

Aspects of confinement from QCD correlation functions

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We discuss the properties of ghost and gluon propagators in Landau gauge Yang-Mills theory and their relation to the confinement problem. In general two types of infrared behavior of these functions are allowed from their functional equations: scaling and decoupling. Both solutions show positivity violations in the gluon propagator and lead to a confining Polyakov loop potential. However, only the scaling solution agrees with the Kugo-Ojima confinement scenario and the related formulation of a physical Hilbert space of Yang-Mills theory. Our numerical results for the gluon dressing function agree almost pointwise with the lattice results at all physical momenta.

*8th Conference Quark Confinement and the Hadron Spectrum
September 1-6, 2008
Mainz, Germany*

*Speaker.

1. Global symmetries, confinement and the infrared behavior of Yang-Mills theory

In this talk we are concerned with the infrared behavior of the dressing functions of the ghost and gluon propagators of QCD. There has been much debate in the past years about the zero momentum limit of these functions mainly due to an apparent mismatch between solutions obtained from lattice gauge theory [1, 2] and functional equations in the continuum, i.e. Dyson-Schwinger equations [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and functional renormalization group equations [13, 14]. In these continuum studies the dressing function of the ghost propagator is divergent, whereas the gluon propagator is infrared finite or even vanishing. In terms of the dressing functions $G(p^2)$ and $Z(p^2)$ of the ghost and gluon propagators in Landau gauge

$$D_G(p) = -\frac{G(p^2)}{p^2}, \quad D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}, \quad (1.1)$$

and in terms of a power-law expansion in the infrared the dressing functions are related by

$$Z(p^2) \sim (p^2)^{2\kappa - \frac{d}{2} + 2}; \quad G(p^2) \sim (p^2)^{-\kappa} \quad (1.2)$$

with dimension d and positive and potentially irrational exponent κ . These power laws are part of an all-order analytical analysis of both the whole tower of DSEs and FRGs in the infrared [9, 10]. They agree with a set of conditions formulated within a framework for confinement of covariantly gauge fixed Yang-Mills theory set up by Kugo and Ojima [15].

The Kugo-Ojima scenario rests on well-defined charges related to unbroken global gauge symmetries. In particular it assumes global BRST symmetry. The related well defined charge operator has been used to identify the positive definite space \mathcal{H}_{phys} of physical states within the total state space \mathcal{V} of QCD. An unbroken global gauge symmetry is then crucial to show that the states in \mathcal{H}_{phys} contributing to the physical S-matrix of QCD are indeed colorless. They also argued that this setup guarantees the disappearance of the 'behind-the-moon' problem, i.e. a colorless bound state with colored constituents cannot be delocalized into colored lumps [15]. This then implements the confining phase of Yang-Mills theory. In Landau gauge a direct consequence of the well defined global color charge is the infrared enhancement of the ghost dressing function $G(p^2)$ [15]. Such a behavior is obtained in eqs.(1.2) if and only if $\kappa > 0$.

In functional methods this enhancement can be implemented as an infrared renormalization condition for the ghost dressing function. This condition leads to a unique [10] (scaling) solution of the whole tower of functional equations for the one-particle irreducible Green's functions of Yang-Mills theory. In turn, given the Kugo-Ojima scenario an infrared divergent ghost implicitly defines the unique gauge fixing with well-defined global BRST-charges [12].

In lattice calculations, however, the behavior (1.2) with $\kappa > 0$ is notoriously difficult to obtain. In the two dimensional theory (1.2) is nicely satisfied [16]. In three dimensions first hints of (1.2) have been found in a formulation with an improved (evolutionary) gauge fixing algorithm [17], whereas in four dimensions one obtains (1.2) in the strong coupling limit $\beta \rightarrow 0$ for not too small momenta [2]. In general, however, lattice calculations return the different behavior

$$Z(p^2) \sim (p^2)^{1/2}; \quad G(p^2) \sim (p^2)^0 \quad (1.3)$$

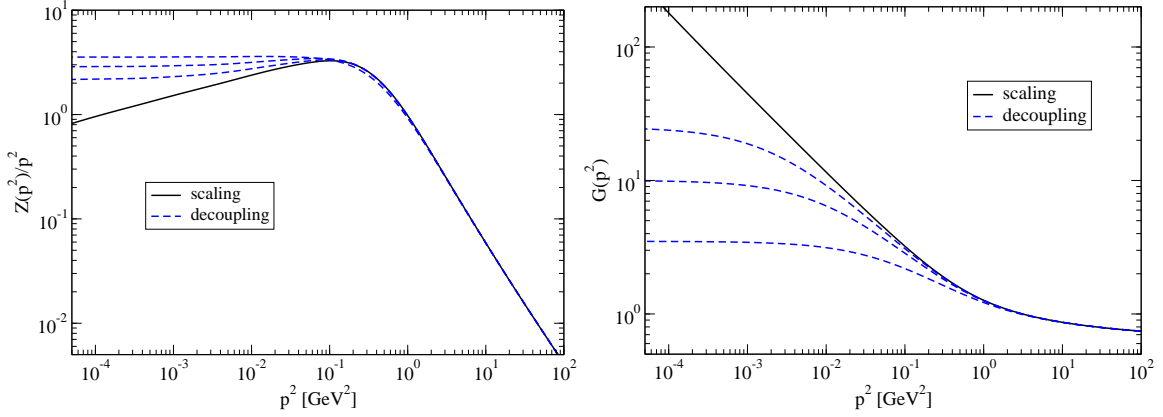


Figure 1: Numerical solutions for the gluon propagator $D(p^2) = Z(p^2)/p^2$ and the ghost dressing function $G(p^2)$ with different boundary conditions $G(0)$.

even for very large lattices [1]. This limit for $p^2 \rightarrow 0$, however, corresponds to a finite ghost dressing function and is therefore not in agreement with the Kugo-Ojima scenario. Within functional methods also this 'decoupling' type of solution can be implemented by suitable boundary conditions in the infrared [6, 12, 18, 19, 20]. Up to logarithms, eqs.(1.2) and (1.3) completely exhaust the possible infrared solutions of the functional equations of Yang-Mills theory.

Given confinement, an infrared solution with finite ghost at zero momentum (termed 'decoupling' below) implies broken global gauge and BRST symmetries [2, 12]. Indeed, all known BRST-quantizations that are compatible with an infrared finite ghost even break off-shell BRST, see e.g. [20] and references therein. The only possibility for the decoupling solution to coexist with a globally well-defined BRST charge is in a Higgs phase, where the breaking of global color symmetry implies the existence of super-selection sectors. Certainly, this is not what is seen in lattice simulations of QCD and therefore one may conclude that BRST-symmetry is indeed broken on the lattice [21]. Regarding global symmetries, the status of the decoupling solutions is therefore clearly different from the scaling solution: whereas scaling agrees with well-defined BRST and global color charges decoupling does not. Note, however, that both scaling and decoupling agree with the confinement criterion developed in [22]: both lead to a confining, nonperturbative Polyakov loop potential. Furthermore, in both cases the gluon propagator exhibits positivity violation [12].

2. The ghost and gluon dressing functions

As already mentioned above the functional continuum equations, DSEs and FRGs, can display both types of solutions, scaling (1.2) and decoupling (1.3). These are distinguished by a boundary condition for the ghost dressing function at zero momentum, $G(0)$. In [12] we demonstrated this behavior using a truncation scheme for the DSEs developed to guarantee the transversality and multiplicative renormalizability of the gluon DSE. As an alternative we also employed a truncation for the corresponding equations in the FRG-framework, which has been developed to minimize truncation artefacts in the mid-momentum region.

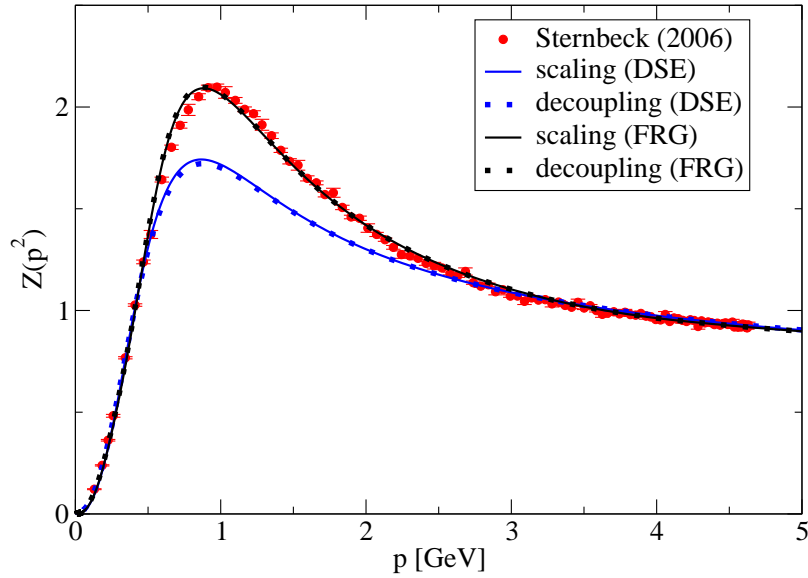


Figure 2: Both type of solutions compared to lattice results in minimal Landau gauge from [23].

Our numerical solutions for the ghost and gluon dressing functions are shown in Fig. 1. The boundary condition $G(0) = \infty$ results in the scaling solution, eq. (1.2), with a diverging ghost dressing function in the infrared and an infrared vanishing gluon propagator. The corresponding critical exponent κ in eq. (1.2) is given by $\kappa = \kappa_C = (93 - \sqrt{1201})/98 \approx 0.595353$ [6]. A finite value $G(0) = \text{const.}$, however, produces a continuous set of decoupling solutions with an infrared finite ghost dressing function. The corresponding gluon propagator is massive in the sense that $D(0) = \lim_{p^2 \rightarrow 0} Z(p^2)/p^2 = \text{const.}$ for decoupling. In the ultraviolet momentum region, both types of solutions are almost identical, as expected.

Finally we wish to emphasize that the question of scaling vs. decoupling only concerns global properties of the theory as the (non-)conservation of charges. The behavior (1.2) or (1.3) sets in at scales $p^2 \ll \Lambda_{QCD}^2$. In contradistinction all dynamics of the theory takes part on scales around or larger than Λ_{QCD} . Certainly, from a phenomenological point of view the behavior of the ghost and gluon dressing function at scales $p^2 \geq \Lambda_{QCD}^2$ is much more relevant than the behavior in the deep infrared. In Fig. 2 we compare the solutions from functional equations with the lattice results of ref. [23] for the gluon dressing function. It is very satisfactory that our numerical solution of the functional renormalization group equations almost pointwise matches the corresponding lattice results in the phenomenologically important mid-momentum region.

Acknowledgments: We thank Reinhard Alkofer, Jens Braun, Jeff Greensite, Joannis Papavasiliou, Olivier Pene, Kai Schwenzer and Lorenz von Smekal for useful discussions. A. M. was supported by the FWF under grant number P20330, C. F. by the Helmholtz-University Young Investigator Grant number VH-NG-332 and J. M. P. by the ExtreMe Matter Institute (EMMI).

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