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Hawking radiation, anomalies and W-infinity algebra

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Higher spin currents that satisfy W_∞ algebra can be used to determine moments of energy distribution of Hawking radiation and hence the spectrum. In certain regularization scheme it seems that these currents do not posses anomalies (except for spin 2, i.e. the case of energy momentum tensor). Using cohomological methods, and therefore independently of the regularization scheme, it is shown that spin four current does not possess trace and diff consistent anomalies.

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1. Introduction

Hawking discovered that black holes radiate as black bodies as seen from infinity. The original derivation [1, 2] involves explicit calculation of particle content in particular vacuum in the context of QFT in curved space-time. Hawking radiation seems to be universal effect that accompanies the existence of the horizon. It has not been seen experimentally yet. Nevertheless, it seems robust in the sense that: different independent derivations agree about the temperature of the radiation; Hawking radiation exists for all types of black holes and even for horizons which are not related to black holes (such as cosmological horizons); it is kinematical effect and independent of the gravity theory under consideration; it completes the picture of black hole thermodynamics by justifying the identification of temperature with surface gravity. See e.g. [3], [4] for further discussion.

Some time ago Hawking's radiation was derived using trace anomalies [5] (see also [6, 7, 8]) and more recently using diff anomalies [9]. Diff anomaly approach was further developed in [10, 11, 12] (for a review, see e.g. [13]). One hopes that anomaly approach can explain the robustness of Hawking radiation. In the following we review the trace anomaly approach, the recent [14, 15, 16, 17] calculation of spectrum of Hawking radiation using anomalies and note the nonexistence of the anomalies for higher spin currents [16, 17]. We consider a Schwarzschild black hole dimensionally reduced to two space-time dimensions.

2. Trace anomaly method for calculating Hawking radiation

We work in the background given by

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 \tag{2.1}$$

The conservation equation and trace equation are anomalous and have the form:

$$\nabla_\mu T^\mu{}_\nu = \frac{\hbar}{48\pi}\frac{c_R - c_L}{2}\varepsilon_{\nu\mu}\partial^\mu R \tag{2.2}$$

and

$$T^\alpha_\alpha = \frac{\hbar}{48\pi}(c_R + c_L)R \tag{2.3}$$

By integrating (2.2) and (2.3) we get

$$T_{uu}(u,v) = \frac{\hbar}{24\pi}c_R\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right) + T_{uu}^{(hol)}(u) \tag{2.4}$$

where $\varphi = \log f$, $u = t - r_*, v = t + r_*$. The quantity $T_{uu}^{(hol)}(u)$ is an integration "constant" with respect to v.

Now, consider the case when $c_R = c_L$, which leaves us only with the trace anomaly. We introduce Kruskal coordinates $U = -e^{-\kappa u}$ and $V = e^{\kappa v}$. Under this transformation we have

$$T_{UU}^{(hol)}(U) = \left(\frac{1}{\kappa U}\right)^2\left(T_{uu}^{(hol)}(u) + \frac{\hbar c_R}{24\pi}\{U,u\}\right) \tag{2.5}$$

We put in boundary conditions: (1) $T^{(hol)}_{UU}(U)$ regular at the horizon $U = 0$ (which implies in particular that $T_{uu}(r = r_H) = 0$) (2) $T_{vv} = 0$ at infinity (since we consider the situation when there is no incoming flux):

$$\begin{aligned} &T^{(hol)}_{UU}(U) \text{ is regular at horizon} \\ &T_{vv} = 0 \text{ at infinity} \end{aligned} \tag{2.6}$$

Now the flux at infinity can be calculated (see e.g. [14])

$$\langle T^r_t \rangle = \langle T_{uu} \rangle - \langle T_{vv} \rangle = \frac{\hbar\kappa^2}{48\pi} c_R \tag{2.7}$$

3. Spectrum of Hawking radiation and W_∞-algebra

Iso, Morita and Umetsu [14] noticed that the vacuum expectation values of fluxes of certain higher spin currents at infinity reproduce the moments

$$F_s = \frac{g_*}{4\pi} \int_{-\infty}^{+\infty} dk \frac{\omega k^{s-2}}{e^{\beta\omega} - 1} \tag{3.1}$$

of the energy distribution of the blackbody spectrum in the same way as spectrum of spin 2 current (i.e. energy momentum tensor) reproduces total energy i.e. F_2 moment. This higher spin currents need to be somehow prescribed. In [14] prescriptions for bosonic higher spin currents are given, but there remains an arbitrariness in relative constants of the terms with the same number of derivatives.

To fix better the definition of these currents, one can choose the currents that satisfy certain properties, such as symmetries. To this end, the currents determined by W_∞-algebra were used in [16]. The currents are defined by (see [18] and also [19, 20, 21])

$$j^{(s)}_{z\ldots z}(z) = B(s) \sum_{k=1}^{s-1} (-1)^k A^s_k :\partial^k_z \phi(z) \partial^{s-k}_z \overline{\phi}(z): \tag{3.2}$$

where ϕ is a complex scalar field with

$$\begin{aligned} \langle \phi(z_1)\overline{\phi}(z_2) \rangle &= -\log(z_1 - z_2) \\ \langle \phi(z_1)\phi(z_2) \rangle &= 0 \\ \langle \overline{\phi}(z_1)\overline{\phi}(z_2) \rangle &= 0 \end{aligned} \tag{3.3}$$

and

$$B(s) = (-\frac{i}{4})^{s-2} \frac{2^{s-3} s!}{(2s-3)!!}, \qquad A^s_k = \frac{1}{s-1} \binom{s-1}{k} \binom{s-1}{s-k} \tag{3.4}$$

They satisfy a W_∞ algebra [18]. It is worth recalling that this W_∞ algebra has a unique central charge, which corresponds to the central charge of the Virasoro subalgebra. The first few currents are

$$
\begin{aligned}
j^{(2)}_{zz} &= -\, :\partial_z\phi\partial_z\overline{\phi}: \\
j^{(3)}_{zzz} &= \frac{i}{2}\left(:\partial_z\phi\partial_z^2\overline{\phi}: -\, :\partial_z^2\phi\partial_z\overline{\phi}:\right) \\
j^{(4)}_{zzzz} &= \frac{1}{5}\left(:\partial_z\phi\partial_z^3\overline{\phi}: -3:\partial_z^2\phi\partial_z^2\overline{\phi}: +\, :\partial_z^3\phi\partial_z\overline{\phi}:\right) \\
j^{(5)}_{zzzzz} &= -\frac{i}{14}\left(:\partial_z\phi\partial_z^4\overline{\phi}: -6:\partial_z^2\phi\partial_z^3\overline{\phi}: +6:\partial_z^3\phi\partial_z^2\overline{\phi}: -\, :\partial_z^4\phi\partial_z\overline{\phi}:\right) \\
j^{(6)}_{zzzzzz} &= -\frac{1}{42}\left(:\partial_z\phi\partial_z^5\overline{\phi}: -10:\partial_z^2\phi\partial_z^4\overline{\phi}: +20:\partial_z^3\phi\partial_z^3\overline{\phi}: -10:\partial_z^4\phi\partial_z^2\overline{\phi}: +\, :\partial_z^5\phi\partial_z\overline{\phi}:\right)
\end{aligned} \tag{3.5}
$$

where normal ordering is defined as

$$
:\partial^n\phi\partial^m\overline{\phi}:= \lim_{z_2\to z_1}\left\{\partial^n_{z_1}\phi(z_1)\partial^m_{z_2}\overline{\phi}(z_2) - \partial^n_{z_1}\partial^m_{z_2}\langle\phi(z_1)\overline{\phi}(z_2)\rangle\right\} \tag{3.6}
$$

As usual in the framework of conformal field theory, the operator product in the RHS is understood to be radial ordered.

Now, in order to use the trace anomaly approach we need to find covariant version of these currents. This was done in [16] following [14]. According to the recipe explained there, the covariant counterpart $J^{(s)}_{u\ldots u}$ of $j^{(s)}_{u\ldots u}$ should be constructed using prescription

$$
:\partial^n_u\phi\partial^n_u\overline{\phi}:\to e^{(n+m)\varphi(u)}\lim_{\varepsilon\to 0}\left\{e^{-n\varphi(u_1)-m\varphi(u_2)}\nabla^n_{u_1}\phi\nabla^m_{u_2}\overline{\phi} - \frac{c_{n,m}\hbar}{\varepsilon^{n+m}}\right\} \tag{3.7}
$$

where $c_{m,n} = (-)^m(n+m-1)!$ are numerical constants determined in such a way that all singularities cancel on the right hand side.

After some algebra one gets

$$
\begin{aligned}
J^{(2)}_{uu} &= j^{(2)}_{uu} - \frac{\hbar}{6}\mathfrak{T} \\
J^{(3)}_{uuu} &= j^{(3)}_{uuu} \\
J^{(4)}_{uuuu} &= j^{(4)}_{uuuu} + \frac{\hbar}{30}\mathfrak{T}^2 + \frac{2}{5}\mathfrak{T}J^{(2)}_{uu} \\
J^{(5)}_{uuuuu} &= j^{(5)}_{uuuuu} + \frac{10}{7}\mathfrak{T}J^{(3)}_{uuu}
\end{aligned} \tag{3.8}
$$

and

$$
\begin{aligned}
J^{(6)}_{uuuuuu} = &\left(-\frac{2\hbar}{63}\mathfrak{T}^3 + \frac{5\hbar}{504}(\partial_u\mathfrak{T})^2 - \frac{\hbar}{126}\mathfrak{T}\partial_u^2\mathfrak{T}\right. \\
&-\frac{2}{3}\mathfrak{T}^2J^{(2)}_{uu} - \frac{1}{21}\mathfrak{T}\nabla_u^2J^{(2)}_{uu} - \frac{1}{21}\left(\partial_u^2\mathfrak{T}\right)J^{(2)}_{uu} + \frac{5}{42}(\partial_u\mathfrak{T})\nabla_uJ^{(2)}_{uu} \\
&\left.-\frac{5}{21}\Gamma\mathfrak{T}\nabla_uJ^{(2)}_{uu} - \frac{5}{21}\Gamma^2\mathfrak{T}J^{(2)}_{uu} + \frac{5}{21}\Gamma(\partial_u\mathfrak{T})J^{(2)}_{uu}\right) - \frac{5}{24}\mathfrak{T}J^{(4)}_{uuuu} + j^{(6)}_{uuuuuu}
\end{aligned} \tag{3.9}
$$

where

$$
\mathfrak{T} = \partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2 \tag{3.10}
$$

These equations are the higher spin analogs of (2.4). It is possible to calculate analogs of (2.2) and (2.3) for these higher spin currents i.e. to calculate their trace and diff anomalies (see [15] for a discussion). Putting in boundary condition analogous to (2.6), one is able to calculate fluxes at infinity corresponding to these currents. The fluxes reproduce correctly the moments F_s (3.1) of Hawking radiation. It should be noted that the currents (3.8) and (3.9) do not (for $s > 2$[1]) possess trace anomalies (in the regularization scheme (3.7) used). Instead, the Hawking radiation and its moments derive only from trace anomaly of energy momentum tensor $s = 2$, together with transformation properties of higher currents (details in [16]). It was shown in [16], using cohomological methods, that there could be no trace anomalies for $s = 4$ current. Furthermore in [17], it was shown, that there could be no consistent diff anomalies for $s = 4$ currents. In the following we outline the proof for the case of trace anomalies and, very briefly, the proof for the case of diff anomalies.

4. Absence of trace and diff anomalies for $s = 4$ current

The covariant form of the current discussed in the previous section does not give rise to any trace anomaly. This is at variance with ref.[15], where the fourth order covariantized current exhibits a trace anomaly which is a superposition of three terms: $\nabla_\mu \nabla_\nu R$, $g_{\mu\nu}\Box R$ and $g_{\mu\nu}R^2$. It is therefore important to clarify whether these are true anomalies or whether they are some kind of artifact of the regularization used to derive the results.

We look at the possible anomalies of the fourth order current $J^{(4)}_{\mu\nu\lambda\rho}$ which couples in the action to the background field $B^{(4)}_{\mu\nu\lambda\rho} \equiv B_{\mu\nu\lambda\rho}$, both being completely symmetric tensors. The relevant Weyl transformations are as follows. The gauge parameters are the usual Weyl parameter σ and the new Weyl parameters $\tau_{\mu\nu}$ (symmetric in μ,ν). The variation δ_τ acts only on $B_{\mu\nu\lambda\rho}$ (see [22])

$$\delta_\tau B_{\mu\nu\lambda\rho} = g_{\mu\nu}\,\tau_{\lambda\rho} + g_{\mu\lambda}\,\tau_{\nu\rho} + g_{\mu\rho}\,\tau_{\nu\lambda} + g_{\nu\lambda}\,\tau_{\mu\rho} + g_{\nu\rho}\,\tau_{\mu\lambda} + g_{\lambda\rho}\,\tau_{\mu\nu} \tag{4.1}$$

while δ_σ acts on $g_{\mu\nu}$, $\tau_{\mu\nu}$ and $B_{\mu\nu\lambda\rho}$ in the following way

$$\begin{aligned} \delta_\sigma g_{\mu\nu} &= 2\,\sigma\, g_{\mu\nu} \\ \delta_\sigma \tau_{\mu\nu} &= (x-2)\,\sigma\,\tau_{\mu\nu} \\ \delta_\sigma B_{\mu\nu\lambda\rho} &= x\,\sigma\, B_{\mu\nu\lambda\rho} \end{aligned} \tag{4.2}$$

where x is a free numerical parameter. The transformation (4.2) of τ and B are required for consistency with (4.1). The actual value of x turns out to be immaterial.

Note that Ward identity for δ_σ for energy momentum tensor gives the trace anomaly equation. In the same way for spin 4 current the Ward identity for δ_τ would give the corresponding trace anomaly equation.

Next step is to promote σ and τ to anticommuting fields:

$$\begin{aligned} &\sigma^2 = 0 \\ &\tau_{\mu\nu}\,\tau_{\lambda\rho} + \tau_{\lambda\rho}\,\tau_{\mu\nu} = 0 \\ &\sigma\,\tau_{\mu\nu} + \tau_{\mu\nu}\,\sigma = 0 \end{aligned}$$

[1] We verified this up to order 10.

and verify that the corresponding variations are nilpotent:

$$\delta_\sigma^2 = 0, \qquad \delta_\tau^2 = 0, \qquad \delta_\sigma\,\delta_\tau + \delta_\tau\,\delta_\sigma = 0$$

The anomalies are by definition variations of the one–loop quantum action $\Gamma^{(1)}$ (which is a nonlocal quantity)

$$\delta_\sigma \Gamma^{(1)} = \hbar \Delta_\sigma, \qquad \delta_\tau \Gamma^{(1)} = \hbar \Delta_\tau, \tag{4.3}$$

Acting with the variations once again, using nilpotency, one finds that the candidates for anomalies Δ_σ and Δ_τ must satisfy the conditions

$$\begin{aligned} &\delta_\sigma\,\Delta_\sigma = 0 \\ &\delta_\tau\,\Delta_\sigma + \delta_\sigma\,\Delta_\tau = 0 \\ &\delta_\tau\,\Delta_\tau = 0 \end{aligned} \tag{4.4}$$

which are the Wess-Zumino consistency conditions.

We have to make sure that Δ_σ and Δ_τ are true anomalies, that is that they are nontrivial. In other words there must not exist a local counterterm C in the action such that

$$\Delta_\sigma \;=\; \delta_\sigma \int d^2x C \tag{4.5}$$

$$\Delta_\tau \;=\; \delta_\tau \int d^2x C \tag{4.6}$$

If such a C existed we could redefine the quantum action by subtracting these counterterms and get rid of the (trivial) anomalies.

We start by expanding candidate anomalies as linear combinations of curvature invariants

$$\Delta_\sigma = \int d^2x \sqrt{-g} \sum_{i=2}^{11} c_i\, I_i \tag{4.7}$$

$$\Delta_\tau = \int d^2x, \sqrt{-g} \sum_{k=1}^{3} b_k\, K_k \tag{4.8}$$

where I_i are linear in $B^{\mu\nu\lambda\rho}$ and σ:

$$\begin{aligned} I_1 &= \sigma R \\ I_2 &= B^{\mu\nu\lambda\rho}\, \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\rho \sigma \\ I_3 &= B^{\mu\nu}\, R\, \nabla_\mu \nabla_\nu \sigma \\ I_4 &= B^{\mu\nu}\, \nabla_\mu \nabla_\nu \Box \sigma \\ I_5 &= B^{\mu\nu}\, \nabla_\mu \nabla_\nu R \sigma \\ I_6 &= B \Box R\, \sigma \\ I_7 &= B R^2\, \sigma \\ I_8 &= B^{\mu\nu} \nabla_\mu R \nabla_\nu \sigma \\ I_9 &= B R \Box \sigma \\ I_{10} &= B g^{\mu\nu}\, \nabla_\mu R\, \nabla_\nu \sigma \\ I_{11} &= B\, \Box^2 \sigma \end{aligned} \tag{4.9}$$

($B^{\mu\nu} = B^{\mu\nu\lambda\rho} g_{\lambda\rho}$, $B = B^{\mu\nu} g_{\mu\nu}$). The term I_1 corresponds to the usual anomaly of the energy–momentum trace (which is consistent and nontrivial). Therefore in the sequel we disregard it and limit ourselves to the other terms which contain 4 derivatives. Similarly K_k are independent curvature invariants that are linear in $\tau_{\mu\nu}$ and contain 4 derivatives:

$$\begin{aligned} K_1 &= \nabla_\mu \nabla_\nu R\, \tau^{\mu\nu} \\ K_2 &= R^2\, \tau \\ K_3 &= \Box R\, \tau \end{aligned} \tag{4.10}$$

where $\tau = g^{\mu\nu}\, \tau_{\mu\nu}$.

Now, the idea is to see what constraints to the form of coefficients do WZ consistency conditions (4.4) give. It turns out that of all coefficients c_i and b_k, only 3 of them (say c_9, c_{10} and c_{11}) become independent. Furthermore, for any choice of these remaining coefficients counterterm C can be found. The counterterm C is a linear combination

$$C = \int d^2x \sqrt{-g} \sum_{j=5}^{7} d_j C_j \tag{4.11}$$

of the following curvature invariants

$$\begin{aligned} C_5 &= B^{\mu\nu}\, \nabla_\mu \nabla_\nu R \\ C_6 &= B \Box R \\ C_7 &= B R^2 \end{aligned} \tag{4.12}$$

where d_j are determined in terms of c_9, c_{10} and c_{11}.

Our conclusion is therefore that not only the trace anomalies found in [15] are trivial, but that there cannot be any anomaly whatsoever in $J^{(4)\mu}{}_{\mu\lambda\rho}$.

For the case of diff anomalies $\Delta_\xi^{(4)}$ and $\Delta_\tau^{(4)}$ the WZ conditions we need to solve are

$$\delta_\xi \Delta_\xi^{(4)} = 0 \tag{4.13}$$

$$\delta_\tau \Delta_\tau^{(4)} = 0 \tag{4.14}$$

with the cross condition

$$\delta_\tau \Delta_\xi^{(4)} + \delta_\xi \Delta_\tau^{(4)} = 0 \tag{4.15}$$

where ξ^μ is the vector field generator of diffeomorphisms (corresponding to the conservation of energy momentum tensor) and $\tau_{\mu_1\mu_2\mu_3}$ is completely symmetric and traceless tensor field (corresponding to the conservation of higher spin current $J^{(4)}$). The variations δ_ξ act as (see [17] for discussion)

$$\begin{aligned} &\delta_\xi \xi^\mu = \xi^\lambda \partial_\lambda \xi^\mu \\ &\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \\ &\delta_\xi \tau_{\mu\nu\rho} = \xi^\lambda \partial_\lambda \tau_{\mu\nu\rho} + \partial_\mu \xi^\lambda \tau_{\lambda\nu\rho} + \partial_\nu \xi^\lambda \tau_{\mu\lambda\rho} + \partial_\rho \xi^\lambda \tau_{\mu\nu\lambda} \\ &\delta_\xi B^{(4)}_{\mu_1\ldots\mu_4} = \xi^\lambda \partial_\lambda B^{(4)}_{\mu_1\ldots\mu_4} + \partial_{\mu_1} \xi^\lambda B^{(4)}_{\lambda\ldots\mu_4} + \ldots + \partial_{\mu_4} \xi^\lambda B^{(4)}_{\mu_1\ldots\lambda} \end{aligned} \tag{4.16}$$

and the variations δ_τ act as

$$\begin{aligned}
&\delta_\tau \xi^\mu = 0 \\
&\delta_\tau g_{\mu\nu} = 0 \\
&\delta_\tau \tau_{\mu\nu\lambda} = 0 \\
&\delta_\tau B^{(4)}_{\mu_1\mu_2\mu_3\mu_4} = \nabla_{\mu_1}\tau_{\mu_2\mu_3\mu_4} + cycl.
\end{aligned} \tag{4.17}$$

Again, as for the trace anomaly case, ξ^μ and $\tau_{\mu\nu\lambda}$ are promoted to anticommuting fields and it is verified that $\delta_\xi^2 = 0$, $\delta_\tau^2 = 0$, $\delta_\xi\delta_\tau + \delta_\tau\delta_\xi = 0$. One proceeds as in the trace anomaly case, finding the consequences on the candidates for the anomalies $\Delta_\xi^{(4)}$ and $\Delta_\tau^{(4)}$. Here, one cannot assume from the start that the anomalies are covariant. Instead a rather technical analysis is needed. It is presented in Appendix B of [17], and uses results from [23, 24, 25]. Fortunately the result of the analysis is simple: the solution to (4.13) is trivial i.e. $\Delta_\xi^{(4)} = \delta_\xi C^{(4)}$. Consequently we can rewrite (4.15) as $\delta_\xi(\Delta_\tau^{(4)} - \delta_\tau C^{(4)}) = 0$. This means that any solution to (4.14) can be written in a diff–covariant form i.e. one can use the covariant candidates for $\Delta_\tau^{(4)}$. They can be easily enumerated, and it turns out, as a consequence of tracelessness of $\tau_{\mu\nu\lambda}$, that they are trivial.

We conclude, therefore, that there are no non–trivial consistent anomalies in the divergence of the fourth order current. Extending this proof to all s would require solving generalizations of (4.13) for higher s. It is plausible to assume that its solution, analogously to $s = 4$ case, would enable the use of covariant candidates. Under this assumption it is was shown [17] that for all the higher (even[2]) s there is no diff anomaly.

5. Conclusion

Our calculations indicate that there are no trace and diff anomalies for higher spin currents[3] $s > 2$. This was shown for $s = 4$ using cohomological methods. Under plausible assumptions the cohomological proof for diff anomalies was extended to all s [17]. Also it was shown for W_∞-algebra currents in the regularization scheme defined by Eq. (3.7) for s up to 10. These results are obtained using 2 spacetime dimensions.

The moments of Planck spectrum of Hawking radiation are reproduced if the high spin currents are taken to be the covariantised W_∞-algebra currents [16].

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[2]We consider only even s since for odd s the fluxes F_s are vanishing [14].

[3]Energy momentum tensor itself ($s = 2$), can have, as is well known, trace and diff anomalies.

References

[1] S. W. Hawking, *Particle Creation By Black Holes,* Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)].

[2] G. W. Gibbons and S. W. Hawking, *Action Integrals And Partition Functions In Quantum Gravity,* Phys. Rev. D **15**, 2752 (1977).

[3] R. M. Wald, *"The thermodynamics of black holes,"* Living Rev. Rel. **4** (2001) 6 [arXiv:gr-qc/9912119].

[4] M. Visser, *"Essential and inessential features of Hawking radiation,"* Int. J. Mod. Phys. D **12** (2003) 649 [arXiv:hep-th/0106111].

[5] S. M. Christensen and S. A. Fulling, *Trace Anomalies And The Hawking Effect,* Phys. Rev. D**15** (1977) 2088.

[6] P.C.W.Davies, S.A.Fulling and W.G.Unruh, *Energy Momentum Tensor Near An Evaporating Black Hole,* Phys. Rev. D**13** (1976) 2720.

[7] A. Strominger, *Les Houches lectures on black holes,* arXiv:hep-th/9501071.

[8] L. Thorlacius, *Black hole evolution,* Nucl. Phys. Proc. Suppl. **41** (1995) 245 [arXiv:hep-th/9411020].

[9] S. P. Robinson and F. Wilczek, *A relationship between Hawking radiation and gravitational anomalies,* Phys. Rev. Lett. **95** (2005) 011303 [arXiv:gr-qc/0502074].

[10] S. Iso, H. Umetsu and F. Wilczek, *Hawking radiation from charged black holes via gauge and gravitational anomalies,* Phys. Rev. Lett. **96** (2006) 151302

[11] R. Banerjee and S. Kulkarni, *Hawking Radiation and Covariant Anomalies,* Phys. Rev. D **77** (2008) 024018 [arXiv:0707.2449 [hep-th]].

[12] R. Banerjee and S. Kulkarni, *Hawking Radiation, Effective Actions and Covariant Boundary Conditions,* Phys. Lett. B **659** (2008) 827 [arXiv:0709.3916 [hep-th]].

[13] S. Iso, *"Hawking Radiation, Gravitational Anomaly and Conformal Symmetry - the Origin of Universality -,"* Int. J. Mod. Phys. A **23** (2008) 2082 [arXiv:0804.0652 [hep-th]].

[14] S. Iso, T. Morita and H. Umetsu, *Higher-spin currents and thermal flux from Hawking radiation,* Phys. Rev. D **75** (2007) 124004 [arXiv:hep-th/0701272].

[15] S. Iso, T. Morita and H. Umetsu, *Higher-spin Gauge and Trace Anomalies in Two-dimensional Backgrounds,* Nucl. Phys. B **799** (2008) 60 [arXiv:0710.0453 [hep-th]].

[16] L. Bonora and M. Cvitan, *Hawking radiation, W-infinity algebra and trace anomalies,* JHEP **0805** (2008) 071 [arXiv:0804.0198 [hep-th]].

[17] L. Bonora, M. Cvitan, S. Pallua and I. Smolić, *Hawking Fluxes, W_∞ Algebra and Anomalies*, JHEP **0812** (2008) 021 [arXiv:0808.2360 [hep-th]].

[18] I. Bakas and E. Kiritsis, *Bosonic realization of a universal W algebra and Z(infinity parafermions,* Nucl. Phys. B **343** (1990) 185 [Erratum-ibid. B **350** (1991) 512].

[19] A. Bilal, *A Remark On The N $\to$ Infinity Limit Of W(N) Algebras*, Phys. Lett. B **227**, 406 (1989).

[20] C. N. Pope, L. J. Romans and X. Shen, *The Complete Structure of W(Infinity),* Phys. Lett. B **236**, 173 (1990).

[21] C. N. Pope, L. J. Romans and X. Shen, *W(infinity) and the Racah-Wigner algebra*, Nucl. Phys. B **339**, 191 (1990).

[22] C. M. Hull, *W Geometry,* Commun. Math. Phys. **156** (1993) 245 [arXiv:hep-th/9211113].

[23] L. Bonora, P. Pasti and M. Tonin, *The Anomaly Structure Of Theories With External Gravity,* J. Math. Phys. **27** (1986) 2259.

[24] L. Bonora, P. Pasti and M. Tonin, *Gravitational And Weyl Anomalies,* Phys. Lett. B **149** (1984) 346.

[25] L. Bonora, P. Pasti and M. Bregola, *Weyl Cocycles,* Class. Quant. Grav. **3** (1986) 635.