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Nuclear Lattice Simulations with Chiral Effective Field Theory

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We present recent results on lattice simulations using chiral effective field theory. In particular we discuss lattice simulations for dilute neutron matter at next-to-leading order and three-body forces in light nuclei at next-to-next-to-leading order.

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Lattice simulations based on the framework of effective field theory have been used in studies of nuclear matter [1] and neutron matter [2, 3, 4]. The method has also been used to study light nuclei in pionless effective field theory [5] and chiral effective field theory at leading order (LO) [6]. More recently next-to-leading order (NLO) calculations have been carried out for the ground state of neutron matter [7, 8]. A review of lattice effective field theory calculations can be found in Ref. [9]. In this proceedings article we describe some new results which, at the time of this writing, are not yet published.

At leading order in chiral effective field theory the nucleon-nucleon effective potential is

$$V_{\rm LO} = V + V_{I^2} + V^{\rm OPEP}. \tag{1}$$

V, V_{I^2} are the two independent contact interactions at leading order in the Weinberg power counting scheme, and $V^{\rm OPEP}$ is the instantaneous one-pion exchange potential. The interactions in $V_{\rm LO}$ can be described in terms of their matrix elements with two-nucleon incoming and outgoing momentum states. For bookkeeping purposes we label the amplitude as though the two interacting nucleons were distinguishable, A and B. In the following $\vec{q}$ denotes the t-channel momentum transfer while $\vec{k}$ is the u-channel exchanged momentum transfer. We use τ_I with $I = 1,2,3$ to represent Pauli matrices acting in isospin space and σ_S with $S = 1,2,3$ to represent Pauli matrices acting in spin space.

For the two leading-order contact interactions the amplitudes are

$$\mathscr{A}(V) = C, \tag{2}$$

$$\mathscr{A}\left(V_{I^2}\right) = C_{I^2} \sum_I \tau_I^A \tau_I^B. \tag{3}$$

For the one-pion exchange potential,

$$\mathscr{A}\left(V^{\rm OPEP}\right) = -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{\sum_I \tau_I^A \tau_I^B \sum_S q_S \sigma_S^A \sum_{S'} q_{S'} \sigma_{S'}^B}{q^2 + m_\pi^2}. \tag{4}$$

For our physical constants we take $m = 938.92$ MeV as the nucleon mass, $m_\pi = 138.08$ MeV as the pion mass, $f_\pi = 93$ MeV as the pion decay constant, and $g_A = 1.26$ as the nucleon axial charge.

In Ref. [6] two different lattice actions were considered which were later denoted LO_1 and LO_2 [7]. The interactions in V_{LO_1} include one-pion exchange and two zero-range contact interactions corresponding with amplitude

$$\mathscr{A}\left(V_{\mathrm{LO}_1}\right) = C + C_{I^2} \sum_I \tau_I^A \tau_I^B + \mathscr{A}\left(V^{\rm OPEP}\right). \tag{5}$$

The interactions in V_{LO_2} consist of one-pion exchange and two Gaussian-smeared contact interactions,

$$\mathscr{A}\left(V_{\mathrm{LO}_2}\right) = C f(\vec{q}) + C_{I^2} f(\vec{q}) \sum_I \tau_I^A \tau_I^B + \mathscr{A}\left(V^{\rm OPEP}\right), \tag{6}$$

where $f(\vec{q})$ is a lattice approximation to a Gaussian function. The smeared interactions in LO_2 are used to better reproduce S-wave phase shifts for nucleon momenta up to the pion mass. The coefficients C and C_{I^2} are tuned to reproduce the physical S-wave scattering lengths. In Ref. [7]

nucleon-nucleon phase shifts were calculated for these two lattice actions using the spherical wall method [10] at spatial lattice spacing $a = (100\ \mathrm{MeV})^{-1}$ and temporal lattice spacing $a_t = (70\ \mathrm{MeV})^{-1}$. For each case NLO corrections were also computed perturbatively and the unknown operator coefficients determined by fitting to low-energy scattering data.

In Ref. [8] the ground state energy for dilute neutron matter was computed using the lattice action LO_2 and auxiliary-field Monte Carlo. Next-to-leading-order corrections to the energy were also calculated perturbatively. In this calculation the largest source of systematic error was the large size of NLO corrections for Fermi momenta larger than 100 MeV. This was due to attractive *P*-wave interactions generated by Gaussian smearing in LO_2 that needed to be cancelled at next-to-leading order. In systems with both protons and neutrons this *P*-wave correction is numerically small when compared with the strong binding produced by *S*-wave interactions. For pure neutron matter, however, the *S*-wave interactions produce much less binding due to Fermi repulsion. Therefore on a relative scale, the *P*-wave interactions are not as small an effect in neutron matter.

These problems have been resolved using a new leading-order action LO_3 [20]. The interactions in V_{LO_3} correspond with the amplitude,

$$\begin{aligned}\mathscr{A}\left(V_{\mathrm{LO}_3}\right) = {} & C_{S=0,I=1} f(\vec{q}) \left(\frac{1}{4} - \frac{1}{4}\sum_S \sigma_S^A \sigma_S^B\right)\left(\frac{3}{4} + \frac{1}{4}\sum_I \tau_I^A \tau_I^B\right) \\ & + C_{S=1,I=0} f(\vec{q}) \left(\frac{3}{4} + \frac{1}{4}\sum_S \sigma_S^A \sigma_S^B\right)\left(\frac{1}{4} - \frac{1}{4}\sum_I \tau_I^A \tau_I^B\right) + \mathscr{A}\left(V^{\mathrm{OPEP}}\right). \end{aligned} \tag{7}$$

The Gaussian-smeared interactions are multiplied by spin and isospin projection operators. Only the $C_{S=0,I=1}$ term contributes in pure neutron matter. Using the LO_3 action with NLO corrections, we have computed the ground state energy for dilute neutrons in a periodic box [20]. For spatial lattice spacing $a = (100\ \mathrm{MeV})^{-1}$ and temporal lattice spacing $a_t = (70\ \mathrm{MeV})^{-1}$ simulations were done with 8, 12, 16 neutrons in periodic boxes with lengths $L = 4, 5, 6, 7$. In Fig. 1 we show results for the ratio of the interacting ground state energy to non-interacting ground state energy, E_0/E_0^{free}, as a function of Fermi momentum k_F. For comparison we show other results from the literature: FP 1981 [11], APR 1998 [12], CMPR $v6$ and $v8'$ [13], SP 2005 [14], GC 2007 [15], and GIFPS 2008 [16].

At next-to-next-to-leading order (NNLO) in chiral effective field theory we find contributions due to three-nucleon forces. These interactions consist of a pure contact interaction, one-pion exchange, and two-pion exchange [17]. The coupling of one or more pions to a single nucleon is constrained by chiral symmetry and the corresponding low energy constants are known [18]. In the limit of exact isospin symmetry there are only two unknown coefficients, one for the three-nucleon contact interaction and one for the two-nucleon-pion vertex involved in the one-pion exchange interaction. At fixed lattice spacing we have determined these two unknown coefficients by fitting to the triton binding energy and spin-doublet nucleon-deuteron scattering phase shifts via Lüscher's finite volume formula [19]. Results for the doublet nucleon-deuteron scattering phase shift are shown in Fig. 2 using the LO_2 lattice action for lattice spacing $a = (100\ \mathrm{MeV})^{-1}$ and temporal lattice spacing $a_t = (150\ \mathrm{MeV})^{-1}$ [20].

Having determined the NNLO three-body forces, we have computed the ground state of the alpha particle without Coulomb interactions on a periodic lattice using auxiliary-field projection

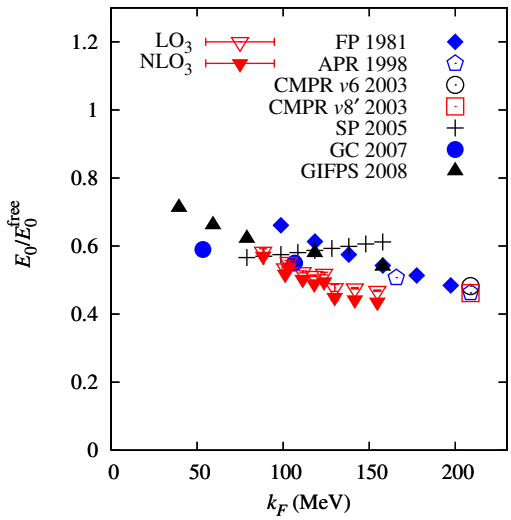

Figure 1: Ground state energy ratio $E_0/E_0^{\rm free}$ for LO_3 and NLO_3 versus Fermi momentum k_F. For comparison we show results for FP 1981 [11], APR 1998 [12], CMPR $v6$ and $v8'$ 2003 [13], SP 2005 [14], GC 2007 [15], and GIFPS 2008 [16].

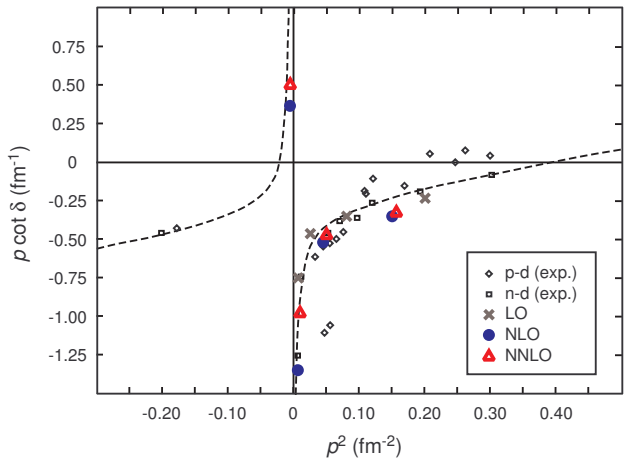

Figure 2: Results for the spin-doublet nucleon-deuteron scattering phase shift at LO, NLO, and NNLO.

Monte Carlo [20]. The NNLO results are within 5% of the actual Coulomb-subtracted alpha binding energy of about 29 MeV. This is consistent with the expected size of errors for our chosen lattice spacing and order in effective field theory.

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