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Chiral symmetry in excited baryons

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Chiral symmetry is expected to be approximately restored in excited baryons, which is a consequence of quite general symmetry based arguments. However, such a restoration cannot be described in the framework of naive constituent quark models. Then chiral potential quark model (Generalised Nambu-Jona-Lasinio model) is used to draw a microscopic picture of such a restoration: the axial charge operator is built explicitly, the behaviour of the diagonal and off-diagonal axial baryon charges as well as the effective decoupling of pions are discussed in detail.

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There exist strong theoretical arguments to believe that chiral symmetry, which is broken spontaneously in low-lying hadrons, should be approximately asymptotically restored in highly excited states [1] (see also the review [2] and references therein). Indeed, all quantum fluctuations, including the effect of chiral symmetry breaking, should progressively disappear as we climb the excitation number staircase of hadronic states [3, 4]. To gain an insight into mechanisms of such a restoration, one can rely upon models, though such models have to meet, at least, the minimal set of requirements necessary in order to be able to address problems related to chiral symmetry. A suitable model must be relativistic, field-theory-inspired (as opposed to quantum mechanical models with a conserved number of particles), chirally symmetric, and able to describe microscopically the phenomenon of spontaneous breaking of chiral symmetry. In particular, the problems related to chiral symmetry are well suited to be treated with the help of the Generalised Nambu–Jona-Lasinio (GNJL) model [5, 6]. A pattern of chiral symmetry restoration in excited heavy–light mesons in the framework of GNJL model was studied in detail in [7, 8, 9] and its connection to the classical limit of the model was discussed in [4]. The Hamiltonian of the model has the form:

$$H=\sum_{i=u,d}\int d^3x\psi_i^\dagger(\mathbf{x})\left(-i\boldsymbol{\alpha}\nabla+\beta m_i\right)\psi_i(\mathbf{x})+\frac12\sum_{i,j=u,d}\int d^3xd^3y\,J^a_{i\mu}(\mathbf{x})K^{ab}_{\mu\nu}(\mathbf{x}-\mathbf{y})J^b_{j\nu}(\mathbf{y}), \qquad (1)$$

where $J^a_{i\mu}(x)=\bar\psi_{i\alpha}(x)\gamma_\mu\left(\frac{\lambda^a}{2}\right)^\alpha_\beta\psi_i^\beta(x)$ and $K^{ab}_{\mu\nu}(x-y)=\delta^{ab}K_{\mu\nu}(|\mathbf{x}-\mathbf{y}|)$ is a confining quark kernel. The quark field $\psi_i^\alpha(\mathbf{x})$ is given by

$$\psi_i^\alpha(\mathbf{x})=\sum_{s=\uparrow,\downarrow}\int\frac{d^3p}{(2\pi)^3}e^{i\mathbf{px}}[b^\alpha_{ips}u_s(\mathbf{p})+d^{\alpha\dagger}_{ips}v_s(-\mathbf{p})], \qquad (2)$$

with the quark bispinors $u(\mathbf{p})$ and $v(\mathbf{p})$,

$$\begin{aligned} u(\mathbf{p}) &= \frac{1}{\sqrt2}\left[\sqrt{1+\sin\varphi_p}+(\boldsymbol{\alpha}\hat{\mathbf{p}})\sqrt{1-\sin\varphi_p}\right]u_0(\mathbf{p}),\\ v(-\mathbf{p}) &= \frac{1}{\sqrt2}\left[\sqrt{1+\sin\varphi_p}-(\boldsymbol{\alpha}\hat{\mathbf{p}})\sqrt{1-\sin\varphi_p}\right]v_0(-\mathbf{p}), \end{aligned} \qquad (3)$$

defined through the rest-frame bispinors $u_0(\mathbf{p})$ and $v_0(-\mathbf{p})$ and a function of the interquark momentum φ_p called the chiral angle. The latter is conveniently defined such that $-\pi/2<\varphi_p\leqslant\pi/2$ and $\varphi(0)=\pi/2$, $\varphi(p\to\infty)\to0$. It may differ for different flavours of quarks. In the meantime, in what follows we assume an exact $SU(2)_f$ symmetry and work in the chiral limit, so that $m_u=m_d=0$ and $\varphi_p^{(u)}=\varphi_p^{(d)}\equiv\varphi_p$. If the chiral angle is set equal to zero, then the w.f. (2) describes free quarks, while nontrivial chiral angles describe dressed quarks. The actual profile of the chiral angle is defined by the requirement that the Hamiltonian (1), normally ordered in terms of the quark creation and annihilation operators, does not contain off-diagonal terms of the form $b^\dagger d^\dagger$ and db. The corresponding equation for the $\varphi_p$is known as the mass-gap equation [5, 6]. For the trivial solution of the mass-gap equation $\varphi_p^{\rm triv}\equiv0$ the Hamiltonian (1) is invariant under the chiral transformation $\psi_i\to[\exp(i\alpha\gamma_5\tau^a/2)]^{ij}\psi_j$. However, it was found long ago [5, 6] that the true vacuum state of the theory (1) is given by a nontrivial solution $\varphi_p\neq0$. This true vacuum has the form of a coherent-like state with condensed ${}^3P_0$ quark–antiquark pairs. The corresponding condensate,

$$\langle\bar\psi\psi\rangle_u=\langle\bar\psi\psi\rangle_d=-\frac{N_C}{\pi^2}\int_0^\infty dp\,p^2\sin\varphi_p\neq0, \qquad (4)$$

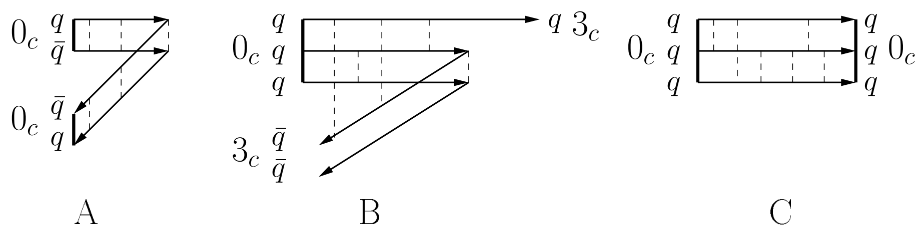

Figure 1: Diagram A: a typical allowed (singlet $0_c \to 0_c$) quark-antiquark pair annihilation transition from mesonic positive-energy to negative-energy Salpeter amplitudes; Diagram B: a similar pair annihilation cannot proceed as it will involve non-singlets 3_c in colour; Diagram C: a typical diagram pertaining to the Dyson ladder for baryons.

violates chiral symmetry and is nothing but the standard chiral condensate. In this vacuum, the Hamiltonian (1), takes a diagonal form

$$H = E_{\rm vac} + \sum_{i=u,d} \int \frac{d^3p}{(2\pi)^3} E_p [b^\dagger_{i\alpha ps} b^\alpha_{ips} + d^{i\alpha\dagger}_{ps} d_{i\alpha ps}] + \ldots, \tag{5}$$

where E_p stands for the dressed-quark dispersive law and the ellipsis denotes the terms which are responsible for the formation of bound states of dressed quarks — hadrons. One can see therefore, that the GNJL model gives an explicit microscopic description of the effect of spontaneous breaking of chiral symmetry.

Hadronic states can be built now from dressed quarks through a generalised Bogoliubov transformation [10] or with the help of a Bethe–Salpeter equation [5, 6]. Notice however an important difference between bound-state equations for mesons and baryons. Indeed, the quark currents interaction contained in the Hamiltonian (1) couples time-forward (positive-energy) and time-backward (negative-energy) amplitudes for the quark–antiquark pair in a meson (see Fig. 1A), so that the corresponding bound–state equation has the form of a system of two coupled equations for a two-component meson w.f. [5, 6]. In the meantime, similar diagrams are forbidden for baryons (see Fig. 1B), so that only diagrams of the type depicted in Fig. 1C are allowed in case of baryons. The Bethe–Salpeter equation reduces then to a single Schrödinger-like equation for the baryon w.f.

$$\Psi_B = \Psi_{\rm colour} \otimes \Psi_{\rm flavor} \otimes \Psi_{\rm spin} \otimes \Psi_{\rm space}, \quad \Psi_{\rm colour} = \frac{1}{3!}\varepsilon_{\alpha\beta\gamma} q^\alpha q^\beta q^\gamma. \tag{6}$$

Let us study now the problem of chiral symmetry restoration in highly excited baryons in detail. To this end one can follow a straightforward procedure of building the spectrum of bound states [7, 8, 11]. However, in this work [12], we choose a different strategy and study the behaviour of the Noether charge for the global chiral symmetry,

$$\mathscr{Q}_5 \equiv \mathscr{Q}_5^3 = \sum_{n=1}^{3} Q^3_{5n}, \tag{7}$$

where index n numerates quarks in the baryon, so that the baryon total axial charge is given by the sum of three individual charges, one for each quark, written in terms of the quark field (from now

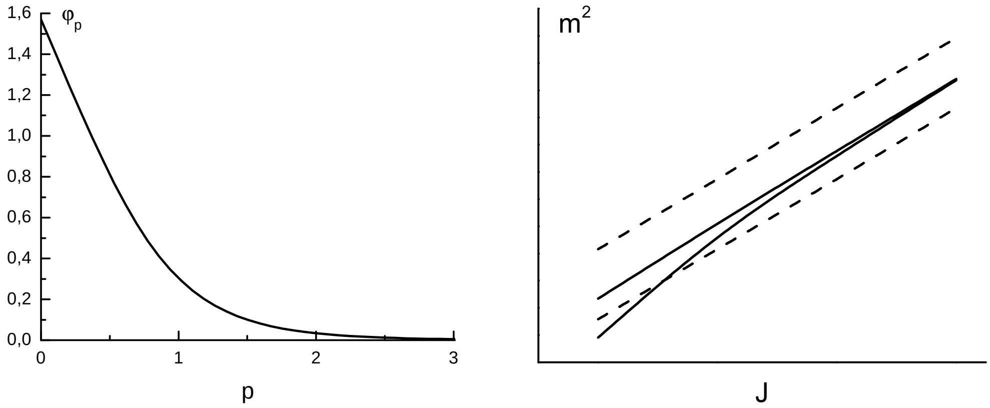

Figure 2: Left plot: a typical profile of the chiral angle — solution to the mass-gap equation which defines the broken vacuum. The momentum p is measured in the units of strength of the interquark potential. Right plot: a typical behaviour of the Regge trajectories for chiral partners in a naive quark model (dashed lines) and in GNJL (solid lines)

onwards a stands for the index of the flavor τ matrices):

$$Q_5^a = \int d^3x\, \bar{\psi}_i \gamma_0 \gamma_5 \left(\frac{\tau^a}{2}\right)^{ij} \psi_j = \left(\frac{\tau^a}{2}\right)^{ij} \int \frac{d^3p}{(2\pi)^3} \Big[\cos\varphi_p (\boldsymbol{\sigma}\hat{\mathbf{p}})_{ss'} \left(b^\dagger_{i\alpha ps} b^\alpha_{jps'} + d^{\alpha\dagger}_{jps} d_{i\alpha ps'}\right) + \sin\varphi_p (i\sigma_2)_{ss'} \left(b^\dagger_{i\alpha ps} d^{\alpha\dagger}_{jps'} + d_{i\alpha ps} b^\alpha_{jps'}\right)\Big]. \quad (8)$$

The axial charge (8) creates a nontrivial state, $Q_5^a|0\rangle = |\pi^a\rangle$. Being a Noether charge, the axial charge commutes with the Hamiltonian, $[Q_5^a, H] = 0$, which ensures that the state $|\pi^a\rangle$ is degenerate in energy with the vacuum. This is the Goldstone boson — the chiral pion [13].

It is obvious from Eq. (8) that this axial charge has the properties ($B_{1,2}$ being baryons):

$$\mathscr{Q}_5^\dagger = \mathscr{Q}_5, \quad \langle B_2|\mathscr{Q}_5^2|B_1\rangle \propto \langle B_2|B_1\rangle = \delta_{B_1B_2} \quad (9)$$

and acts on baryonic states in a two–fold way, which can be schematically written in the form ($|\pi\rangle$ denotes the neutral pion):

$$\mathscr{Q}_5|B\rangle = |B'\rangle + |B\pi\rangle, \quad (10)$$

where the relative strength of the first and second terms in (10) is given by $\cos\varphi_p$ and $\sin\varphi_p$, respectively, integrated with the corresponding baryonic w.f. In the left panel of Fig. 2 a typical profile of the chiral angle — solution to the mass-gap equation is depicted. Clearly, for highly excited baryons, the mean interquark momentum grows and the chiral angle decreases. Therefore, the chiral pion decouples from excited baryons (see also [14] for a detailed discussion of the pion decoupling from excited hadrons in the framework of GNJL) and we approach the limit:

$$\mathscr{Q}_5|B^\pm\rangle = G^A_{\pm\mp}|B^\mp\rangle, \quad (11)$$

with $B^\pm$ representing baryons with the parity $\pm$ and $G^A_{\pm\mp}$ being a c-number axial charge. The latter relation, together with the fact that $[\mathscr{Q}_5, H] = 0$, ensures that, in the chiral limit, the two states $|B^+\rangle$ and $|B^-\rangle$ must be nearly degenerate in mass. They form an approximate chiral doublet. In the right panel of Fig. 2 we show a typical behaviour of the Regge trajectories corresponding to chiral partners in a naive — for example, given by a simple Hamiltonian $H=\sum_{n=1}^{3}\sqrt{p_n^2}+V(r_1,r_2,r_3)$ — quark model (dashed line) and in the GNJL (solid line). The phenomenon of chiral restoration is clearly demonstrated by merging GNJL Regge trajectories, while naive quark models are simply unable to describe this effect. Finally, from (11), we arrive at the following approximate relations for the baryonic diagonal and off-diagonal axial charges:

$$G^A_{+-}=G^A_{-+}\simeq 1,\quad G^A_{++}=G^A_{--}\simeq 0, \tag{12}$$

which have been obtained microscopically.

We conclude therefore that the GNJL model gives a clear and selfconsistent pattern of effective chiral symmetry restoration in excited baryons (see the discussion in [2]) and, what is more, it provides a full *microscopic* picture of this phenomenon, which, as a matter of principle, cannot be reproduced by any naive quark model or approach [15].

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