PROCEEDINGS OF SCIENCE

Elliptic Flow and Shear Viscosity in a Parton Cascade Approach

Graziella Ferini[*]
INFN-LNS, Via S. Sofia 62, I-95125 Catania, Italy
E-mail: `ferini@lns.infn.it`

Maria Colonna
INFN-LNS, Via S. Sofia 62, I-95125 Catania, Italy
E-mail: `colonna@lns.infn.it`

Massimo Di Toro
INFN-LNS, Via S. Sofia 62, I-95125 Catania, Italy
Dipartimento di Fisica e Astronomia, Via S. Sofia 64, I-95125 Catania, Italy
E-mail: `ditoro@lns.infn.it`

Vincenzo Greco
INFN-LNS, Via S. Sofia 62, I-95125 Catania, Italy
Dipartimento di Fisica e Astronomia, Via S. Sofia 64, I-95125 Catania, Italy
E-mail: `greco@lns.infn.it`

Within a parton cascade approach we investigate the scaling of the differential elliptic flow $v_2(p_T)$ with eccentricity ε_x and system size and its sensitivity to finite shear viscosity. A central issue we address here is the relation between $v_2(p_T)/\varepsilon_x$ and $v_2(p_T)/\langle v_2 \rangle$ scalings, as raised by recent experimental observations. We show that, with the inclusion of a freeze out condition, a cascade approach for a plasma at finite η/s can supply a reliable tool for the description of several observables on elliptic flow such as the breaking of $v_2(p_T)/\varepsilon_x$ scaling and the persistence of $v_2(p_T)/\langle v_2 \rangle$ scaling. Results are moreover compatible with a coalescence plus fragmentation hadronization mechanism acting at intermediate p_T.

High-pT Physics at LHC - Tokaj'08
March 16 - 19 2008
Tokaj, Hungary

---

[*] Speaker.

PoS(HIGH-pTLHC)019

1. Introduction

The first stage at the Relativistic Heavy Ion Collider (RHIC) has successfully shown the formation of hot and dense matter which behaves like a nearly perfect fluid. Such a conclusion mainly relies on the large value of the elliptic flow v_2 [1, 2] together with a mass dependent radial flow effect on v_2 and p_T spectra. However there is a growing evidence of a breakdown of the ideal hydrodynamical behaviour especially in the intermediate p_T region ($1.5 < p_T < 5$ GeV) and in peripheral collisions, questioning the assumption of vanishing shear viscosity. A currently debated topic is in particular the effect of the conjectured minimal shear viscosity to entropy density ratio $\eta/s \geq 1/4\pi$ [3] on collective flows. Indeed a first recent evaluation of shear viscosity in lattice QCD (lQCD) is consistent with the lower bound [4, 5] and shows a mild evolution with temperature in the range covered by RHIC [5]. Furthermore, the description of the elliptic flow, the strong scattering of heavy quarks, the measurements of p_T fluctuations, all point [6] to a η/s that should be close to the bound and much smaller than the one expected in a pQCD regime [7], (about 5-10 times the lower bound). This is also confirmed by an estimation based on Knudsen number analysis of v_2/ε_x [8] and by early calculations within viscous hydrodynamics [9, 10] and parton cascade [11] approaches, which both indicate a significant reduction of v_2 already at finite η/s close to the lower bound. However a detailed investigation of dissipative effects on differential elliptic flow $v_2(p_T)$ within a transport theory has been only recently started [12, 13, 14].

Here we present a study of the elliptic flow and its scaling properties as a function of p_T at finite η/s in the range $(4\pi)^{-1} < \eta/s < \pi^{-1}$. The analysis is based on a parton cascade approach for massless particles and it is mainly focused on the intermediate p_T region. The main idea is to keep the η/s of the medium constant during the collision dynamics by rearranging the parton cross section according to the local density and mean momentum values as suggested in [12]. Simulations have been carried out for a large range of impact parameters in $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV. Some simulations have been performed also for $Cu+Cu$ for a first investigation of the system size dependence. A first issue that we discuss is the scaling of the v_2 with the spatial eccentricity $\varepsilon_x = \langle y^2 - x^2 \rangle / \langle x^2 + y^2 \rangle$ and the system size. We show that if η/s is kept constant down to the thermal freeze-out ($\varepsilon \sim 0.2$ GeV/fm^3) a parton cascade exhibits a v_2/ε_x scaling in the whole p_T range investigated (up to 3.5 GeV). Therefore the scaling is present also at finite η/s and hence it is not a unique feature of ideal hydrodynamics [15]. However experimentally the (in)dependence of v_2/ε_x on the centrality of the collision and on the system size is indeed a delicate issue as raised by recent publications from PHENIX [16] and STAR [17]. We point out that once a suitable freeze-out condition is introduced at $\varepsilon_c \sim 0.7$ GeV/fm^3 a cascade approach at finite viscosity can account for the breaking of the scaling for $v_2(p_T)/\varepsilon_x$ together with a persisting scaling for $v_2(p_T)/\langle v_2 \rangle$, as experimentally observed. Furthermore the shape of $v_2(p_T)$ at $\eta/s \sim 1/2\pi$ is similar to the one conjectured in coalescence models [18]. We finally emphasize that a definitive evaluation of η/s is entangled with the observation of quark number scaling in the same p_T range hence hadronization by coalescence plus fragmentation has to be self-consistently included.

2. The parton cascade

We have developed a $3+1$ dimensional Montecarlo cascade [13] for on-shell partons based on the stochastic interpretation of the transition rate. Such an interpretation is free from several unphysical drawbacks and particularly suitable for an extension to multiparticle collisions as pointed out by Z. Xu and C. Greiner [19]. The evolution of parton distribution function from initial conditions through elastic two-body scatterings is governed by the Boltzmann equation

$$p_\mu \partial^\mu f_1 = \int\!\!\!\int\!\!\!\int_{2\,1'\,2'} (f_{1'} f_{2'} - f_1 f_2) |\mathcal{M}_{1'2' \to 12}|^2 \delta^4 (p_1 + p_2 - p'_1 - p'_2) \tag{2.1}$$

where $\int_j = \int_j d^3 p_j / \left[(2\pi)^3 2E_j\right]$, $\mathcal{M}$ denotes the transition matrix for the elastic processes and f_j are the particle distribution functions.

For the numerical implementation, we discretize the space into cells small respect to the system size and we use such cells to calculate all the local quantities. Several checks have been performed as in [19] to test the validity of the code and to choose a good discretization for convergency of the results for the elliptic flow that is the main observable analyzed in the present work.

In kinetic theory under ultra-relativistic conditions the shear viscosity can be expressed as [20]

$$\eta = \frac{4}{15} \rho \langle p \rangle \lambda \tag{2.2}$$

with ρ the parton density, λ the mean free path and $\langle p \rangle$ the average momentum. Therefore considering that the entropy density for a massless gas is $s = \rho(4 - \mu/T)$, μ being the chemical potential, we get:

$$\eta/s = \frac{4}{15} \frac{\langle p \rangle}{\sigma_{tr} \rho (4 - \mu/T)} \tag{2.3}$$

where σ_{tr} is the transport cross section, i.e. the $\sin^2\theta$ weighted cross section. We use a pQCD inspired cross section with the infrared singularity regularized by Debye thermal mass m_D [21]:

$$\frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{(t + m_D^2)^2} \left(\frac{1}{2} + \frac{m_D^2}{2s} \right) \tag{2.4}$$

where s, t are the Mandelstam variables and $m_D = 0.7$ GeV.

Our approach is to artificially keep the η/s of the medium constant during the dynamics of the collisions in a way similar to [12, 14], but evaluating locally in space and time the strength of the cross section needed to keep the η/s constant. From Eq. (2.3) we see that assuming locally the thermal equilibrium this can be obtained evaluating in each cell the cross section according to:

$$\sigma_{tr} = \frac{4}{15} \frac{\langle p \rangle}{\rho (4 - \mu/T)} \frac{1}{\eta/s} \tag{2.5}$$

with η/s set from 1 to 4 in units of the minimum value. We notice that an indication on the temperature and time dependence of the cross section can be obtained considering the simple case of a free massless gas for which $s = g\frac{2\pi^2}{45}T^3$ and therefore neglecting μ in Eq. (2.5) one gets $\sigma_{tr} \sim T^{-2}$ for $\eta/s = 1/4\pi$. Furthermore a simple Bjorken expansion which means $T \sim \tau^{-1/3}$ gives $\sigma_{tr} \propto \tau^{2/3}$ which is the approximate prescription adopted in [12, 14]. In our approach σ_{tr} is instead evaluated locally in space and time, however in the central region of the fireball our results show a time and temperature dependence in agreement with the above estimate, see Fig. 1.

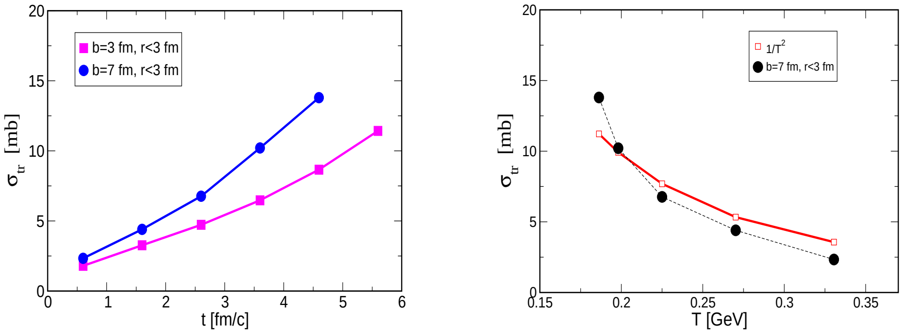

Figure 1: Left: Time dependence of the cross section in a central region of rapidity ($|y| < 0.2$) and transverse radius $r < 3$ fm. Right: Temperature dependence of the cross section in the central region compared with the T^{-2} dependence.

Partons are initially distributed according to a mixture of the density of participant nucleons (80%) and of binary collisions (20%) calculated with a Glauber model. The eccentricity is therefore similar to the one used in standard calculations [22]. We also start our simulation like in hydrodynamics at a time $t = 0.6$ fm, assuming free-streaming evolution from $t = t_0$ to $t = 0.6 fm$. Partons with $p_T < p_0 =$2 GeV are distributed according to a thermalized spectrum, while for $p_T > p_0$ we take the spectrum of non-quenched minijets as calculated in [23]. Three-body scatterings [19] and inelastic processes are not taken into account since, looking at collective modes like the elliptic flow, we implicitly assume that the results do not depend significantly on the details of the collision kinematics once the η/s value has been fixed. Furthermore, the hadronic re-scattering together with the formation and decay of the resonances are neglected. This is justified by the fact that the bulk of $v_2(p_T)$ develops in the early stage of the reaction, i.e. well before hadronization sets in, as found by several theoretical approaches [24, 25, 1] and more recently confirmed experimentally [26, 27].

3. Results

A first objective of our study is to investigate the scaling behavior of the elliptic flow with the initial eccentricity and the system size to see if such a scaling typical of a hydrodynamical behavior [15] persists also in a cascade approach. The interest for such a behavior is triggered by the recent observation by the PHENIX Collaboration [16] of a scaling of $v_2(p_T)/\langle v_2\rangle$ up to $p_T \sim 3$ GeV, a region usually considered out of the range where hydrodynamics should work. In Fig. 2 our results for the parton $v_2(p_T)/\varepsilon_x$ (left panel) and $v_2(p_T)/k\langle v_2\rangle$ (right panel) in the central rapidity region ($|y| < 0.35$) are shown for different impact parameters and the two systems Au+Au (filled symbols) and Cu+Cu (open symbols) at 200 AGeV when η/s is kept constant at $1/4\pi$. The dot-dashed and dashed lines refer to Au+Au at b=7 fm with $\eta/s = 1/2\pi$ and $\eta/s = 1/\pi$ respectively.

A first important result is the clear observation of a scaling as a function of centrality and system size for the $v_2(p_T)/\langle v_2\rangle$ which approximately holds also for $v_2(p_T)/\varepsilon_x$. This indicates that the scaling $v_2(p_T)/\langle v_2\rangle$, which is advocated as a signature of the hydrodynamical behavior [16], is a more general property that holds also at finite mean free path or shear viscosity at least for values close to the lower bound. Moreover the scaling is shown to persist also at higher p_T (~ 3

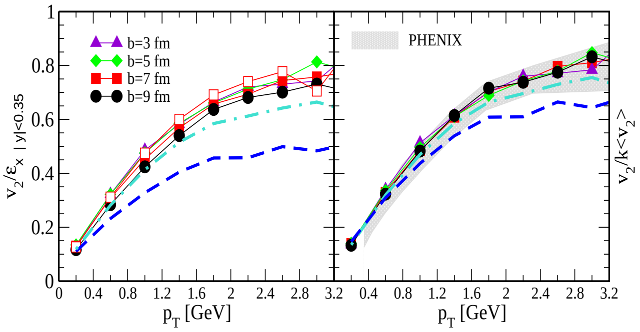

Figure 2: Parton $\frac{v_2}{\varepsilon_x}$ (left panel) and $\frac{v_2}{k\langle v_2\rangle}$ (right panel) in the central rapidity region ($|y| < 0.35$) for Au+Au (filled symbols) and Cu+Cu (open symbols) collisions at $\sqrt{s} = 200 AGeV$. Different symbols refer to cascade simulations at various impact parameters for $\eta/s = 1/4\pi$. In both panels also results for Au+Au at b=7 fm with $\eta/s = 1/2\pi$ (dot-dashed line) and $\eta/s = 1/\pi$ (dashed line) are shown. The grey band in the right panel refers to charged particles data in the pseudorapidity range $|\eta| \leq 0.35$ with the constant $k = 3.1$ as in [16].

GeV) where not only the scaling but also the saturation shape is correctly reproduced by the parton cascade approach. We mention that our results are in qualitative agreement with predictions from viscous hydrodynamics [9, 10], however we are not aware of an explicit investigation of $v_2(p_T)/\varepsilon_x$ and $v_2(p_T)/\langle v_2\rangle$ scaling within hydrodynamics. Our simulations show a good sensitivity to the shear viscosity especially at intermediate p_T where $v_2(p_T)/\varepsilon_x$ drops of about 40% when increasing η/s by a factor 4 above the lower bound. Obviously a smaller sensitivity is observed for the ratio $v_2/\langle v_2\rangle$, being both quantities affected by the η/s.

The equivalence between $v_2(p_T)/\langle v_2\rangle$ and $v_2(p_T)/\varepsilon_x$ scalings has been questioned by latest results from STAR [17], which show that the $v_2(p_T)$ scaled by the participant eccentricity ε_x is not independent on centrality while a good scaling with centrality is still observed for $v_2(p_T)/\langle v_2\rangle$ ratio. In the following we will show that both these observations can be recovered in our approach.

3.1 Effect of QGP freeze-out

In this subsection we investigate the effect of a freeze-out condition on the elliptic flow. Freeze-out conditions are justified by the fact that, at a critical value for energy density, hadronization sets in and parton dynamics is no longer acting. To take into account such an effect we stop the interactions among partons as the local energy density drops below 0.7 GeV/fm^3, an intermediate value in the range corresponding to a mixed quark-hadron phase [1]. The calculations previously discussed were performed with a freeze-out condition at $\varepsilon = 0.2$ GeV/fm^3 which corresponds to the end of a mixed phase or roughly to a hadronic-thermal freeze-out. We have checked that a freeze-out at 0.2 GeV/fm^3 is practically identical to consider no energy density freeze-out at all. This is in agreement with the observation that both theoretically and experimentally the elliptic flow does not develop significantly during the hadronic stage [1, 24, 25, 26, 27], hence we will refer to such a calculation as the one without freeze-out.

When the freeze-out condition is implemented a sizeable reduction for the elliptic flow is observed (see Fig. 2 in [13]), especially for the most peripheral collisions and at larger p_T. Correspondingly the scaling of elliptic flow with the initial spatial eccentricity is broken (see filled symbols in left panel of Fig. 3). In particular v_2/ε_x varies of nearly $40-50\%$ from b=3 fm to b=9 fm in the intermediate p_T region (<3 GeV). The amount of such a spreading is consistent with the data reported by [17] for the centrality selections $0-10\%$ and $10-40\%$, with central collisions exhibiting a bigger elliptic flow to eccentricity ratio than the peripheral ones. We are therefore driven to the conclusion that the breaking of the $v_2(p_T)/\varepsilon$ scaling, as observed in Fig. 3, traces back to the freeze out physics, which deserves a deeper investigation. On the other hand, the scaling of $v_2/\langle v_2 \rangle$ with the impact parameter is still observed (see right panel in Fig. 3). This is partially due to the fact that, at variance with ideal hydrodynamics predictions, $\langle v_2 \rangle/\varepsilon_x$ does not exactly scale with centrality, as pointed out by STAR and PHOBOS [28, 17] measurements and also qualitatively confirmed by our approach ([13]).

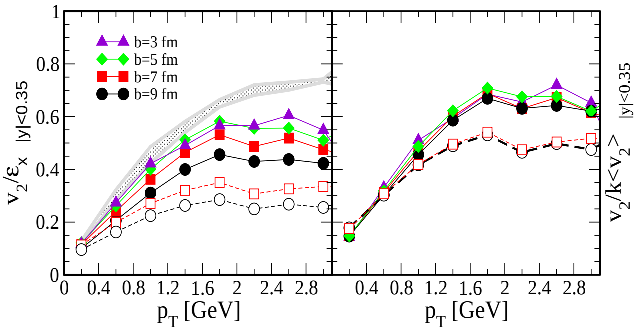

Figure 3: Same as in Fig.2 with freeze out condition: filled symbols refer to calculations at $\eta/s = 1/4\pi$; open symbols are for b=7 fm (squares) and b=9 fm (circles) calculations at $\eta/s = 1/\pi$. The grey band in the left panel refers to the results of simulations without freeze out (see left panel in Fig. 2).

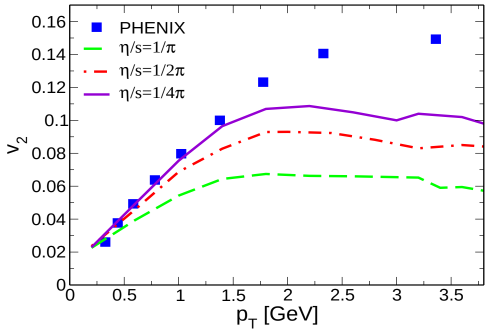

Figure 4: Differential elliptic flow for Au+Au collisions at b=5 fm, $|y| \leq 0.35$ and $\eta/s = 1/4\pi$ (solid curve), $\eta/s = 1/2\pi$ (dot-dashed curve) and $\eta/s = 1/\pi$ (dashed curve). Results from cascade are compared with data from [16] (squares).

As a last point we discuss how information on the viscosity to entropy density ratio of the RHIC plasma can be inferred from the $v_2(p_T)$ absolute value. One has to consider that at intermediate p_T there are several evidences for hadronization via coalescence and it has been shown that due to a coalescence mechanism the parton v_2 translates into a nearly doubled hadron v_2 [18]. Therefore a definite evaluation of η/s from $v_2(p_T)$ data needs a further development of the parton cascade approach that includes self-consistently the coalescence and fragmentation process in order to account for the baryon-meson quark number scaling. However we refrain from using simple naive coalescence formula [29] here, considering that it has been shown that space-momentum correlation and the freeze-out hypersurface can significantly affect the relation between quark and hadron v_2 [30]. Nonetheless from Fig.4 we notice that for $\eta/s = \pi^{-1}$ the parton elliptic flow, with a quite small slope at low p_T, saturates at about 6%. Even assuming a coalescence mechanism in the hadronization phase, this value appears to be too low to reproduce the baryon and meson v_2. This provides anyway an indication that a η/s as high as 4 times the minimum value should be ruled out for the RHIC fluid and the viscosity is therefore quite smaller than pQCD estimates [7]. On the other hand the results with both $\eta/s = 1/4\pi$ and $\eta/s = 1/2\pi$ could be quite close to the experimental data within a coalescence picture. Such a range of values, to be narrowed in the next future, is sligthly larger than the first estimates with viscous hydrodynamics [9, 10], slightly below the one based on Knudsen number analysis [8] and compatible with the best present evaluation in lQCD [5]. However we notice that for a quantitative estimate we also need to include the effect of a non vanishing $\varepsilon - 3p$ related to the cross-over transition and that is known to lower the sound velocity and hence the $v_2(p_T)$ [10].

4. Summary and Conclusions

We have investigated the dependence on the shear viscosity of the elliptic flow $v_2(p_T)$ and its scaling properties. As a first result we find that the approximate scaling of $v_2(p_T)/\varepsilon_x$ advocated as a signature of the perfect hydrodynamical behavior [16] can still hold also at finite viscosity and in a parton cascade approach. However such a scaling versus centrality and system size is present only if one makes the fireball evolve down to energy density $\varepsilon \sim 0.2$ GeV/fm^3 corresponding typically to the end of a mixed phase. If a freeze-out condition for the partonic dynamics is put at $\varepsilon \sim 0.7$ GeV/fm^3 then a sizeable breaking of the $v_2(p_T)/\varepsilon_x$ scaling is seen while $v_2(p_T)/\langle v_2 \rangle$ still scales. This is in qualitative agreement with the recent experimental data from STAR [17] indicating that freeze-out of QGP dynamics with the consequent change of η/s should be more thoroughly investigated. As a final remark we notice that without any freeze-out condition the $v_2(p_T)$ at parton level would be close to the data for $\eta/s = 1/4\pi$, see Fig.2. On the other hand we consider such an agreement misleading because it would not be compatible with the enhancement of v_2 due to coalescence and the observation of quark number scaling [18]. Instead the freeze-out condition, leading to a reduction of v_2, seems to pave the way for a consistency among the different available observations on elliptic flow: the breaking of $v_2(p_T)/\varepsilon_x$, the persistence of $v_2(p_T)/\langle v_2 \rangle$ scaling and the presence of a coalescence plus fragmentation hadronization mechanism acting at intermediate p_T. We therefore conclude that a safe evaluation of the shear viscosity to entropy density ratio from the available data on $v_2(p_T)$ necessitates a cascade approach that includes self-consistently hadronization by coalescence and fragmentation.

5. Acknoweldgments

We thank B. Muller for stimulating comments on the manuscript.

References

[1] P. F. Kolb and U. W. Heinz, arXiv:nucl-th/0305084; P. Houvinen, arXiv:nucl-th/0305064; in *Quark Gluon Plasma 3,* R.C. Hwa and X.N. Wang (Eds.), World Scientific, Singapore, 2004.

[2] K. H. Ackermann *et al.* [STAR Collaboration], Phys. Rev. Lett. **86** (2001) 402

[3] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **94** (2005) 111601

[4] A. Nakamura and S. Sakai, Phys. Rev. Lett. **94** (2005) 072305

[5] H. B. Meyer, Phys. Rev. D **76** (2007) 101701

[6] R. A. Lacey *et al.*, Phys. Rev. Lett. **98** (2007) 092301

[7] P. Arnold, G. D. Moore and L. G. Yaffe, JHEP **0305** (2003) 051

[8] H. J. Drescher, A. Dumitru, C. Gombeaud and J. Y. Ollitrault, Phys. Rev. C **76** (2007) 024905

[9] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99** (2007) 172301

[10] H. Song and U. W. Heinz, Phys. Lett. B **658** (2008) 279; Phys. Rev. C **78** (2008) 024902

[11] Z. Xu, C. Greiner and H. Stocker, Phys. Rev. Lett. **101** (2008) 082302

[12] D. Molnar in N. Armesto *et al.*, J. Phys. G **35** (2008) 054001

[13] G. Ferini, M. Colonna, M. Di Toro and V. Greco, arXiv:0805.4814 [nucl-th].

[14] D. Molnar, arXiv:0806.0026 [nucl-th].

[15] R. S. Bhalerao, J. P. Blaizot, N. Borghini and J. Y. Ollitrault, Phys. Lett. B **627** (2005) 49

[16] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **98** (2007) 162301

[17] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. C **77** (2008) 054901

[18] R.J. Fries, V. Greco, P. Sorensen, Ann. Rev. Nucl. Part. Sci., **58** (2008) 177, arXiv:0807.4939 [nucl-th]

[19] Z. Xu and C. Greiner, Phys. Rev. C **71** (2005) 064901

[20] S.R. De Groot et al., *Relativistic Kinetic Theory*, North-Holland, Amsterdam, 1980.

[21] D. Molnar, and M. Gyulassy, Nucl. Phys.**A697** (2002) 495; Erratum in Nucl. Phys. **A703** (2002) 893.

[22] We note that our eccentricity is usually quoted in literature as ε_{std}. However, we use a standard Glauber model with no fluctuations (except for the numerical noise) and hence ε_{std} does not significantly differ from the so-called participant eccentricity ε_{part} like in hydro calculations.

[23] Y. Zhang, G. I. Fai, G. Papp, G. G. Barnafoldi and P. Levai, Phys. Rev. C **65** (2002) 034903

[24] B. Zhang, M. Gyulassy and C. M. Ko, Phys. Lett. B **455** (1999) 45

[25] Z. w. Lin and C. M. Ko, Phys. Rev. C **65** (2002) 034904

[26] S. Afanasiev *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **99** (2007) 052301

[27] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. Lett. **99** (2007) 112301

[28] B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **98** (2007) 242302

[29] P. F. Kolb, L. W. Chen, V. Greco and C. M. Ko, Phys. Rev. C **69** (2004) 051901

[30] S. Pratt and S. Pal, Phys. Rev. C **71** (2005) 014905; D. Molnar, arXiv:nucl-th/0408044; V. Greco and C. M. Ko, arXiv:nucl-th/0505061.

PoS(HIGH-pTLHC)019