant analogue. Considering a full Volterra expansion of the exponent, leaving out all odd powers of ϕ in the $(\phi \to -\phi)$ symmetric case, the Schrödinger equation for the wave functional leads to a tower of equations for the coefficient functions in the Volterra expansion similar to Dyson-Schwinger equations upon equating the coefficients of corresponding powers of ϕ multiplying the exponential on both sides of the Schrödinger equation.

This tower of equations can be solved iteratively in a perturbative expansion in a unique way, starting with the vacuum functional of the noninteracting theory (which provides the bare "propagator" for the equal-time correlation functions) and assuming that the lowest-order contribution to the coefficient function of the power ϕ^{2k} is of the order λ^{k-1}. Although it is difficult to formulate the diagrammatic Feynman rules for this perturbative expansion, it is important for the following to associate diagrams to the different contributions in order to keep track of them. This can be done in a natural way, and one obtains a one-to-one correspondence of the expressions arising in the iterative solution with the connected diagrams of covariant perturbation theory.

The perturbative series for the vacuum wave functional can now be used to calculate the equal-time correlation functions, also in a perturbative expansion. The procedure is analogous to covariant perturbation theory, only that one has an infinite set of vertices given by the coefficient functions in the Volterra expansion of the exponent of the (absolute) square of the vacuum wave functional. Every one of these vertices has a perturbative expansion by itself that involves all orders in powers of λ. Then to a given n-point equal-time correlation function to a fixed order λ^ℓ there are contributions from several vertex functions to different orders. The diagrammatic representation of the correlation functions in terms of the "propagator" and vertices coming from the Volterra expansion can be merged with the diagrammatic representation of the coefficients of the latter expansion. As a result, we get several different contributions in this merged representation that correspond to one and the same Feynman diagram in covariant perturbation theory. Quite amazingly, summing the different contributions that correspond to the same diagram leads to a remarkable simplification in the mathematical expressions. It even seems possible to establish simple diagrammatic rules for the sums of the contributions corresponding to the same Feynman diagrams in the covariant theory, contrary to the individual contributions themselves. These rules involve a formal manipulation that we have termed the "E operator" which does not seem to have arisen in perturbation theory before.

We have determined the explicit expressions for the equal-time correlation functions in $\lambda\phi^4$ theory up to two-loop order for the 2-point and to one-loop order for the 4-point functions, and in Coulomb gauge Yang-Mills theory using the Christ-Lee Hamiltonian, for the gluon and ghost 2-point functions up to one-loop order. We have compared the results for these n-point equal-time correlation functions with the corresponding covariant correlation functions calculated within covariant perturbation theory (to the same loop order) and projected to equal times ($t = 0$) by integrating over the energy variables in the momentum representation. In the case of Coulomb gauge Yang-Mills theory, we have used the recent results [4] for the covariant correlation functions (of course, covariance is broken explicitly by the Coulomb gauge condition). It is worth mentioning that the actual determination of the mathematical expressions for the n-point equal-time correlation functions for $n \geq 4$ is *much* simpler in the way we have described here than by integrating the expressions for the covariant correlation functions over the energy variables. If the simple rules involving the E operator can be shown to extend to all loop orders and all n-point functions, the mathematical expressions for the equal-time correlation functions can be written down at once.

Our last comment concerns the possibility of writing down equations of Dyson-Schwinger type for equal-time correlation functions which is of interest particularly in the deep IR limit of QCD. In principle, such equations can directly be derived from the generating functional described above. However, the infinite number of vertices stemming from the Volterra expansion leads to an infinite number of terms in every equation (in the infinite tower of equations). On the other hand, the possible simplification by use of the E operator may lead to the formulation of a tower of equations involving a finite number of terms at the cost of a slightly more complicated struture.

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