PROCEEDINGS OF SCIENCE

Coulomb gauge ghost propagator and the Coulomb potential*

Markus Quandt†
University of Tübingen
E-mail: quandt@tphys.physik.uni-tuebingen.de

Giuseppe Burgio
University of Tübingen
E-mail: burgio@tphys.physik.uni-tuebingen.de

Songvudhi Chimchinda
University of Tübingen and Suranaree University of Technology
E-mail: chimchinda@tphys.physik.uni-tuebingen.de

Hugo Reinhardt
University of Tübingen
E-mail: hugo.reinhardt@uni-tuebingen.de

The ghost propagator and the Coulomb potential are evaluated in Coulomb gauge on the lattice, using an improved gauge fixing scheme which includes the residual symmetry. This setting has been shown to be essential in order to explain the scaling violations in the instantaneous gluon propagator. We find that both the ghost propagator and the Coulomb potential are insensitive to the Gribov problem or the details of the residual gauge fixing, even if the Coulomb potential is evaluated from the A_0–propagator instead of the Coulomb kernel. In particular, no signs of scaling violations could be found in either quantity, at least to well below the numerical accuracy where these violations were visible for the gluon propagator. The Coulomb potential from the A_0-propagator is shown to be in qualitative agreement with the (formally equivalent) expression evaluated from the Coulomb kernel.

8th Conference Quark Confinement and the Hadron Spectrum
September 1-6, 2008
Mainz. Germany

*Supported by DFG under contract *Re-856/6-1,2*
†Speaker.

1. Introduction

Yang–Mills theory in the Coulomb gauge has recently drawn a renewed attention, both in the continuum [1, 2, 3] and on the lattice [5, 4, 6, 7, 8]. This is mainly due to the fact that Gauß' law can be resolved explicitly in this gauge, which allows for a neat Hamiltonian formulation with the transversal part of the remaining vector potential $\mathbf{A}^{\perp}$ as the only physical degree of freedom. Much of the intuition and techniques from ordinary quantum mechanics can thus be carried over to the YM case. In particular, recent variational approaches in the Schrödinger picture, based on the notion of a weakly interacting constitutent gluon and the Gribov–Zwanziger confinement scenario [12], proved to be very successfull [3]; similar calculations are presently carried out in the renormalisation flow approach.

All these continuum formulations, in one way or the other, give rise to relations between low-order Green functions of the constituent gluon $\mathbf{A}$ and the Faddeev–Popov ghosts. It is therefore important to obtain non-perturbative information on such correlators from the lattice. Careful studies of the equal–times gluon propagator, for instance, reveal strong scaling violations and a UV behaviour at odds with simple dimensional arguments [5, 7, 6]. These surprising results reflect the renormalisation problems for instantaneous correlators in the continuum. One possible explanation of the lattice findings [9] is based on the idea that the residual gauge freedom left over by the Coulomb condition must be fixed in such a way that it resembles the Hamiltonian formulation as closely as possible.[1] A careful study of the energy dependence of the gluon propagator then allows to manipulate the data such that perfect scaling is observed even on finite lattices.

For the confinement scenario layed out by Gribov and Zwanziger[12], the more important correlators are, of course, the ghost propagator and, in particular, the Coulomb potential. Furthermore, the ghost form factor has been shown to represent the inverse of the colour dielectric function of the Yang–Mills vaccum [13], and is therefore of direct physical relevance. Initial studies of the ghost and Coulomb propagator for the gauge group $G = SU(2)$ with simple Coulomb and no residual gauge fixing [5] found no scaling violations at low momenta, but had inconclusive results about the Coulomb string tension in the deep infrared. Moreover, these results were partially at odds with more careful $SU(3)$ studies using a residual gauge fixing different from ours [11], which featured a peculiar saddle-like behaviour in the Coulomb potential at low momenta. In the present talk, I will report about recent $SU(2)$ calculations of ghost form factors and the Coulomb potential, using exactly the same gauge fixing techniques which proved essential for the resolution of the scaling violations in the gluon propagator.

2. Gauge Fixing

Our gauge fixing procedure employs both simulated annealing and the microcanonical flip procedure layed out in [7] as a preconditioning with subsequent (over)relaxation to complete the gauge fixing within machine precision. To reduce the Gribov noise and bring the lattice configs closer to the fundamental modular region, we perform up to 40 restarts with random gauge transformations as starting points, and take the copy with the best minimum of the gauge fixing functional. While this procedure proved to be important for the correct extraction of the gluon propagator in

[1] For the first-order formalism in the continuum, renormalisability has been proven algebraically [10].

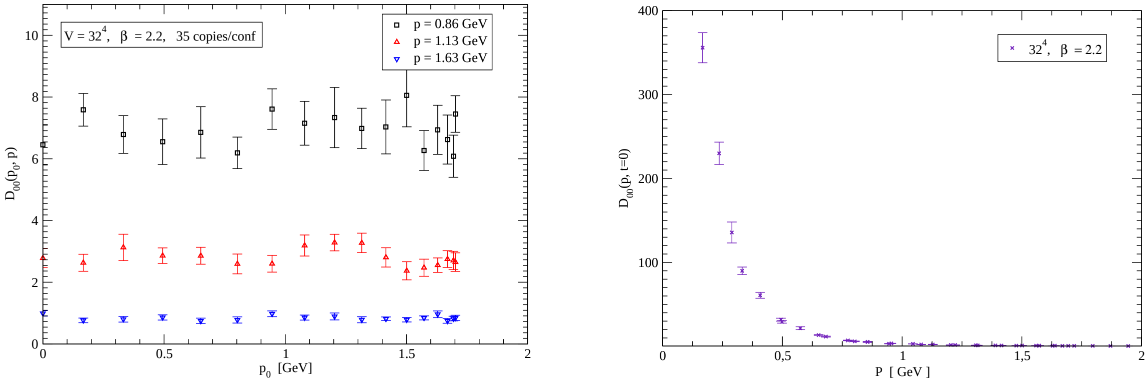

Figure 1: Left panel: Energy dependence of the A_0–propagator $D_{00}(\mathbf{p}, p_0)$ after improved Coulomb and resiudal gauge fixing, for various spatial momenta $|\mathbf{p}|$. Right panel: The equal-times A_0–propagator $D_{00}(\mathbf{p}, t = 0)$ as a function of the spatial momentum $|\mathbf{p}|$.

the deep infrared [7], the ghost correlators exhibit a much weeker dependence on the quality of gauge fixing. This can be clearly seen in the left panel of fig. 2: The value of the ghost propagator at the lowest diagonal momentum $\hat{\mathbf{p}} = (1,1,1)$ is only very slightly suppressed as the number n of Gribov restarts is increased, and the optimum is already reached for n as low as $n \approx 2..3$. All this is in constrast to the corresponding findings for the gluon propagator, where a 20% effect was seen that required up to $n = 40$ for saturation.

The second important ingredient is the residual gauge fixing. To make contact with the Hamiltonian approach in Weyl gauge, we would like to put the spatial average $u(t) = L^{-3}\sum_{\mathbf{x}} U_0(t,\mathbf{x})$ to unity. However, periodic boundary conditions only allow us to make $u(t)$ time-independent, $u(t) \equiv \overline{U}_0 = \text{const.}$ In the infinite volume limit (and in praxis also for $L \geq 32$), $\overline{U}_0$ approaches unity. Although this only enforces $\partial_0 U_0 = 0$ on the spatial average, the A_0–propagator is, within statistical errors, independent of energy (see left panel of fig. 1). In the right panel of fig. 1, we thus plot only the instantaneous A_0–propagator which is strongly enhanced in the infrared. This result will be related to the Coulomb potential below.

3. Results

The right panel of figure 2 shows our results for the ghost propagator and its form factor,

$$G(p) = \langle \bar{c}(-\mathbf{p})\, c(\mathbf{p}) \rangle = L^{-3} \sum_{\mathbf{x}} e^{i\mathbf{p}\mathbf{x}} \langle \mathsf{M}(\mathbf{x},\mathbf{0})^{-1} \rangle \equiv \frac{d(|\mathbf{p}|)}{\mathbf{p}^2} \tag{3.1}$$

where $\mathsf{M} \equiv (-\nabla \mathbf{D})$ is the Faddeev–Popov operator and the ghost form factor $d(p)$ measures the deviation from the perturbative result. The form factor is infrared enhanced, which agrees with the horizon condition $d^{-1}(0) = 0$ necessary in the Zwanziger confinement criterion [12]. Our infrared exponent $\kappa \approx 0.22$ for the divergence $d(p) \sim 1/(p^2)^\kappa$ is slightly smaller than the one obtained with naive gauge fixing [5], but agrees well with recent improved studies in $SU(3)$ [11].

Even more directly related to the confinement problem is the so-called Coulomb potential V_c, i.e. the response of the gluon vacuum to static colour charges. Since the constituent gluon $\mathbf{A}$ and its wave functional are gauge-dependent, V_c is not directly the physical potential between static

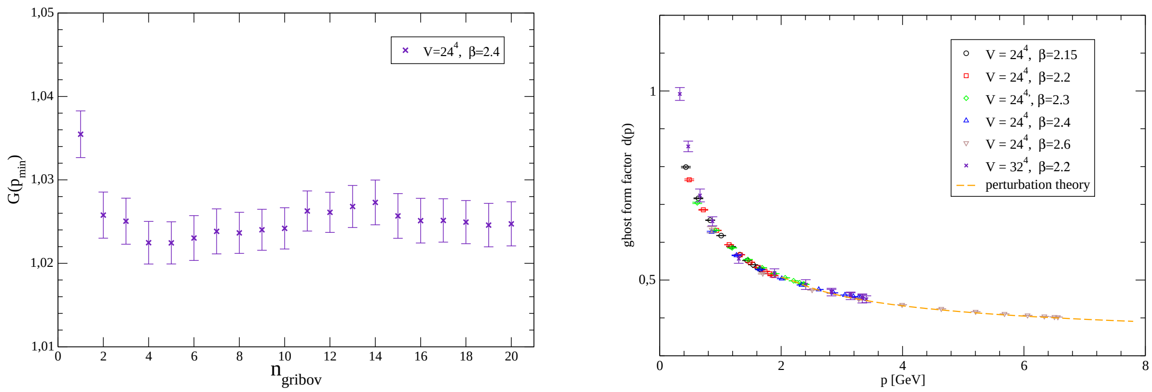

Figure 2: Left panel: The ghost propagator at the lowest diagonal momentum $\hat{\mathbf{p}} = (1,1,1)$ as a function of the number of Gribov copies considered in the Coulomb gauge fixing. (Note the scale on the y–axis.) Right panel: The ghost form factor $d(p)$ as a function of the spatial momentum $|\mathbf{p}|$.

quarks (as extracted from Wilson loops or Polyakov lines), but an upper bound, $V_c(r) \geq \frac{4}{3} V(r)$. This implies that there is no confinement without Coulomb confinement [12], but a linear Coulomb potential may persist even in the deconfined phase.

Formally, $V_c(r)$ can be computed in one of two equivalent ways,

$$V_c(|\mathbf{x}-\mathbf{y}|) = \langle A_0(t,\mathbf{x}) A_0(t,\mathbf{y}) \rangle = g^2 \, \langle \left(\mathsf{M}^{-1} \cdot \Delta \cdot \mathsf{M}^{-1} \right)_{\mathbf{x},\mathbf{y}} \rangle \,. \tag{3.2}$$

The formal equivalence of these two expressions can be shown in the first order formalism upon explicitly resolving Gauß' law [10, 14]. This leaves possible renormalisation issues aside and the lessons learned from the scaling violations in the gluon propagator indicate that some caution is required when connecting bare instantaneous correlators. Of course, the A_0–propagator is numerically much simpler than the complicated Coulomb kernel involving two inversions of the FP operator.

The strong Ward identities in Coulomb gauge [10] imply that the special combination in momentum space

$$\mathbf{p}^2 V_c(\mathbf{p}) \sim g^2(p) \tag{3.3}$$

is a renormalisation group invariant which can be taken as a definition of the running coupling constant. Simulations with different β should thus fall on top of each other without further multiplicative renormalisation. We have tested this conjecture for numerous values of β on relatively small 16^4 lattices. (On 32^4 lattices, we have only been able to complete the analysis of the complicated Coulomb kernel for a single value of β). The β–invariance was much better for the A_0–correlator, while V_c constructed from the Coulomb kernel still showed noticable scaling violations. At present, it is not known whether these deviations are pure numerical or finite volume effects, or if they have any more significant meaning. (Similar observations were made in ref. [11]). Simulations with improved statistics on larger lattices have to be conducted to resolve this issue.

Finally, the most direct approach to the confinement issue is given by the expression

$$\mathbf{p}^4 V_c(\mathbf{p}) \,. \tag{3.4}$$

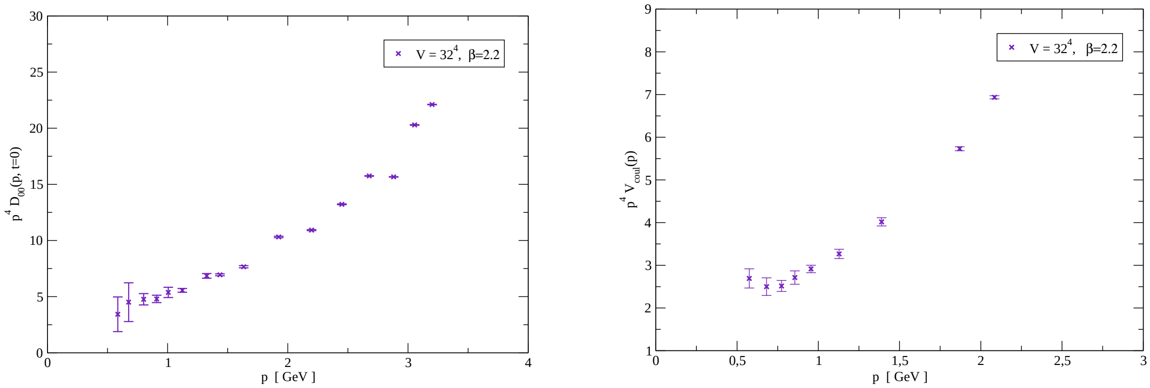

Figure 3: Left panel: The combination $\mathbf{p}^4 V_c(|\mathbf{p}|)$ with the Coulomb potential V_c extracted from the A_0–propagator $D_{00}(\mathbf{p}, t = 0)$. Right panel: The same quantity, with V_c extracted from the Coulomb kernel.

From the Fourier transformation of a linear potential, $V_c(r) = \sigma_c\, r$, it is readily seen that

$$\mathbf{p}^4 V_c(\mathbf{p}) \to 8\pi\sigma_c\,, \qquad |\mathbf{p}| \to 0\,.$$

The *Coulomb string tension* σ_c is an upper bound for the real string tension σ extracted from Wilson loops. Previous and current lattice studies are inconclusive as to whether $\sigma_c = \sigma$, since the approach to $|\mathbf{p}| \to 0$ is not as uniform as expected: Early simulations without improved/residual gauge fixing saw a slight but noticeable rise in the quantity (3.4) below $|\mathbf{p}| \approx 1\,\mathrm{GeV}$, which seemed compatible with σ_c/σ anywhere in the range $1\ldots3$. More recent computation for the gauge group $G = SU(3)$ prefer a value $\sigma_c/\sigma \approx 1.6$, but the extrapolation to zero momentum is again uncertain due to a peculiar "bump" in the quantity (3.4) at momenta between $0.1\ldots1\,\mathrm{GeV}$.

Our result on a $V = 32^4$ lattice in figure 3 using all improved gauge fixing techniques give reliable results (for cylinder cut momenta) only down to $|\mathbf{p}| \simeq 0.5\,\mathrm{GeV}$. In this range, the results for (3.4) are compatible with V_c computed either from the A_0–propagator or from the Coulomb kernel. The latter result show a more pronounced plateau at the smallest momenta, which is reminiscent of the slight rise observed in [5]. However, the numerical data can equally well be fitted with a constant. (V_c from the A_0–propagator is compatible with the Coulomb kernel results within statistical errors). For both definitions of V_c, we do not see the "bump" reported for $SU(3)$ in ref. [11]. While the approach to a constant seems promising, better statistics and larger lattices are required for a reliable extrapolation of σ_c/σ.

4. Conclusion

The computation of ghost correlators and the Coulomb potential in $G = SU(2)$ show qualitative agreement with continuum calculations in the variational approach [2, 3]. The scaling violations observed previously for the equal-times gluon propagator $D(\mathbf{p})$ have no counter part in the ghost correlators studied here. In particular, the dependence on the Gribov noise and the details of the improved gauge fixing are negligable. Likewise, the residual gauge fixing, which is essential for the resolution of the scaling violations in $D(\mathbf{p})$, seems to have little or no influence on the ghost propagator or Coulomb potential, even when the latter is extracted from the the A_0–propagator.

Our residual gauge fixing removes the energy dependence on A_0 not only in the spatial average, but effectively for arbitrary A_0–correlators. There is thus no issue with renormalisation and the results for the instantaneous A_0–correlators resemble the ones with unfixed residual symmetry. (Similar observations are made for the ghost propagator and the Coulomb potential as extracted from the Coulomb kernel.) It is therefore not surprising that our findings agree with other calculations, even if these fixed the Coulomb gauge naively, or left the residual symmetry unfixed.

The statistics in the deep infrared are not sufficient to make reliable quantitative extrapolations for the Coulomb string tension σ_c, or the Coulomb form factor $f(p)$ whose infrared behaviour is an important ingredient in the variational approaches [2, 3]. We intend to improve on this and accumulate data for 32^4 lattices with various β, and A_0–correlators on even larger lattices. These results will be published in a forthcoming paper.

References

[1] A.P. Szczepaniak and E.S. Swansen, Phys. Rev. **D65** (2002) 025015, [hep-ph/0107078].

[2] C. Feuchter and H. Reinhardt, Phys. Rev. **D70** (2004) 105021, [hep-th/0408236]; H. Reinhardt and C. Feuchter, Phys. Rev. **D71** (2005) 105002.

[3] D. Epple, H. Reinhardt and W. Schleifenbaum, Phys. Rev. **D75** (2007) 045011; W. Schleifenbaum, M. Leder and H. Reinhardt, Phys. Rev. **D73**, (2006) 125019.

[4] A. Cucchieri and D. Zwanziger, Phys. Rev. **D65** (2002) 014001, [hep-lat/0008026].

[5] K. Landfeld and L. Moyaerts, Phys. Rev. **D70** (2004) 074507, [hep-lat/0406024].

[6] A. Voigt et al. , *PoS* **LAT2007**, (2007) 338; [arXiv:0709.4585].

[7] M. Quandt et al. , *PoS* **LAT2007** (2007) 325, [arXiv:0710.0549].

[8] I. L. Bogolubsky et al. , Phys. Rev. **D74** (2006) 034503, [hep-lat/0511056] ; I. L. Bogolubsky et al. , Phys. Rev. **D77** (2008) 014504, [arXiv:0707.3611].

[9] G. Burgio, M. Quandt and H. Reinhardt, [arXiv:0807.3291] (2008).

[10] D. Zwanziger, Nucl. Phys. **B518** (2002) 014001.

[11] A. Voigt, M. Ilgenfritz, M. Müller-Preussker, Phys. Rev. **D78** (2008) 14501.

[12] D. Zwanziger, Nucl. Phys. **B485** (1997) 185 ; [help-th/9603203].

[13] H. Reinhardt, Phys. Rev. Lett. **101** (2008) 061602, [arXiv:0803.0504].

[14] P. Watson, H. Reinhardt, Phys. Rev. **D76** (2007) 125016, [arXiv:0709.0140].