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Dressed Polyakov loops and center symmetry from Dirac spectra

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We construct a novel observable for finite temperature QCD that relates confinement and chiral symmetry. It uses phases as boundary conditions for the fermions. We discuss numerical and analytical aspects of this observable, like its spectral behavior below and above the critical temperature, as well as the connection to chiral condensate and center symmetry.

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The main phenomena in QCD at the finite temperature transition are deconfinement and chiral symmetry restoration. There has always been the question whether there is a mechanism connecting the two, in particular since in the quenched theory the corresponding phase transitions occur at the same critical temperatures [1]. Here we define an observable that indeed links the two and discuss numerical findings from quenched lattice configurations [2].

The Polyakov loop, on the one hand, is the order parameter of confinement, being traceless in the confined phase and moving towards the center of the gauge group at temperatures above T_c. The spectral density of the Dirac operator at the origin, $\rho(0)$, on the other hand, is the order parameter of chiral symmetry. It is related to the condensate by the famous Banks-Casher relation [3] $\langle\bar{\psi}\psi\rangle = -\pi\rho(0)$ and vanishes above T_c. How does confinement leave a trace in the Dirac spectrum? The answer lies in the dependence on temporal boundary conditions, as we will show now.

We use the lattice as a regulator. The (untraced) Polyakov loop is the product of temporal links. For the lattice Dirac operator we use the staggered one [4], which can be viewed as hopping by one link. It is well-known that the k-th power of the Dirac operator at the same argument, $D^k(x,x)$, contains all products of links along closed loops of length k starting at x. To distinguish the Polyakov loop from 'trivially closed' loops (like the plaquette), one needs phase boundary conditions for the fermions [5], $\psi(x_0+\beta,\vec{x}) = e^{i\varphi}\psi(x_0,\vec{x})$. These boundary conditions amount to an imaginary chemical potential. They can be easily implemented by replacing the temporal links U_0 in some time slice by $e^{i\varphi}U_0$. As a consequence, all loops get a factor $e^{i\varphi q}$ according to their winding number q around the temporal direction, trivially closed loops remain unchanged.

In this way, the Polyakov loop can be reconstructed from the Dirac spectrum by using at least three boundary conditions, see [6]. This 'thin' Polyakov loop, however, has poor renormalization and scaling properties and it turned out that in this approach it is UV dominated [6].

Influenced by the Jena group [7], we instead consider the propagator $1/(m+D)$ with some probe mass m and use a geometric series obtaining all powers of the Dirac operator. Denoting the Dirac operator at a particular boundary condition φ by D_φ, i.e., including the factors of $e^{i\varphi}$, the propagator is given as a product of links along all closed loops l,

$$\mathrm{tr}\,\frac{1}{m+D_\varphi} = \frac{1}{m}\sum_{\text{loops }l}\frac{\mathrm{sign}(l)}{(2am)^{|l|}}\,e^{i\varphi q(l)}\,\mathrm{tr}_c\prod_{(x,\mu)\in l}U_\mu(x)\,, \tag{1}$$

where $|l|$ is the length of the loop and $\mathrm{sign}(l)$ comes from the staggered factor.

Of importance in (1) is the phase factor, since one can project onto a particular winding q by a Fourier transform w.r.t. the boundary angle, $\int_0^{2\pi} d\varphi\, e^{-i\varphi q}\ldots$. Specifying to a single winding, $q=1$, like for the Polyakov loop, we arrive at [2]

$$\tilde{\Sigma} \equiv \int_0^{2\pi}\frac{d\varphi}{2\pi}\,e^{-i\varphi}\frac{1}{V}\left\langle \mathrm{tr}\,\frac{1}{m+D_\varphi}\right\rangle = \frac{1}{mV}\sum_{q(l)=1}\frac{\mathrm{sign}(l)}{(2am)^{|l|}}\left\langle \mathrm{tr}_c\prod_{(x,\mu)\in l}U_\mu(x)\right\rangle. \tag{2}$$

We refer to the new observable $\tilde{\Sigma}$ as the 'dual condensate', because it is obtained through a Fourier transform from the trace of the propagator. Indeed, in the massless limit (after the infinite volume limit as usual) we obtain the chiral condensate integrated with a phase factor over the boundary conditions and, using the Banks-Casher relation at every individual angle φ, the corresponding representation in terms of the eigenvalue densities $\rho(0)_\varphi$. The right hand side of (2) represents the

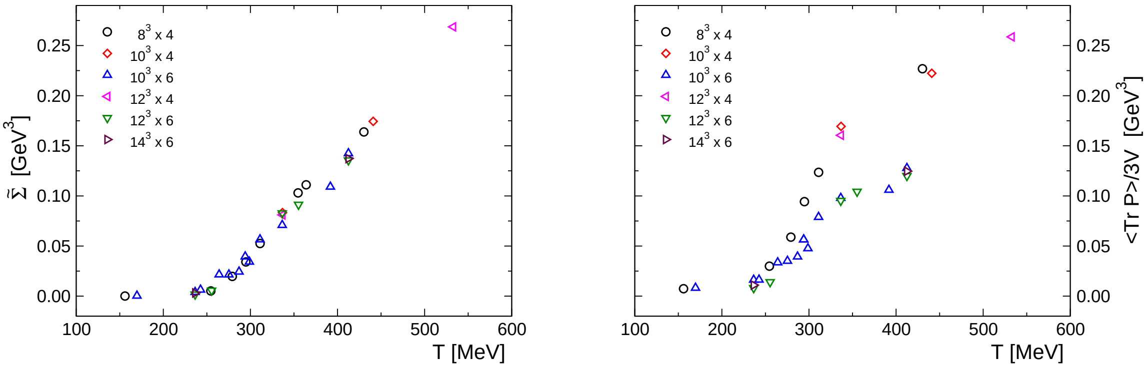

Figure 1: Left: The dressed Polyakov loop $\tilde{\Sigma}$ as a function of T at various lattices (for $m = 100$ MeV). Right: The corresponding plot for the conventional Polyakov loop.

'dressed Polyakov loop', that is the set of all loops which wind once around the temporal direction. In the infinite mass limit, detours become suppressed and only the straight Polyakov loop survives as it is the shortest possible loop in this set.

Fig. 1 shows that $\tilde{\Sigma}$ is indeed an order parameter. Keeping the mass m fixed, $\tilde{\Sigma}$ vanishes below the critical temperature (which is about 280 MeV in the quenched case) and develops an expectation value for higher temperatures. One finds that the results (when expressed in physical units) obtained for different volumes and with different resolution essentially fall on a universal curve. This illustrates the good renormalization properties of our observable, which are inherited from $\langle\bar{\psi}\psi\rangle$ and may be viewed as an effect of the dressing rendering such loops less UV dominated.

In a spectral representation, the Dirac operator in $\tilde{\Sigma}$ is replaced by an eigenvalue sum,

$$\tilde{\Sigma} = \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi}\, \frac{1}{V} \Big\langle \sum_i \frac{1}{m + \lambda_\phi^{(i)}} \Big\rangle . \tag{3}$$

As the eigenvalues appear in the denominator, we expect the sum to be dominated by the IR modes. As Fig. 2 shows, this is confirmed by our lattice data.

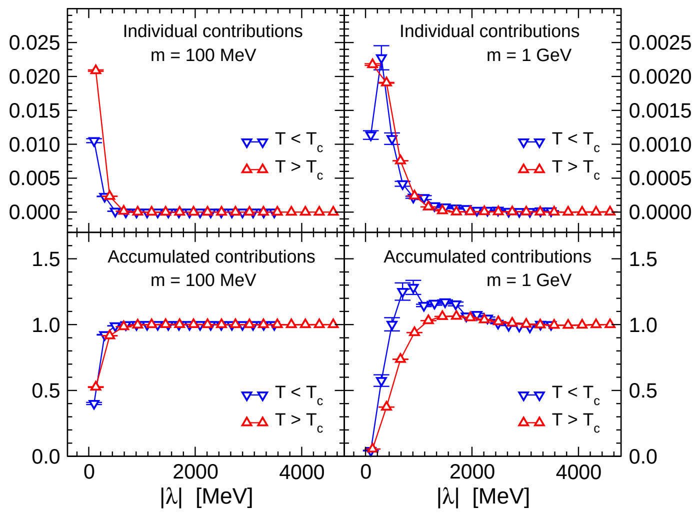

Figure 2: Individual and accumulated contributions to the spectral sum (3) at two different masses.

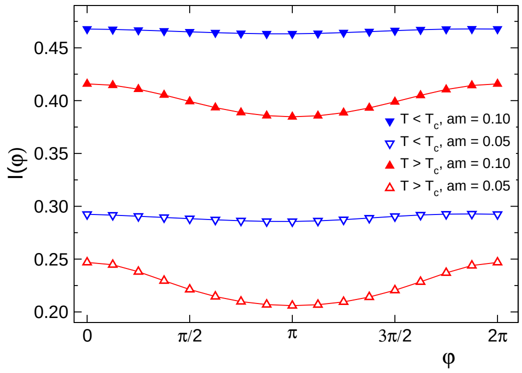

Figure 3: The integrand $\langle \sum_i (m+\lambda_\varphi^{(i)})^{-1}\rangle/V$ as a function of the boundary condition φ for two values of *am* and two temperatures, for configurations with real Polyakov loop.

How is a finite resp. vanishing order parameter $\tilde{\Sigma}$ built up by the eigenvalues? Fig. 3 shows that they respond differently to the boundary conditions in the confined vs. deconfined phase. The eigenvalues are independent of the boundary condition in the confined phase, which leads to a vanishing order parameter $\tilde{\Sigma}$. In the deconfined phase, on the other hand, the eigenvalues show a typical cosine-type of modulation. Together with the Fourier factor this yields a nonvanishing $\tilde{\Sigma}$.

The chiral condensate has to behave essentially in the same way, as it is the integrand in the massless limit. Although the chiral condensate is finite in the confined phase, it is independent of the boundary condition φ and hence results in a vanishing dual condensate. The situation in the deconfined phase might be more confusing at first glance as the spectrum has a gap there. For boundary conditions in line with the original Polyakov loop, however, the chiral condensate persists above the critical temperature T_c, which has been seen in several studies [8]. This mechanism is needed to ensure a finite $\tilde{\Sigma}$ and should actually be at work for all $T > T_c$.

In the quenched case the conventional Polyakov loop is the order parameter for center symmetry. Under center transformations the dressed Polyakov loop behaves in the same way. Hence, $\tilde{\Sigma}$ is an order parameter for center symmetry, which is underneath our numerical findings and which we therefore expect to be independent of the choice of the lattice Dirac operator.

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