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The gluon propagator in Coulomb gauge from the lattice

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We show that in the lattice Hamiltonian limit the static transverse propagator $D(|\vec{p}|) \propto \int dp_0 D(|\vec{p}|, p_0)$ satisfies multiplicative renormalizability. We give a procedure to calculate $D(|\vec{p}|)$ on available lattices at finite temporal spacing. The result agrees at all momenta with the Gribov formula $D(|\vec{p}|) \propto (|\vec{p}|^2 + M^4|\vec{p}|^{-2})^{-\frac{1}{2}}$, with $M = 0.88(1)\text{GeV} \simeq 2\sqrt{\sigma}$.

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1. Introduction

The original Gribov-Zwanziger confinement scenario [1, 2] predicts an IR vanishing static gluon propagator $D(|\vec{p}|)$ in Coulomb gauge. The gluon propagator is also at the heart of variational estimates to the ground state wave function [3, 4]. A cross check of continuum results with lattice calculations had, however, long failed. A first study for SU(2) at fixed $\beta = 2.2$ indicated compatibility with Gribov's formula in the IR but was inconclusive in the UV [5]. Later studies in SU(2) and SU(3) showed for $D(|\vec{p}|)$ strong scaling violations and a UV behaviour at odds with simple dimensional arguments [6–8]. All these works calculate $D(|\vec{p}|)$ fixing the Coulomb gauge only at a given time-slice, neglecting the residual gauge freedom of temporal links. Also one takes for granted that multiplicative renormalizability holds for the full propagator $D(|\vec{p}|, p_0)$, although perturbative results point at a more complex picture [9]. We report here the results first obtained in [10], where a different strategy was adopted, fixing analytically the residual gauge and then studying the renormalization of the full spatial propagator $D(|\vec{p}|, p_0)$. We show that the Gribov formula [1] perfectly describes the lattice data for the static propagator. We refer to our original paper [10] for details about conventions and notations. To achieve a good gauge fixing we adapt the algorithms developed in [11, 12].

2. Results

The first observation made in [10] is that the lattice bare propagator $D_\beta(|\vec{p}|, p_0)$ factorizes as:

$$D_\beta(|\vec{p}|, p_0) = \frac{f_\beta(|\vec{p}|)}{|\vec{p}|^2} \frac{g_\beta(z)}{1+z^2} \qquad z = \frac{p_0}{|\vec{p}|} . \tag{2.1}$$

The denominator $|\vec{p}|^2(1+z^2)$ explicitly accounts for dimensions. Without loss of generality we can choose $g_\beta(0) = 1$. The data for $g_\beta(z) = (1+z^2)D_\beta(|\vec{p}|, p_0)D_\beta(|\vec{p}|, 0)^{-1}$ are shown in Fig. 1 for $L = 24$. Their leading behaviour can be well described by a power law $(1+z^2)^\alpha$. For $\beta \gtrsim 2.3$ the functions g_β vary consistently with L, violating multiplicative renormalizability. However as $L \to \infty$ all values of α are compatible with 1 within one or two σ i.e. $D_\beta(|\vec{p}|, p_0)$ might eventually be p_0 independent. Eq. (2.1) has deep consequences on the "naive" calculation of $D(|\vec{p}|) \propto \sum_{p_0} D(|\vec{p}|, p_0)$ as in [6–8]. Consider different lattice cut-offs for space and time $\frac{a_s}{a_t} = \xi > 1$ and define $\hat{p} = a_s|\vec{p}|$. Neglecting subleading terms, for large L we approximate the sum over p_0 by an integral yielding:

$$\begin{aligned} D_\beta(|\vec{p}|) &\simeq \int_{-\frac{2}{a_t}}^{\frac{2}{a_t}} \frac{dp_0}{2\pi} D_\beta(|\vec{p}|, p_0) = \frac{f_\beta(|\vec{p}|)}{|\vec{p}|} \int_0^{\frac{2\xi}{\hat{p}}} \frac{dz}{\pi} \left(1+z^2\right)^{\alpha-1} = \frac{f_\beta(|\vec{p}|)}{|\vec{p}|} I(\frac{2\xi}{\hat{p}}, \alpha) ; \\ I(\frac{2\xi}{\hat{p}}, \alpha) &= \frac{1}{2\pi} B(\frac{4\xi^2}{4\xi^2+\hat{p}^2}, \frac{1}{2}, -\alpha+\frac{1}{2}) , \end{aligned} \tag{2.2}$$

where $B(z,a,b)$ is the incomplete beta function. In the lattice Hamiltonian limit, corresponding to $\xi \to \infty$ [13], I becomes $|\vec{p}|$ independent[1], $I(\frac{2\xi}{\hat{p}}, \alpha) \to \frac{1}{2\sqrt{\pi}} \frac{\Gamma(\frac{1}{2}-\alpha)}{\Gamma(1-\alpha)}$. Then $D_\beta(|\vec{p}|) \propto \frac{f_\beta(|\vec{p}|)}{|\vec{p}|}$ and multiplicative renormalizability relies solely on $f_\beta(|\vec{p}|)$. In a standard lattice formulation, however, $\xi \equiv 1$ and the extra $|\vec{p}|$ dependence $B(\frac{4}{4+\hat{p}^2}, \frac{1}{2}, -\alpha+\frac{1}{2})$ cannot be avoided. Fig. 2 shows, in spite of approximations, nearly perfect agreement between Eq. (2.2) and the slopes observed in the UV for the naive definition of $D(|\vec{p}|)$ as in [7].

[1] I can be analytically continued if $\alpha > \frac{1}{2}$, $\alpha - \frac{1}{2} \notin \mathbb{N}$.

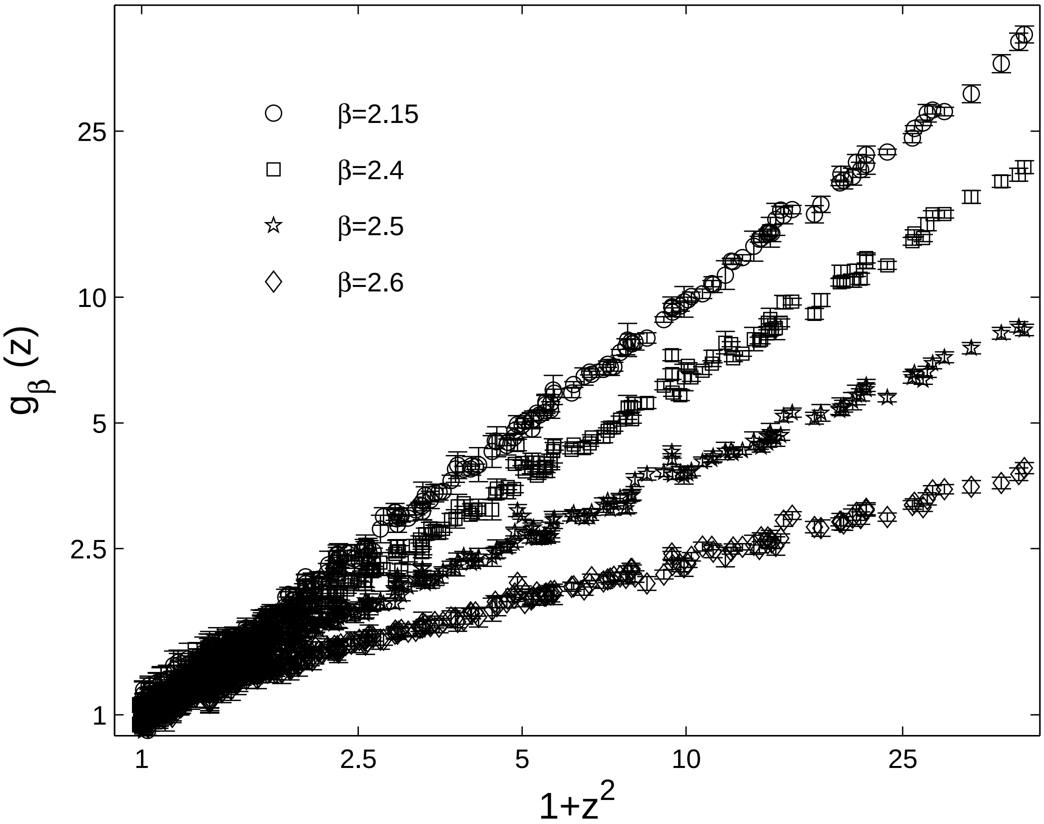

Figure 1: Data for $g_\beta(z)$ vs $1+z^2$ in log-log scale, $L=24$. For sake of readability not all β are shown.

The above discussion makes clear that the static propagator should be defined as $D_\beta(|\vec{p}|) = \frac{f_\beta(|\vec{p}|)}{|\vec{p}|}$. To extract it at available L and β we fit $g_\beta(z)$ and define:

$$f_\beta(|\vec{p}|) = D_\beta(|\vec{p}|, p_0)\frac{1+z^2}{g_\beta(z)} =: \tilde{D}_\beta(|\vec{p}|, p_0) \tag{2.3}$$

which is now independent of p_0, up to noise. To improve the signal we average over p_0, yielding:

$$\tilde{f}_\beta(|\vec{p}|) := \frac{1}{L}\sum_{p_0}\tilde{D}_\beta(|\vec{p}|, p_0)\ , \quad D_\beta(|\vec{p}|) := \frac{\tilde{f}_\beta(|\vec{p}|)}{|\vec{p}|}\ , \tag{2.4}$$

Fig. 3 shows the resulting $D_\beta(|\vec{p}|)$, which is multiplicatively renormalizable. Fitting a power law in the IR, $|\vec{p}|^q M^{-1-q}$, and a power law plus logarithmic corrections in the UV, $M^{r-1}|\vec{p}|^{-r}|\log|\vec{p}||^{-s}$ gives $q = 0.99(1)$, $r = 1.002(3)$, $s = 0.002(2)$ and $M = 0.88(1)$ GeV, with χ^2/d.o.f. all in the range 3.2-3.3, in agreement with the UV and IR analysis in [4, 14, 15]. We thus constrain $q = r = 1$, $s = 0$ and fit the whole result through the Gribov formula:

$$D(|\vec{p}|) = \frac{1}{2\sqrt{|\vec{p}|^2 + \frac{M^4}{|\vec{p}|^2}}} \tag{2.5}$$

We find just as good agreement (χ^2/d.o.f. $= 3.3$) again with $M = 0.88(1)\mathrm{G}eV \simeq 2\sqrt{\sigma}$.

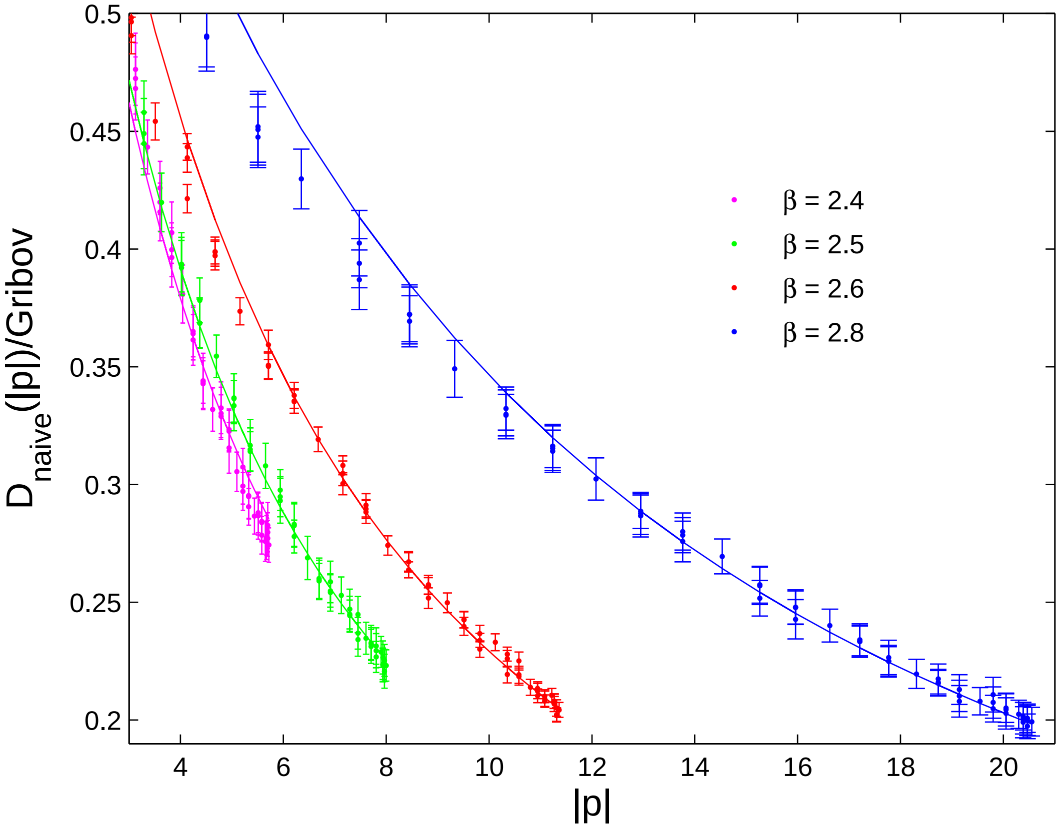

Figure 2: Comparison between UV deviations from the Gribov formula in the MC data [7] for the naive static propagator $\sum_{p_0} D_\beta(|\vec{p}|, p_0)$ ($\bullet$) and our prediction for the leading term $B(\frac{4}{4+\hat{p}^2}, \frac{1}{2}, -\alpha + \frac{1}{2})$ (—).

3. Conclusions

We have shown that on the lattice the static transverse gluon propagator is multiplicatively renormalizable, IR-UV symmetric and can be well described by Gribov's formula over the whole momentum range. Its infrared and ultraviolet behaviours are in good agreement with the results obtained in the variational approach to continuum Yang-Mills theory in Coulomb gauge [4, 14, 15].

Acknowledgments

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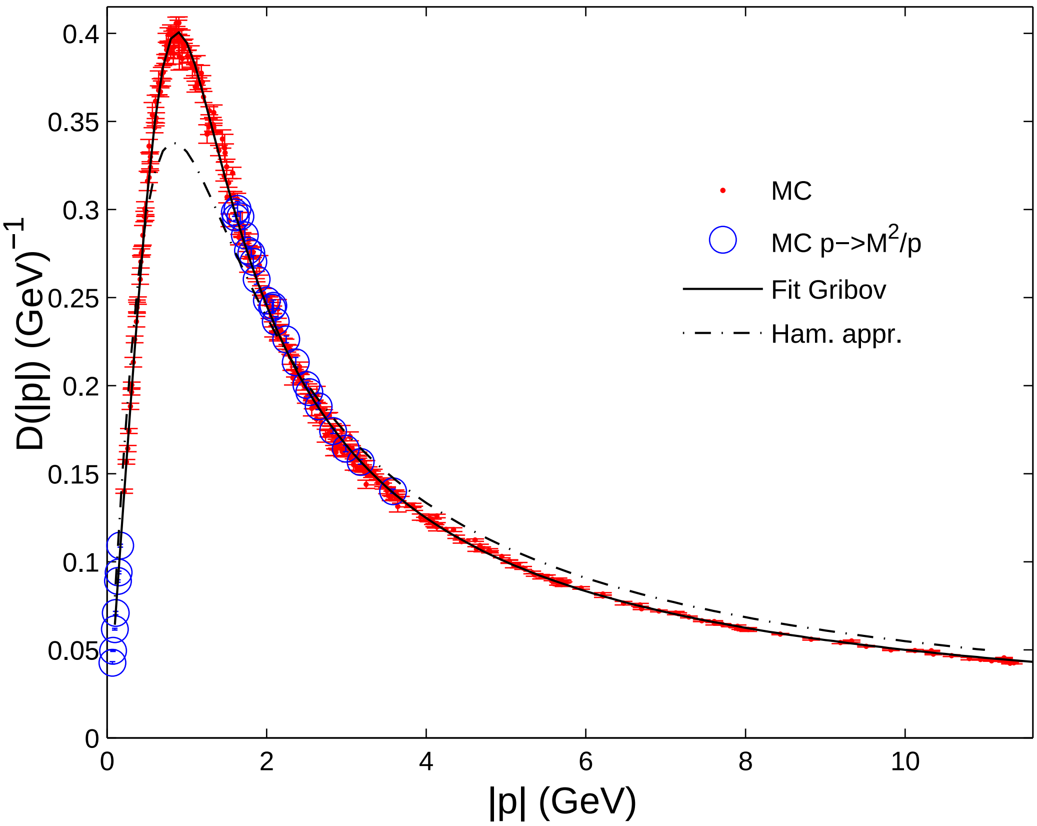

Figure 3: The gluon propagator: MC data (•), (a few) data for $|\vec{p}| \to M^2|\vec{p}|^{-1}$ (◯), the fit to Gribov's formula (—) and the result of the Hamiltonian approach [14] (·−).

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