

Recent Results in AdS/QCD

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AdS/QCD is an extra-dimensional approach to modeling the light hadronic resonances in QCD. AdS/QCD models are generally successful at reproducing low-energy observables with around 10-20% accuracy, depending on the details of the model. We discuss the motivation for these models, their intrinsic limitations, and some recent results.

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1. What is AdS/QCD?

AdS/QCD is an extra-dimensional approach to modeling the light hadronic resonances in QCD, motivated by the AdS/CFT correspondence in string theory [1]. AdS/QCD models combine several features of previous approaches to modeling the spectrum and interactions of light hadrons, including: chiral symmetry breaking, hidden local symmetry [2], Large-*N*, and the Weinberg sum rules.

There are two complementary classes of AdS/QCD models: top-down models rooted in string theory, and phenomenological bottom-up models. The benefit of top-down models is that both sides of the AdS/CFT duality are often well understood. The benefit of bottom-up models is that there is more freedom to build in properties of QCD.

1.1 Top-Down AdS/QCD and the AdS/CFT Corresponence



Figure 1: Brane configuration in the Sakai-Sugimoto model. The D4-branes create a spacetime horizon, forcing the D8 and $\overline{D8}$ -branes to intersect.

In the top-down approach, a brane configuration in string theory is engineered whose lowenergy spectrum of open-string fluctuations has a known field-theoretic interpretation. Via the AdS/CFT correspondence, for some brane constructions describing large-*N* gauge theories with large 't Hooft coupling g^2N , a dual description exists in terms of supergravity on a fixed spacetime background [1]. The basic dictionary between the dual theories was mapped out independently by Witten [3] and Gubser, Klebanov and Polyakov [4]. There now exists a large number of examples of field theories with supergravity duals. Conformal invariance and supersymmetry are not essential. The field theory can be confining with chiral symmetry breaking, which in those respects is similar to QCD. Some examples of confining theories with known supergravity duals are the $\mathcal{N}=1^*$ theory of Polchinski and Strassler [5], the Klebanov-Strassler cascading gauge theory [6], the D3-D7 system of Kruczenski *et al.* [7], and the D4-D8 system of Sakai and Sugimoto [8].

The Sakai-Sugimoto model is so far the example of the AdS/CFT correspondence most closely related to QCD. Ignoring the gravitational backreaction, the system includes a stack of *N* D4-branes wrapped on a circle on which fermions satisfy antiperiodic boundary conditions which break supersymmetry. From an effective 3+1 dimensional point of view, the massless spectrum includes the SU(*N*) gauge fields, but the fermions and scalars are massive. So far this is a model constructed by Witten shortly after the initial AdS/CFT conjecture [9]. N_f D8-branes and N_f D8-branes transverse

to the circle on which the D4-branes wrap intersect the D4-branes on 3+1 dimensional manifolds at definite positions along the circle, as in Fig. 1. The massless fluctuations of open strings connecting the D4 and D8 or $\overline{D8}$ -branes at their intersections describe 3+1 dimensional chiral fermions, with opposite chirality at the D8 and $\overline{D8}$ -branes. This is the Sakai-Sugimoto model [8].

In the supergravity limit with $N_f \ll N$, the D4-branes generate a horizon which effectively cuts off the spacetime geometry. The location of this horizon sets the scale of masses of those fluctuations in the 3+1 dimensional theory, which will have the interpretation of hadron masses. Because of the horizon, the D8 and $\overline{\text{D8}}$ -branes intersect, which reflects the breaking of the $SU(N_f) \times SU(N_f)$ chiral symmetry in the theory. For the geometry to be smooth the location of the horizon is correlated with the size of the compact circle on which the D4-branes wrap [7, 9]. The perturbative massless spectrum is that of SU(N) QCD with N_f flavors of quarks. However, Kaluza-Klein modes associated with the circle direction have masses comparable to the confining scale in this theory, so the massive spectrum of QCD-like bound states cannot be separated (in mass) from the spectrum of non-QCD-like Kaluza-Klein modes. The five-dimensional nature of the effective theory on the D4-branes becomes apparent at the same scale as the hadron masses we are interested in. This is an important distinction between this theory and QCD.

If we ignore the Kaluza-Klein modes around the circle, the fluctuations of 4+1 dimensional $SU(N_f) \times SU(N_f)$ gauge fields on the D8 and $\overline{D8}$ -branes are identified with vector mesons, axialvector mesons, and pions. The quantum numbers of the corresponding states can be identified with symmetries of the D-brane system. 3+1 dimensional parity, for example, is identified with a 4+1 dimensional parity, which also exchanges the two sets of $SU(N_f)$ gauge fields. If we ignore the extra circle direction, then the effective 3+1 dimensional action on the D8-branes describes the effective action for the light mesons, and easily allows for the calculation of decay constants (f_{π} , F_{ρ} , etc.) and couplings (e.g. $g_{\rho\pi\pi}$). Most results agree relatively well with experimental data (at around the 25% level). The light baryons in AdS/QCD have been identified with solitonic configurations of the 4+1 dimensional fields, which are closely analogous to the baryons of the Skyrme model [8].

2. Bottom-Up AdS/QCD

In the bottom-up approach, we begin with the observation that the Kaluza-Klein modes of fields in an extra dimension might be identified with the radial excitations of hadrons in a confining gauge theory. As in the top-down approach, the quantum numbers of those excitations are identified with the transformations of the Kaluza-Klein modes under corresponding symmetries. In the hard-wall model [10, 11], we begin with a 5D SU(2)×SU(2) gauge theory. To reproduce the discrete spectrum of hadronic excitations, the spacetime geometry must be such that the spectrum of Kaluza-Klein gauge fields is discrete. The AdS₅ metric can be written,

$$ds^{2} = \frac{R^{2}}{z^{2}} \left(dx_{\mu} \eta^{\mu\nu} dx_{\nu} - dz^{2} \right), \qquad (2.1)$$

where $\eta_{\mu\nu}$ is th 3+1 dimensional Minkowski metric with components diag(1,-1,-1,-1); *R* is the AdS curvature, which we will set to 1. For simplicity, we take as the geometry a slice of AdS₅ between an ultraviolet cutoff scale $z = \varepsilon$ and an infrared scale $z = z_m$ related to Λ_{QCD} . Other choices for

the geometry may better match various aspects of QCD (such as running of the QCD coupling and the Regge spectrum [12]). The SU(2)×SU(2) gauge invariance is related to the approximate SU(2)×SU(2) chiral symmetry of the up and down quarks. (This may be extended to SU(3)×SU(3) in order to include the strange quark.) In order to break the chiral symmetry we introduce a scalar field that transforms in the bifundamental representation of the chiral symmetry, in analogy to the operator $\bar{q}q$. If this field is arranged to have a nonvanishing background profile, the gauge invariance is spontaneously broken. The 4+1 dimensional action takes the form,

$$S = \int d^5 x \sqrt{-g} \left(-\frac{1}{2g_5^2} \operatorname{Tr} \left(L_{MN} L^{MN} + R_{MN} R^{MN} \right) + \operatorname{Tr} \left(|D_M X|^2 + m_X^2 |X|^2 \right) \right), \qquad (2.2)$$

where L_{MN} and R_{MN} are the field strengths of the two sets of SU(2) gauge fields and m_X^2 is the squared mass of the bifundamental field X; contractions of indices are by the AdS₅ metric.

According to the AdS/CFT correspondence there is a relationship between the scaling dimension of a 3+1 dimensional operator and the mass of the corresponding field in the 4+1 dimensional dual theory. If we want we can fix the mass of the bifundamental scalar field X by this dictionary, although to do so we temporarily ignore running of the scaling dimension. The scaling dimension of the operator $\bar{q}q$ in the ultraviolet is three. By the AdS/CFT dictionary the mass of a field X with scaling dimension Δ is given by $m_X^2 = \Delta(\Delta - 4)$ in units of the AdS curvature, or in our case $m_X^2 = -3$. The negative squared mass does not lead to instability as a result of the curvature of Antide Sitter space, since it satisfies the Breitenlohner-Freedman bound $m_X^2 \ge -4$ in these units [13]. It is important to note that the choice $m_X^2 = -3$ is made here for definiteness, but it is not necessary to fix m_X^2 in this way. The AdS/CFT correspondence in the classical limit is not valid for QCD with finite N. However, for definiteness we may still choose to use the AdS/CFT correspondence to fix parameters in the model.

The equations of motion for the *z*-dependent scalar field background, with the gauge fields turned off, are,

$$\partial_z \left(\frac{1}{z^3} \partial_z X(z) \right) + \frac{3}{z^5} X(z) = 0.$$
(2.3)

The solutions for the scalar field background are,

$$X(z) = \left(\frac{m_q}{2}z + \frac{\sigma}{2}z^3\right),\tag{2.4}$$

where m_q and σ are arbitrary. By the AdS/CFT correspondence, the coefficient of the solution with divergent action has the interpretation of the source for the corresponding operator [3, 4]. We can think of the quark mass as a source for the operator $\bar{q}q$, so m_q has the interpretation of the quark mass (which we assume to be isospin-preserving). Similarly, the coefficient of the finiteaction solution has the interpretation of the expectation value of the corresponding operator, so σ has the interpretation of the chiral condensate $\langle \bar{q}q \rangle$. However, since the hard-wall model is phenomenological and does not follow from a precise AdS/CFT correspondence, these physical identifications of the parameters m_q and σ are not to be taken precisely.

If we want, we can fix g_5 by comparison with perturbative QCD at large $-q^2$, although the hard-wall model is not expected to be valid at high energies. The hard-wall prediction for the vector current two-point function is [10, 11],

$$i \int d^4x e^{iq \cdot x} \langle J^a_\mu(x) J^b_\nu(0) \rangle = \sum \frac{F_n^2}{q^2 - m_n^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_n^2} \right) \delta^{ab},$$
(2.5)

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Observable	Measured	Model
	(Central Value - MeV)	(MeV)
m_{π}	139.6	141
$m_{ ho}$	775.8	832
m_{a_1}	1230	1220
f_{π}	92.4	84.0
$F_{ ho}^{1/2}$	345	353
$F_{a_1}^{1/2}$	433	440
<i>8</i> ρππ	6.03	5.29

Table 1: Best fit of hard-wall model to seven observables, from Ref. [10].

Observable	Measured	Model
	(Central Value - MeV)	(MeV)
m_{K^*}	892	897
m_{ϕ}	1020	994
m_{K_1}	1272	1290
m_K	498	411
f_K	113	117
m_{f_2}	1275	1236
m_{ω_3}	1667	1656
m_{f_4}	2025	2058
m_{η}	548	520
m'_n	958	867

Table 2: Additional predictions of hard-wall model with three quark flavors, from Ref. [15].

where m_n is the mass of the n^{th} Kaluza-Klein mode, and the decay constants F_n are determined by the kinetic mixing between the zero-mode gauge field source for the vector current and the n^{th} excited vector meson in the Kaluza-Klein decomposition of the action. The perturbative one-loop result in SU(N) QCD with two flavors, valid for large $-q^2$, is

$$i \int d^4 x e^{iq \cdot x} \langle J^a_\mu(x) J^b_\nu(0) \rangle = \left(q_\mu q_\nu - g_{\mu\nu} q^2 \right) \delta^{ab} \frac{N}{24\pi^2} \log(-q^2).$$
(2.6)

Using the bulk-to-boundary propagator, the AdS/CFT correspondence gives us a trick for performing the sum in Eq. (2.5) for large $-q^2$. For more details in the context of AdS/QCD, I refer the reader to Refs. [10, 11]. The result is that the AdS/QCD prediction of the vector current two-point function is precisely of the perturbative form of Eq. (2.6) if we set $g_5^2 = \frac{24\pi^2}{N} = 8\pi^2$ when N = 3. It is not necessary to fix g_5 by matching to the ultraviolet, just as it was not necessary to fix the mass of the field X by matching to the conformal dimension of the operator $\bar{q}q$. These choices are made for definiteness, but they may be relaxed. If we do fix m_X^2 and g_5 this way, the model has three remaining parameters. A root-mean-squared fit of the remaining parameters to the central values of experimental and lattice data for seven observables gives $z_m = 1/(346 \text{ MeV})$, $\sigma = (308 \text{ MeV})^3$, $m_q = 2.3$ MeV [10]. The hard-wall AdS/QCD predictions with these values for the parameters are given in Table 1. A number of additional predictions are given in Table 2, taken from Emanuel Katz's talk at Lattice 2008 [15]. Some of the latter predictions required fitting an additional parameter analogous to m_q in the profile of the field X corresponding to the strange quark mass. The best fit for that parameter takes the value $m_s = 35$ MeV, which deserves some comment due to its small value. Recall that the parameters in the hard wall are only related to QCD parameters to the extent that the AdS/CFT correspondence is valid for this model. The parameter m_s is analogous to the strange quark mass, but it is not the strange quark mass. Furthermore, in the fit which led to Table 2, the parameter related to the strange quark condensate was assumed for simplicity to be the same as that for the up and down quarks [16]. A relation between observables and the parameters m_q and σ can be derived which resembles the Gell-Mann–Oakes–Renner relation [10],

$$m_\pi^2 f_\pi^2 = 2m_q \sigma. \tag{2.7}$$

This is a reflection of the pattern of chiral symmetry breaking, which is built in to the AdS/QCD model. Hence, if the strange quark condensate violates SU(3) isospin, then the parameter m_s has an even less direct relation to the strange quark mass.

3. Soft-Wall AdS/QCD

The AdS/QCD models described above are not expected to be valid much above the scale of the lightest vector resonances. For heavy resonances, the vector and axial-vector masses in the hard-wall model scale as $m_n^2 \sim n^2$. However, experimental data confirm the Regge behavior $m_n^2 \sim n$. Misha Shifman has stressed this difference between AdS/QCD and QCD [17]. It is possible to produce the Regge spectrum by effectively modifying the AdS₅ geometry [12]. One way to do this is to couple the fields in the AdS/QCD model to a background dilaton:

$$S = \int d^5 x \sqrt{-g} e^{-\Phi(x,z)} \mathscr{L}, \qquad (3.1)$$

where the appropriate background for the dilaton is $\Phi_0(z) \sim z^2$. Low-energy predictions are comparable to, but not the same as, those of the hard-wall model.

4. Other Predictions of AdS/QCD Models

4.1 Form Factors

It is straightforward to calculate form factors in AdS/QCD models, as the AdS/CFT correspondence teaches us that the contribution of a tower of resonances can be summed by use of the bulk-to-boundary propagator (see also Ref. [14]). Several authors have calculated various form factors of the pion, ρ and a_1 mesons. From these form factors were deduced moments of generalizaed parton distributions, charge radii and gravitational radii. For example, in the hard wall model with a particular choice of parameters [18, 19, 20]:



Figure 2: Dependence of observables on $m_X^2 = \Delta(\Delta - 4)$ in hard-wall model, holding fixed f_{π} , m_{π} , and m_{ρ} . Plot from Ref. [29].

4.2 Baryons

If one naively truncates an AdS/QCD model to the lightest modes, the gauge kinetic terms of type $F_{\mu z}F^{\mu z}$ include the Skyrme term with coefficient that depends on the 4+1 dimensional gauge coupling and the spacetime geometry [8]. The light baryons have therefore been identified as Skyrmions in AdS/QCD. There has been some debate as to the stability of predictions for baryons in this approach, as in the original Skyrme model, based on the relative importance of higher-dimension operators that are neglected in this approach. A description of baryons in terms of 4+1 dimensional solitons, as opposed to the Skyrmions in the effective 3+1 dimensional theory, has been discussed by several authors, for example in Refs. [21, 22]. A comparison of these approaches appears in [23]. An alternative description of baryons modeled as fundamental fermions in the extra dimension has also been considered, and has been seen to accurately reproduce the spectrum of excited Delta and nucleon resonances (see, for example, Refs. [24, 25]).

4.3 Light-front wave functions

Stan Brodsky and Guy de Teramond have noticed an intriguing relationship between the equations of motion for Kaluza-Klein modes of fields with general spin and scaling dimension in AdS/QCD models, and equations describing light-front wavefunctions of hadrons with general spin and orbital angular momentum (*e.g.* Ref. [26]). This is an interesting observation, and relates the radial direction of Anti-de Sitter space with partonic momenta inside the hadrons.

5. Universality in AdS/QCD

Certain observables in AdS/CFT models at finite temperature have been found to be completely independent of the details of the model. A famous example is the ratio of shear viscosity η to entropy density *s*, which in natural units is predicted to be [27],

$$\frac{\eta}{s} = \frac{1}{4\pi}.\tag{5.1}$$

A more recent example is the ratio of electrical conductivity σ to charge susceptibility χ , which depends only on the temperature and number of spacetime dimensions [28]. Universal predictions provide the strongest test of the AdS/CFT correspondence as applied to QCD. Despite the absence

of complete universality of most AdS/QCD predictions, it is interesting that the various AdS/QCD models make comparable predictions for low-energy observables. It is worthwhile to better understand which observables are approximately independent of the details of the AdS/QCD model, and which details of the model are unimportant. For example, upon varying the mass of the field X by $\pm 20\%$, as long as the remaining parameters are chosen so as to correctly reproduce a small number of observables $(m_{\pi}, f_{\pi} \text{ and } m_{\rho})$ the remaining low-energy observables of Table 1 were found to vary by only a few percent, as in Fig. 2 [29]. Perhaps the surprising success of AdS/QCD models at low energies is the result of such universality in its predictions.

6. Additional Applications of AdS/CFT models

Models similar in spirit to AdS/QCD have been applied to other strongly-interacting dynamical systems. Holographic technicolor models [31]are based on AdS/QCD, and AdS/CFT methods allow for calculation of precision electroweak observables in these models. These models are examples of Higgsless models of electroweak symmetry breaking, in which unitarity of longitudinal W boson scattering is the result of interactions involving the massive Kaluza-Klein excitations of the electroweak gauge bosons.

An exciting recent application of the AdS/CFT correspondence is to condensed matter systems. An important development in this direction was the construction of a spacetime geometry whose isometries are the same as the nonrelativistic conformal group [32]. There are intriguing similarities of some models to high-temperature superconductors and systems of cold atoms [33].

7. Summary

AdS/QCD models share the features of a number of earlier approaches to modeling QCD at low energies. AdS/QCD models generally predict low-energy observables at the 10-20% level, but do not fare as well at high energies. It is not yet clear why these models work as well as they do, but some predictions have been found to be universal as details of the model are varied.

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References

- J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200].
- [2] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, "Is Rho Meson A Dynamical Gauge Boson Of Hidden Local Symmetry?," Phys. Rev. Lett. 54, 1215 (1985).

- [3] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].
- [4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109].
- [5] J. Polchinski and M. J. Strassler, "The string dual of a confining four-dimensional gauge theory," arXiv:hep-th/0003136.
- [6] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities," JHEP 0008, 052 (2000) [arXiv:hep-th/0007191].
- [7] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, "Towards a holographic dual of large-N(c) QCD," JHEP 0405, 041 (2004) [arXiv:hep-th/0311270].
- [8] T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. 113, 843 (2005) [arXiv:hep-th/0412141]; "More on a holographic dual of QCD," Prog. Theor. Phys. 114, 1083 (2005) [arXiv:hep-th/0507073].
- [9] E. Witten, "Anti-de Sitter space, thermal phase transition, and confinement in gauge theories," Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].
- [10] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, "QCD and a Holographic Model of Hadrons," Phys. Rev. Lett. 95, 261602 (2005) [arXiv:hep-ph/0501128].
- [11] L. Da Rold and A. Pomarol, "Chiral symmetry breaking from five dimensional spaces," Nucl. Phys. B 721, 79 (2005) [arXiv:hep-ph/0501218].
- [12] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, "Linear Confinement and AdS/QCD," Phys. Rev. D 74, 015005 (2006) [arXiv:hep-ph/0602229].
- [13] P. Breitenlohner and D. Z. Freedman, "Positive Energy In Anti-De Sitter Backgrounds And Gauged Extended Supergravity," Phys. Lett. B 115, 197 (1982).
- [14] J. Hirn and V. Sanz, "(Not) summing over Kaluza-Kleins," Phys. Rev. D 76, 044022 (2007) [arXiv:hep-ph/0702005].
- [15] E.Katz, Lattice 2008 plenary talk, http://conferences.jlab.org/lattice2008/
- [16] Emanuel Katz, private communication.
- [17] M. Shifman, "Highly excited hadrons in QCD and beyond," arXiv:hep-ph/0507246.
- [18] H. J. Kwee and R. F. Lebed, "Pion Form Factors in Holographic QCD," JHEP 0801, 027 (2008) [arXiv:0708.4054 [hep-ph]].
- [19] H. R. Grigoryan and A. V. Radyushkin, "Form Factors and Wave Functions of Vector Mesons in Holographic QCD," Phys. Lett. B 650, 421 (2007) [arXiv:hep-ph/0703069].
- [20] Z. Abidin and C. E. Carlson, "Hadronic Momentum Densities in the Transverse Plane," Phys. Rev. D 78, 071502 (2008) [arXiv:0808.3097 [hep-ph]].
- [21] K. Nawa, H. Suganuma and T. Kojo, "Baryons in Holographic QCD," Phys. Rev. D 75, 086003 (2007) [arXiv:hep-th/0612187].
- [22] A. Pomarol and A. Wulzer, "Baryon Physics in Holographic QCD," Nucl. Phys. B 809, 347 (2009) [arXiv:0807.0316 [hep-ph]].
- [23] K. Nawa, H. Suganuma and T. Kojo, "Brane-induced Skyrmion on S³: baryonic matter in holographic QCD," Phys. Rev. D 79, 026005 (2009) [arXiv:0810.1005 [hep-th]].

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- [24] G. F. de Teramond and S. J. Brodsky, "Baryonic states in QCD from gauge / string duality at large N(c)," arXiv:hep-th/0409074.
- [25] D. K. Hong, T. Inami and H. U. Yee, "Baryons in AdS/QCD," Phys. Lett. B 646, 165 (2007) [arXiv:hep-ph/0609270].
- [26] S. J. Brodsky and G. F. de Teramond, "Hadronic spectra and light-front wavefunctions in holographic QCD," Phys. Rev. Lett. 96, 201601 (2006) [arXiv:hep-ph/0602252].
- [27] P. Kovtun, D. T. Son and A. O. Starinets, "Viscosity in strongly interacting quantum field theories from black hole physics," Phys. Rev. Lett. 94, 111601 (2005) [arXiv:hep-th/0405231].
- [28] P. Kovtun and A. Ritz, "Universal conductivity and central charges," arXiv:0806.0110 [hep-th].
- [29] J. Erlich and C. Westenberger, in preparation.
- [30] C. Csaki, M. Reece and J. Terning, "The AdS/QCD Correspondence: Still Undelivered," arXiv:0811.3001 [hep-ph].
- [31] J. Hirn and V. Sanz, "A negative S parameter from holographic technicolor," Phys. Rev. Lett. 97, 121803 (2006) [arXiv:hep-ph/0606086]; C. D. Carone, J. Erlich and J. A. Tan, "Holographic Bosonic Technicolor," Phys. Rev. D 75, 075005 (2007) [arXiv:hep-ph/0612242]; C. D. Carone, J. Erlich and M. Sher, "Holographic Electroweak Symmetry Breaking from D-branes," Phys. Rev. D 76, 015015 (2007) [arXiv:0704.3084 [hep-th]]; D. K. Hong and H. U. Yee, "Holographic estimate of oblique corrections for technicolor," Phys. Rev. D 74, 015011 (2006) [arXiv:hep-ph/0602177].
- [32] D. T. Son, "Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry," Phys. Rev. D 78, 046003 (2008) [arXiv:0804.3972 [hep-th]]; A. Adams,
 K. Balasubramanian and J. McGreevy, "Hot Spacetimes for Cold Atoms," JHEP 0811, 059 (2008) [arXiv:0807.1111 [hep-th]].
- [33] S. S. Gubser and S. S. Pufu, "The gravity dual of a p-wave superconductor," JHEP 0811, 033 (2008) [arXiv:0805.2960 [hep-th]]; M. M. Roberts and S. A. Hartnoll, "Pseudogap and time reversal breaking in a holographic superconductor," JHEP 0808, 035 (2008) [arXiv:0805.3898 [hep-th]].