# Elliptic Flow Without Using Hydrodynamics[*]

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Azimuthal anisotropy is studied by taking into account the ridges created by semi-hard scattering, which is sensitive to the initial spatial configuration in non-central heavy-ion collisions, without requiring rapid thermalization. Phenomenological properties of the bulk and ridge behaviors are used as inputs to determine the elliptic flow of pion and proton at low $p_T$. At intermediate $p_T$ the recombination of shower partons with thermal ones becomes more important. The $\phi$ dependence of shower partons arises from the variation of the in-medium path length of the hard parton that generates the shower. The $p_T$ dependence of $v_2$ is therefore very different at intermediate $p_T$ compared to that at low $p_T$. Because of the difference of $v_2$'s for thermal and shower partons, constituent quark number scaling of $v_2$ for hadrons is violated.

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## 1. Introduction

In relativistic heavy-ion collisions the subjects of transverse momentum ($p_T$) distribution and azimuthal anisotropy have been given prominent attention both experimentally and theoretically from the very beginning [1]-[6]. In the second half of this decade details of the properties of elliptic flow have continued to be studied experimentally with greater accuracy [7]-[13]. The hydrodynamical model at low $p_T$ [14]-[17] and the recombination-coalescence model at intermediate $p_T$ [18]-[21] have described the data on elliptic flow so well that little room seems to exist for further improvement. In this paper we study the problem of azimuthal anisotrophy in the framework of our version of the recombination model [22] in which we consider thermal and shower partons. We take into account the ridges at low $p_T$ generated by semi-hard scattering near the surface and the shower partons at higher $p_T$. We calculate the second harmonic $v_2$ for pion and proton, as well as for a light quark, and show that, when $p_T$ is extended to the intermediate region, there is significant departure from the constituent quark number scaling result suggested by simple consideration of quark recombination [20, 21].

## 2. Azimuthal anisotropy at low $p_T$

Let us first review our approach to elliptic flow at low $p_T$ without rapid thermalization [23]. Instead of assuming the meaningfulness of thermodynamical quantities, like pressure and temperature, at early time, we recognize that when the parton transverse-momentum is around 2 - 3 GeV/c, the rate of low $Q^2$ semi-hard scattering can be high, while the time scale involved is low enough ($\sim 0.1$ fm/c) to be sensitive to the initial spatial configuration of the collision system. When such scattering occurs near the surface of the overlap region in the transverse plane, each semi-hard parton creates a ridge in $\Delta\eta$ and $\Delta\phi$ [24]. Ridges due to the scattering of low-$x$ partons ($< 0.03$) are abundantly produced even if triggers are not used to select events to examine their properties. The effect of such ridges on both the $p_T$ and $\phi$ dependences of the produced particles should not be ignored.

The direction of a scattered parton is random, but the average direction of all outward partons near the surface is normal to the surface. The formation of the ridge of hadrons takes some time for the transverse expansion to complete, but the directions in which the ridge partons flow are determined by spatial configuration at early time. We do not rule out the applicability of hydrodynamics at some point of the expansion process when equilibration is established. However, fast thermalization is not needed if semi-hard scattering can initiate the anisotropic expansion by emitting semi-hard jets normal to the overlap surface at early time.

The overlap region in the transverse plane for two nuclei of radius $R_A$ at impact-parameter $b$ apart is, assuming simple geometry with sharp boundaries, the almond-shaped area bounded by two circular arcs whose maximum angle is $\Phi$, where

$$\cos\Phi = \hat{b} \equiv b/2R_A \, , \tag{2.1}$$

and the angle $\phi$ within the arcs satisfies $\phi \in \mathscr{R}$, which is a set of angles defined by

$$|\phi| < \Phi \qquad \text{and} \qquad |\pi - \phi| < \Phi. \tag{2.2}$$

Semi-hard jets normal to the surface lead to the development of the ridges ($R$) of hadrons in the final state with the $\phi$ dependence given by

$$R(p_T, \phi) = R(p_T)\Theta(\phi) ,\tag{2.3}$$

where

$$\Theta(\phi) = \theta(\Phi - |\phi|) + \theta(\Phi - |\pi - \phi|) .\tag{2.4}$$

The bulk ($B$) medium has no $\phi$ dependence and will be denoted by $B(p_T)$. The single-particle distribution at low $p_T$ is then

$$\frac{dN}{p_T dp_T d\phi} = B(p_T) + R(p_T)\Theta(\phi) .\tag{2.5}$$

The second harmonic in the $\phi$ distribution is

$$v_2(p_T) = \langle \cos 2\phi \rangle = \frac{\int_0^{2\pi} d\phi \, \cos 2\phi \, dN/p_T dp_T d\phi}{\int_0^{2\pi} d\phi \, dN/p_T dp_T d\phi} .\tag{2.6}$$

When Eq. (2.5) is used in the above, we obtain

$$v_2(p_T, b) = \frac{\sin 2\Phi(b)}{\pi B(p_T)/R(p_T) + 2\Phi(b)} .\tag{2.7}$$

At very low $p_T$, $\pi B/R$ can be very large, then we can have the even simpler formula

$$v_2(p_T, b) \simeq \frac{R(p_T)}{\pi B(p_T)} \sin 2\Phi(b) ,\tag{2.8}$$

where the $p_T$ and $b$ dependences are factorized.

## 3. $v_2$ for pion and proton at low $p_T$

Let us now consider the low-$p_T$ behaviors of $B(p_T)$ and $R(p_T)$. If the semi-hard scattering occurs near the surface, one of the scattered partons may emerge, while the recoil parton directed inward gets thermalized. The emerging semi-hard partons along the surface lose extra energy to the medium in addition to those others that cannot escape. Thus there is an enhancement over the bulk, for which the thermal distribution has the same form but with a higher inverse slope $T'$

$$q_0 \frac{dN_q^{B+R}}{dq_T d\phi} = C q_T e^{-q_T/T'}, \qquad \phi \in \mathscr{R} \tag{3.1}$$

It is not necessary for us to specify how long the equilibration time is, since Eq. (3.1) may be regarded as phenomenological input with $T$ and $T'$ to be determined from data.

For pions, neglecting pion mass, we obtain in [22] the pion distribution due to $TT$ recombination

$$B_\pi(p_T) = \frac{dN_\pi^B}{p_T dq_T d\phi} = \frac{C^2}{6} e^{-p_T/T} \tag{3.2}$$

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for the bulk medium at any $\phi$. Starting from Eq. (3.1) we obtain for $\phi \in \mathscr{R}$

$$B_\pi(p_T) + R_\pi(p_T, \phi) = \frac{dN_\pi^{B+R}}{p_T dq_T d\phi} = \frac{C^2}{6} e^{-p_T/T'}. \tag{3.3}$$

For pion one can use $E_T$ in place of $p_T$ in last two equations. Then $R_\pi(p_T)$ has the concrete form

$$R_\pi(p_T) = \frac{C^2}{6} e^{-E_T(p_T)/T'} \left( 1 - e^{-E_T(p_T)/T''} \right) , \tag{3.4}$$

$$\frac{1}{T''} = \frac{1}{T} - \frac{1}{T'} = \frac{\Delta T}{TT'}, \qquad \Delta T = T' - T . \tag{3.5}$$

Experimental data [4] gives $T'$=0.3 GeV for pion, we then obtain $T = 0.255$ GeV when $\Delta T$=45 MeV is adopted,and

$$T_\pi'' = 1.7 \text{ GeV} \tag{3.6}$$

for the pion. Then we have

$$\frac{B_\pi(p_T)}{R_\pi(p_T)} = \frac{1}{e^{E_T(p_T)/T''} - 1} , \tag{3.7}$$

which depends only on $T''$. We assume the validity of this equation for all $p_T < 2$ GeV/c where TT recombination is valid. When $p_T$ is small, Eq. (3.7) can be approximated by $T''/E_T$, so Eq. (2.8) has the simple expression

$$v_2^\pi(p_T, b) = \frac{E_T(p_T)}{\pi T''} \sin 2\Phi(b) . \tag{3.8}$$

The centrality dependence of $v_2$ is shown as a function of $E_T$ in Fig. 1. Good agreement with data can be seen there.

For proton production in central Au-Au collision we have obtained for TTT recombinations [22]

$$\frac{dN_p}{p_T dp_T} = A \frac{p_T^2}{p_0} e^{-p_T/T} \tag{3.9}$$

where

$$A = \frac{C^3}{6} \frac{B(\alpha+2, \gamma+2) B(\alpha+2, \alpha+\gamma+4)}{B(\alpha+1, \gamma+1) B(\alpha+1, \alpha+\gamma+2)} , \tag{3.10}$$

$\alpha = 1.75$ and $\gamma = 1.05$. Here we want to extend Eq. (3.9) to lower $p_T$, still at $y \approx 0$, so to take the mass effect into account we rewrite the equation in the form

$$B_p(p_T) = A \frac{p_T^2}{m_T} e^{-E_T(p_T)/T}. \tag{3.11}$$

Similarly, for bulk + ridge we have

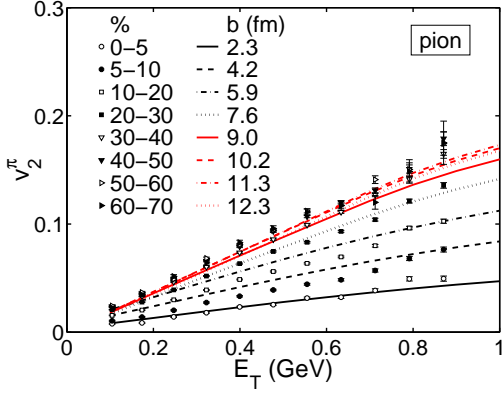$$B_p(p_T) + R_p(p_T, \phi) = A \frac{p_T^2}{m_T} e^{-E_T(p_T)/T'}, \ \phi \in \mathscr{R}. \tag{3.12}$$

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**Figure 1:** (Color online) Comparison of calculated $v_2^\pi$ with data for Au+Au collisions at 200GeV [7] for 8 centrality bins whose corresponding values of $b$ are shown in the legend.
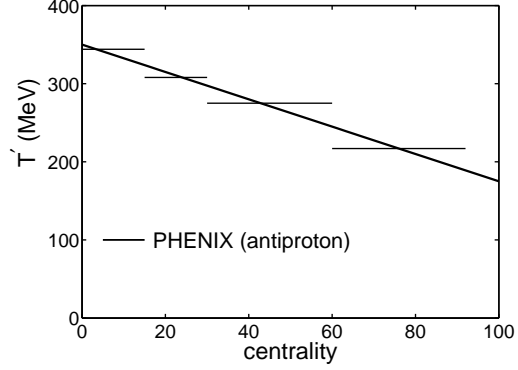


**Figure 2:** Fit of inverse slopes of $\bar{p}$ production for five centrality bins [3].

The ridge solution is then for $\phi \in \mathscr{R}$

$$R_p(p_T) = A\frac{p_T^2}{m_T}e^{-E_T(p_T)/T'}\left(1 - e^{-E_T(p_T)/T''}\right). \tag{3.13}$$

As stated in the above, the slopes determined from the $E_T$ distribution for 0-5% centrality is the value of $T'$. We obtain from the proton distribution in [4] $T' = 0.35$ GeV. Assuming dominance by proton, we use Eq. (3.13) to fit the data by varying $\Delta T$. The best fit is for $\Delta T = 45$ MeV. With $\Delta T = 0.045$ GeV we obtain from Eq. (3.5) $T = 0.305$ GeV, and

$$T_p'' = 2.37 \text{ GeV} \tag{3.14}$$

for 0-10% centrality.

To investigate the centrality dependence of $v_2$ for proton, we go to Ref. [3] and find that, whereas the slope for pion is essentially independent of $c$ (defined as centrality in %), that for proton (and antiproton) decreases with $c$. Since the $E_T$ distribution of proton has a break in slope from $E_T < 1$ GeV to $> 1$ GeV, we choose to consider the tabulated slope for $\bar{p}$, which is independent of the $E_T$ regions and in our view should be the same as for $p$. We find that (identifying $T_p' = T_{\bar{p}}'$)

$$T_p' = 0.35\,(1 - 0.5\hat{c}) \text{ GeV}, \qquad \hat{c} = c/100 \tag{3.15}$$

gives a good fit of the data as shown in Fig. 2. $v_2^p(p_T, b)$ can be calculated using Eq. (2.7). The result is shown in Fig. 3 in rough agreement with the data that have large errors [7].

## 4. $v_2$ at intermediate $p_T$

As $p_T$ is increased to above 2 GeV/c, it is necessary to consider the role played by the shower partons [22]. We now realize, as discussed in the preceding section, that the thermal distribution is $B + R$, since semi-hard scattering is always present. The only effect of this realization is just to relabel $T$ in previous work by $T'$ now, as its value is determined from data. As we proceed
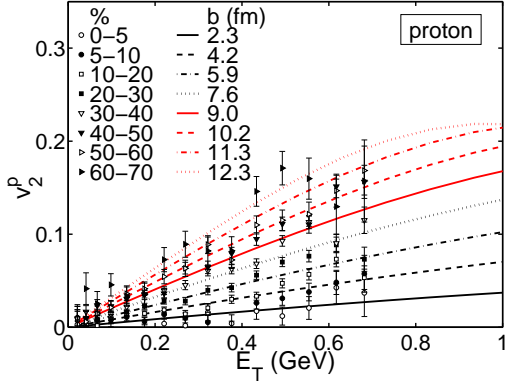
**Figure 3:** (Color online) Comparison of calculated $v_2^p$ with data for Au+Au collisions at 200GeV [7] for 8 centrality bins whose corresponding values of $b$ are shown in the legend.
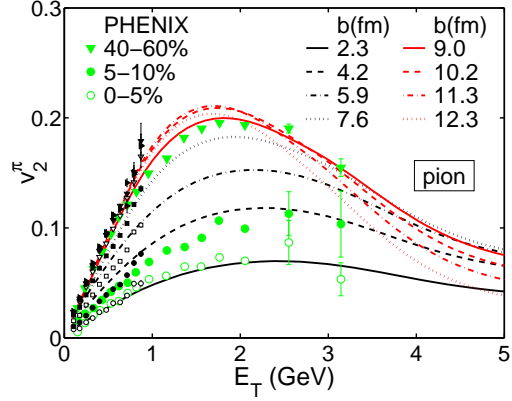
**Figure 4:** (Color online) $v_2^\pi$ for a wide range of $E_T$. The small symbols are the same as those in Fig. 1; the larger symbols (in green) are preliminary data from [26].

to consider TS recombination in this section, it is $T'$ that we shall use for the thermal partons. The condition of $\phi \in \mathscr{R}$ in Eq. (3.1) for $T'$ to be used is for TT recombination. Now for TS recombination, there is a hard parton to generate the shower parton. That hard parton may have any $\phi$. Even if $\phi \notin \mathscr{R}$, the energy loss of the hard parton can enhance the thermal partons near its trajectory, so the new $T'$ can depend on $b$ and $\phi$, characterizing the new ridge associated with hard parton.

The shower parton distribution $S_i^j(z)$ is the invariant probability of finding a parton of type $j$ with momentum fraction $z$ in a shower initiated by a hard parton of type $i$. Its application to TS recombination in central collisions averaged over all $\phi$ is discussed in Ref. [22]. We now consider $\phi$ dependence due to energy loss of the hard parton with varying path length in the dense medium. Let us denote the distribution of hard parton $i$ emerging from the surface of the medium with transverse momentum $k$ at angle $\phi$ by

$$\left.\frac{dN_i^{\text{hard}}}{k\,dk\,dy\,d\phi}\right|_{y=0} = F_i(k, \phi) \ . \tag{4.1}$$

Jet quenching degrades the hard-scattering momentum from the value $k'$ at the point of scattering to the emerging momentum $k$ by an amount $\Delta k$ that depends on the path length $\ell(\phi)$ in the medium. Assuming that the energy loss is proportional to the square root of the initiating parton momentum [25, 19], we write $\Delta k$ in the form

$$\Delta k = \varepsilon(b)\hat{\ell}(b, \phi)\sqrt{k'} \ , \tag{4.2}$$

where $\varepsilon(b)$ is the energy-loss coefficient that may be given a reasonable form [19]

$$\varepsilon(b) = \varepsilon_0 \frac{1 - e^{-2(1-\hat{b})}}{1 - e^{-2}} \ , \tag{4.3}$$

and $\hat{\ell} = \ell_{\text{max}}/2R_A$, since the density decreases with increasing $b$. We determine $\varepsilon_0 = 0.55 \ \text{GeV}^{1/2}$ by fitting the normalization of the data for 0-10% centrality at just one point ($p_T = 4.35 \ \text{GeV}/c$).

Then $v_2$ for both pion and proton at intermediate $p_T$ can be calculated without free parameter. The results are shown in Figs. 4 and 5.
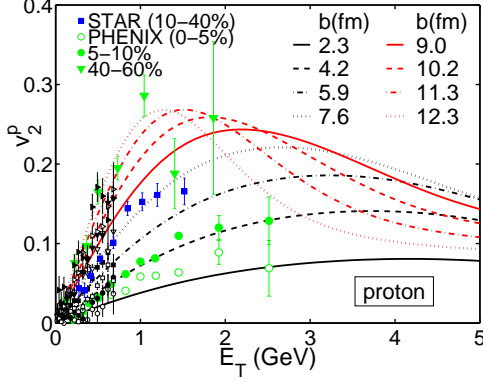


**Figure 5:** (Color online) $v_2^p$ for a wide range of $E_T$. The small symbols are the same as those in Fig. 4. The larger symbols (in green) are preliminary PHENIX data [26] for centralities 0-5% (open circle), 5-10% (full circle), and 40-60% (triangle). The STAR data are for 10-40% (blue square) at $\sqrt{s_{NN}} = 62.4$ GeV [8].
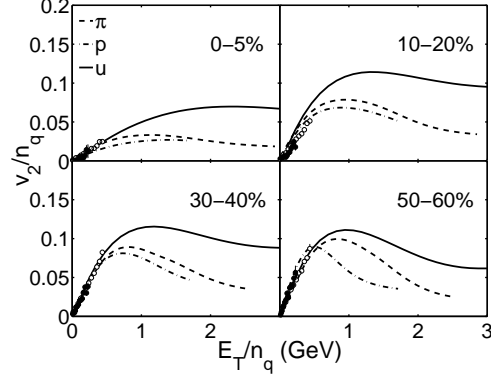
**Figure 6:** Scaled $v_2$ for pion, proton and $u$ quark. For the $u$ quark the thermal $v_2$ is plotted at low $E_T$, changing to shower $v_2$ at high $E_T$. The data points are scaled from those in Fig. 1 (open circles) and Fig. 4 (filled circles) from [7]. At higher $E_T/n_q$ only minimum bias data are available, not suitable for display here.

## 5. Breaking of quark number scaling

In the naive application of the recombination model there is quark number scaling (QNS) of $v_2$, namely the universality of $v_2^h(p_T/n_q)/n_q$ where $n_q$ is the number of constituent quarks in the hadron $h$ [20, 27]. Experimental verification of QNS has evolved to the replacement of $p_T$ by $E_T$ with impressive confirmation of the scaling behavior [8]-[12], at least at low $E_T/n_q$. Since it is known that fragmentation is more important than recombination at very high $p_T$ (or in very peripheral collisions), QNS should break down at some point. The question is at what point. We show here that it occurs rather early, even when TS recombination is still dominant. In fact, at even lower $p_T$ where TT and TTT recombination are more important, QNS is not valid in general for specific centralities. The scaling violation of $v_2$ is shown in Fig. 6 for four centralities. QNS violation can be seen clearly from the figure, more obvious for peripheral collisions. Such a violation is due to the difference of the elliptic flow for thermal $v_2^T$ and shower partons $v_2^S$. In the intermediate $p_T$ region where thermal-shower recombination is dominant, $v2^S > v_2^T$. For this region one can derive

$$\frac{v_2^\pi(E_T)}{v_2^p(3E_T/2)} \simeq \frac{2+\delta}{3+\delta} > \frac{2}{3} \;,\quad \delta = v_2^S(q_+)/v_2^T(q_-) - 1 > 0, \qquad q_\pm \gtrless E_T/2\;. \tag{5.1}$$

However, averaging over all centralities leads to approximate QNS, in agreement with minimum bias data [8, 9, 11, 12].

## 6. Summary

We have demonstrated that the observed features of elliptic flow are on the whole reproduced by ridge consideration at low $p_T$ and thermal-shower recombination at intermediate $p_T$. Constituent quark number scaling of $v_2$ is shown to be violated. In the same way, one can calculate $v_2$ of hadrons in the strange sector. More detailed discussion can be found in [28].

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