

## CP violation in charged Higgs boson decays in the MSSM

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CP violation in  $H^\pm$  decays into the three possible decay modes into ordinary particles, 1)  $H^\pm \rightarrow tb$ , 2)  $H^\pm \rightarrow \nu\tau$  and 3)  $H^\pm \rightarrow W^\pm h^0$  is considered. Analytic expressions and numerical results for the CP violating decay rate asymmetries in the MSSM are obtained. Increasing  $\tan\beta$  the asymmetries for the fermionic decays,  $H^\pm \rightarrow tb$  and  $H^\pm \rightarrow \nu\tau$ , decrease and it increases for  $H^\pm \rightarrow W^\pm h^0$ . The asymmetry of  $H^\pm \rightarrow tb$  is most sensitive to the phase of  $A_t$  and can go up to 20%, the asymmetries of 2) and 3) depend mainly on the phases of  $A_\tau$  and  $M_1$ . The asymmetry of 2) is smaller than 0.5% and of 3) can reach up to 2%.

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## 1. Introduction

If a charged Higgs boson is discovered at LHC, which we believe very much, or at a possible future International Linear Collider (ILC) or at CLIC, it would be ultimately a signal for Physics beyond the Standard Model (SM). The next question would be which Physics beyond the SM is it – almost all extensions of the SM enlarge the Higgs sector of the SM and inevitably predict the existence of a charged Higgs. The effects of CP violation is a possible tool to disentangle the different charged Higgs bosons. Nearly all extensions of the SM contain additional sources of CP violation.

In this note we study CP violation in the Minimal Supersymmetric Standard Model (MSSM) with complex couplings. We consider the processes of  $H^\pm$ -decays into ordinary particles – these are the decays

$$H^\pm \rightarrow tb, \quad H^\pm \rightarrow \nu\tau \quad \text{and} \quad H^\pm \rightarrow W^\pm h^0, \quad (1.1)$$

where  $h^0$  is the lightest neutral Higgs boson.

The CP violating asymmetries that we consider are the decay rate asymmetries

$$\delta_f^{CP} = \frac{\Gamma(H^+ \rightarrow f) - \Gamma(H^- \rightarrow \bar{f})}{\Gamma(H^+ \rightarrow f) + \Gamma(H^- \rightarrow \bar{f})} \quad (1.2)$$

where  $f$  stands for  $tb$ ,  $\nu\tau$  and  $Wh^0$ , respectively. At tree level the partial decay rates are always equal and there is no CP violation.  $\delta_f^{CP}$  is a loop induced effect, in our case these are loops with SUSY particles. However, a CP violating phase and loop corrections are not enough – for  $\delta_f^{CP} \neq 0$  the loop integrals must have absorptive parts, i.e.  $\delta_f^{CP}$  is a threshold effect – for a non-zero value of  $\delta_f^{CP}$  at least one decay channel of  $H^\pm$  into SUSY particles should be open.

In the SM the only source of CP violation is the CKM CP phase. In the MSSM, in addition to it, new phases appear – these are the phase of the higgsino mass parameter  $\mu = |\mu|e^{i\phi_\mu}$ , the phases of the gaugino masses  $M_i = |M_i|e^{i\phi_i}$ ,  $i = 1, 2, 3$  and the phases of the trilinear couplings  $A_f = |A_f|e^{i\phi_f}$ . Of these the phase  $\phi_\mu$  is strongly constrained by measurements of the neutron and electron EDM:  $\phi_\mu \leq 10^{-2}$ . The phases of the trilinear couplings  $A_f$  always occur as  $A_f m_f$ , with  $m_f$  the corresponding fermion mass. They are practically only important for the fermions of the 3-rd generation only. Thus, the phases most relevant to our study are  $\phi_t$ ,  $\phi_b$  and  $\phi_\tau$  – the phases of  $A_t$ ,  $A_b$  and  $A_\tau$ . We also allow for a non-zero phase  $\phi_1$  of  $M_1$ , imposing the GUT relation only for the absolute values,  $|M_1| = \frac{5}{3} \tan \theta_W |M_2|$ , because the phase of  $M_2$  can be rotated away and is not physical.

In refs. [1]–[4] these decay rate asymmetries were studied. Here we give a short review of these papers.

## 2. $H^\pm \rightarrow tb$ decay

The matrix elements for  $H^\pm \rightarrow t\bar{b}$  decays, including loop corrections, can be written as

$$\begin{aligned} \mathcal{M}_{H^+} &= \bar{u}(p_t) [Y_b^+ P_R + Y_t^+ P_L] v(-p_{\bar{b}}), \\ \mathcal{M}_{H^-} &= \bar{u}(p_b) [Y_b^- P_R + Y_t^- P_L] v(-p_{\bar{t}}), \end{aligned} \quad (2.1)$$

$\tan\beta$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\nu}}$
5	142	300	706	706	300	709	166	522	327	377	344	362	344
30	141	296	705	709	296	711	172	519	183	464	295	402	344

**Table 1:** Sparticles masses (in GeV) for the given parameter together with  $\phi_{A_t} = \phi_{A_b} = \pi/2$  and  $\phi_\mu = 0$ .

where  $P_{R,L} = (1 \pm \gamma_5)/2$  and the form factors are

$$Y_i^\pm = y_i \pm \delta Y_i^\pm, \quad \delta Y_i^\pm = \delta Y_i^{inv} \pm \delta Y_i^{CP}, \quad i = t, b, \quad (2.2)$$

$y_i$  are the tree level Yukawa couplings. The loop induced form factors  $\delta Y_i^\pm$  have CP-invariant and CP violating contributions. The CP violating contributions  $\delta Y_i^{CP}$  distinguish the form factors of  $H^+$  and  $H^-$ . Both  $\delta Y_i^{inv}$  and  $\delta Y_i^{CP}$  have real and imaginary (absorptive) parts. In the decay rate asymmetries always the absorptive parts contribute. This can be easily understood following the simple explanation: for having CP violation we need a CP phase, the other phase that we need in order to have a real decay width could come only from absorptive parts of the loop integrals. For the decay rate asymmetry  $\delta_{tb}^{CP}$  we obtain [1]

$$\delta_{tb}^{CP} = \frac{2(m_{H^+}^2 - m_t^2 - m_b^2)(y_t \Re e \delta Y_t^{CP} + y_b \Re e \delta Y_b^{CP}) - 4m_t m_b (y_t \Im e \delta Y_b^{CP} + y_b \Im e \delta Y_t^{CP})}{(m_{H^+}^2 - m_t^2 - m_b^2)(y_t^2 + y_b^2) - 4m_t m_b y_t y_b}. \quad (2.3)$$

At one loop there are two types of SUSY corrections: in the  $H^\pm tb$ -vertex and self-energy corrections on the  $H^\pm$ -line. They were calculated in [1] and analytic expressions for the results are given therein. The performed numerical analysis [1, 2] showed that the contributions from the self-energy loop with  $\tilde{t}\tilde{b}$  give the main contribution to  $\delta_{tb}^{CP}$ , also the  $\tilde{g}\tilde{t}\tilde{b}$ -vertex corrections can give a relevant contribution, the contributions with  $\tilde{\chi}^\pm$ ,  $\tilde{\chi}^0$  and  $\tilde{\tau}$  are totally negligible. The relative importance of the different diagrams is shown on Fig.1a). This suggests that  $\delta_{tb}^{CP}$  will be sensitive to the phases of  $A_t$  and  $A_b$  only. As the contribution of  $A_t$  is enhanced by the large mass of the  $t$ -quark mass,  $m_t = 178$  GeV, the dependence on  $A_b$  should be much weaker. Our numerical analysis showed that there is no sensitivity to the phase of  $A_b$ .

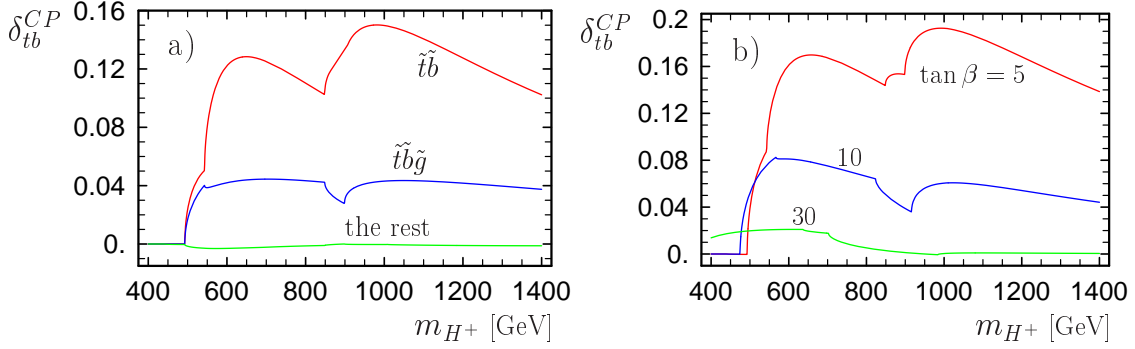
We have studied the dependence of  $\delta_{tb}^{CP}$  on  $m_{H^+}$  and the phases  $\phi_t$  and  $\phi_b$  for different values of  $\tan\beta$ . In order not to vary too many parameters, we fix:  $M_2 = 300$  GeV,  $M_3 = 745$  GeV,  $M_{\tilde{U}} = M_{\tilde{D}} = M_E = M_L = 350$  GeV,  $\mu = -700$  GeV,  $|A_t| = |A_b| = |A_\tau| = 700$  GeV. The relevant sparticle masses for this choices are given explicitly in Table 1,

Fig 1b) shows  $\delta_{tb}^{CP}$  as a function of  $m_{H^+}$  for different values of  $\tan\beta$ . For  $m_{H^+} < m_{\tilde{\nu}} + m_{\tilde{\tau}_1}$ ,  $\delta^{CP}$  is very small,  $\mathcal{O}(10^{-3})$  or smaller. However, once the  $H^+ \rightarrow \tilde{t}\tilde{b}$  channel is open,  $\delta^{CP}$  can go up to roughly 20%. All four thresholds of  $H^+ \rightarrow \tilde{t}_i\tilde{b}_i$  are clearly visible in the figure.

The considered decay mode  $H^\pm \rightarrow tb$  will be traced by the decay products of the  $t$ -quark. Because of its large mass, the  $t$ -quark will decay keeping its momentum and polarization. In ref. [2] the CP violating angular and energy asymmetries of the  $t$ -decay products are considered as well.

### 3. $H^\pm \rightarrow \nu\tau^\pm$ decay

In the previous section we showed that large phases of  $A_t$  can lead to a large CP-violating asymmetry  $\delta_{tb}^{CP}$  in  $H^\pm \rightarrow tb$ , up to of 15–20% for  $m_{H^\pm} > m_{\tilde{t}} + m_{\tilde{b}}$ . In this section we consider the



**Figure 1:** a) the different contributions to  $\delta_{tb}^{CP}$ ,  $\tan\beta = 5$ , and b)  $\delta_{tb}^{CP}$  as a function of  $m_{H^+}$ ,  $\tan\beta = 5, 10$  and 30. For both figures  $\phi_{A_t} = \pi/2$ ,  $\phi_{A_b} = \phi_\mu = \phi_3 = 0$

lepton decay channels of the charged Higgs bosons,  $H^+ \rightarrow \tau^+ \nu_\tau$  and  $H^- \rightarrow \tau^- \bar{\nu}_\tau$  and calculate the CP-violating asymmetry  $\delta_{\nu\tau}^{CP}$  at the one-loop level.

The decay  $H^\pm \rightarrow \tau \nu$  may be important for relatively low masses of  $H^\pm$  when the decay  $H^\pm \rightarrow tb$  is not allowed kinematically. This implies that, as it is always the absorptive parts of the loop integrals that contribute, the loops with  $\tilde{t}$  and  $\tilde{b}$  will not contribute to  $\delta_{\nu\tau}^{CP}$  in this region. Thus, the only relevant phase from the trilinear couplings will be the phase  $\phi_\tau$ . As we also allow for the gaugino mass parameter  $M_1$  to be complex, a non-zero value of  $\delta_{\nu\tau}^{CP}$  would imply non-zero phases  $\phi_\tau$  and/or  $\phi_1$ .

The theoretical consideration is quite similar to the decay  $H^\pm \rightarrow tb$ , but as  $m_\nu = 0$  there is only one form factor in the matrix element:

$$Y_\tau^\pm = y_\tau + \delta Y_\tau^\pm, \quad \delta Y_\tau^\pm = \delta Y_\tau^{inv} \pm \delta Y_\tau^{CP} \quad (3.1)$$

where  $y_\tau$  is the tree level coupling. Both the CP-invariant and the CP-violating contributions have real and imaginary parts and  $\delta_{\nu\tau}^{CP}$  is expressed in terms of the imaginary part in the simple form:

$$\delta_{\nu\tau}^{CP} = \frac{2\mathcal{R}e\delta Y_\tau^{CP}}{y_\tau + 2\mathcal{R}e\delta Y_\tau^{inv}} \simeq \frac{2\mathcal{R}e\delta Y_\tau^{CP}}{y_\tau}. \quad (3.2)$$

The loops that will contribute are: self-energy loops with  $\tilde{\tau}\tilde{\nu}$  and  $\tilde{\chi}^+\tilde{\chi}^0$ , and vertex graphs with  $\tilde{\tau}\tilde{\nu}\tilde{\chi}^0$ ,  $\tilde{\chi}^+\tilde{\chi}^0\tilde{\nu}$  and  $\tilde{\chi}^-\tilde{\chi}^0\tilde{\tau}$ . The explicit expressions for  $\delta Y_\tau^{CP}$  from the different loop diagrams, together with the masses and couplings of staus and sneutrinos, are given in [3].

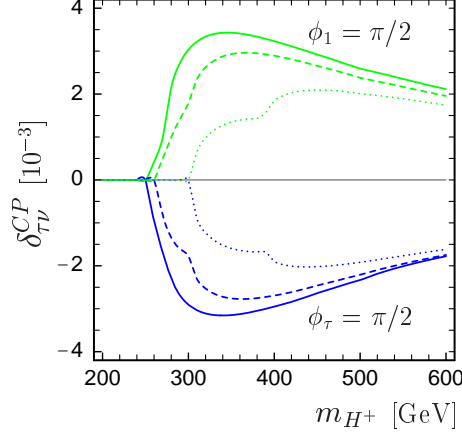
For the numerical analysis we fix  $M_2 = 200$  GeV,  $\mu = 300$  GeV,  $M_{\tilde{E}} = M_{\tilde{L}} - 5$  GeV,  $|A_\tau| = 400$  GeV.

$\tan\beta$	$M_{\tilde{L}}$	$m_{\tilde{\nu}}$	$m_{\tilde{\nu}_2}$	$\theta_{\tilde{\tau}}$
5	138	123	150	$56^\circ$
10	147	132	166	$50^\circ$
30	180	168	221	$47^\circ$

**Table 2:** Parameters and slepton masses in [GeV] used in the analysis for  $\delta_{\nu\tau}^{CP}$ ,  $m_{\tilde{\nu}_1} = 135$  GeV.

Fig. 2 shows  $\delta_{\nu\tau}^{CP}$  as a function of  $m_{H^\pm}$  for the two cases:  $\phi_\tau = \pi/2$ ,  $\phi_1 = 0$  and  $\phi_\tau = 0$ ,  $\phi_1 = \pi/2$  and  $\tan\beta = 5, 10$ , and 30. The corresponding values for  $m_{\tilde{\nu}}$ ,  $m_{\tilde{\nu}_2}$  and  $\theta_{\tilde{\tau}}$  are listed in

Table 2. In our analysis  $|\delta_{\nu\tau}^{CP}|$  goes up to  $\sim 3.5 \times 10^{-3}$  and it is interesting to note that maximal  $\phi_\tau$  and maximal  $\phi_1$  lead to very similar values of  $\delta_{\nu\tau}^{CP}$  but with opposite signs. However, if both phases are maximal, i.e.  $\phi_\tau \sim \phi_1 \sim \pi/2$  or  $3\pi/2$ , they compensate each other and  $\delta_{\nu\tau}^{CP}$  practically vanishes.



**Figure 2:**  $\delta_{\nu\tau}^{CP}$  as a function of  $\phi_\tau$  for  $m_{H^\pm} = 350$  GeV and  $\tan\beta = 5$ . The full, dashed, and dotted lines are for  $\phi_1 = 0, \pi/4,$  and  $\pi/2$ , respectively.

#### 4. $H^\pm \rightarrow W^\pm h^0$ decay

In this section we consider the decay rate asymmetry  $\delta_{Wh^0}^{CP}$  of  $H^\pm \rightarrow W^\pm h^0$  decay. Though the final state  $h^0$  is not observed yet,  $m_{h^0}$  is not an unknown parameter – once  $m_{H^\pm}$  and  $\tan\beta$  are fixed, the SUSY structure of the theory determines uniquely both  $m_{h^0}$  and the branching ratio (BR). Previously this asymmetry was considered in the two-Higgs doublet model [5].

Respecting the experimental lower bound from LEP and the theoretical upper bound, including radiative corrections, we consider  $m_{h^0}$  in the range  $96 \leq m_{h^0} \leq 130$  GeV. In order to keep the value of  $\text{BR}(H^+ \rightarrow W^+ h^0)$  at the level of a few percent, we consider low  $m_{H^\pm}$ ,  $200 \leq m_{H^\pm} \leq 600$  GeV and low  $\tan\beta$ ,  $3 \leq \tan\beta \leq 9$  ( $\tan\beta \leq 3$  being excluded from the Higgs searches at LEP).

The matrix elements of  $H^\pm \rightarrow W^\pm h^0$  is expressed in terms of one form factor only:

$$M_{H^\pm} = ig\varepsilon_\alpha^\lambda(p_W)p_h^\alpha Y^\pm, \quad Y^\pm = y + \delta Y^\pm, \quad \delta Y^\pm = \delta Y^{inv} \pm \delta Y^{CP} \quad (4.1)$$

where  $y = \cos(\alpha - \beta)$  is the tree level coupling. For  $\delta_{Wh^0}^{CP}$  we obtain [4]:

$$\delta_{Wh^0}^{CP} \simeq \frac{2\mathcal{R}e(\delta Y^{CP})}{y}. \quad (4.2)$$

As in the previous section, we assume that the squarks, as suggested by most SUSY models, are heavier and thus will not contribute in the considered range of  $m_{H^\pm}$ .

In accordance with this, there are two types of diagrams that will contribute: with  $\tilde{\nu}, \tilde{\tau}$  and with  $\tilde{\chi}^\pm$  and  $\tilde{\chi}^0$  in the loops. The explicit expressions were obtained in [4]. This implies the sensitivity of  $\delta_{Wh^0}^{CP}$  to the phases  $\phi_\tau$  and  $\phi_1$ . The numerical analysis was performed for

$$M_2 = 250 \text{ GeV}, M_E = M_L - 5 \text{ GeV}, M_L = 120 \text{ GeV}, |A_\tau| = 500 \text{ GeV}, |\mu| = 150 \text{ GeV}. \quad (4.3)$$

and the GUT relation for the absolute values of  $M_1$  and  $M_2$  was assumed.

The numerical analysis showed that the values for  $\delta_{Wh^0}^{CP}$  are typically about  $10^{-2} \div 10^{-3}$ , the main contributions being from  $\tilde{\nu}$  and  $\tilde{\tau}$  for  $m_{H^+} < 300$  GeV, and from  $\tilde{\chi}^+$  and  $\tilde{\chi}^0$  for  $m_{H^+} \geq 300$  GeV. The dependence on different values of  $\tan\beta$  was examined [4].

## 5. Summary

Discussing CP violation in the decay widths, we must keep in mind the branching ratios of the relevant decay modes. The BR of  $H^+ \rightarrow \nu\tau^+$  is dominant below the  $t\bar{b}$  threshold. This determines the sensitivity of  $\delta_{\nu\tau}^{CP}$  to the phases  $\phi_\tau$  and  $\phi_1$  of  $A_\tau$  and  $M_1$ , respectively. The decay rate asymmetry remains always below 0.5%.

If  $m_{H^+}$  is large enough and the  $t\bar{b}$ -threshold is open,  $H^+ \rightarrow t\bar{b}$  will dominate and  $\delta_{t\bar{b}}^{CP}$  will be important. Due to the large top Yukawa coupling and the fact that  $\delta_{t\bar{b}}^{CP}$  goes down for large  $\tan\beta$ , for all values of  $\tan\beta$  it is most sensitive to the phase of  $A_t$ . For large  $m_{H^+}$  and a relatively light gluino,  $m_{\tilde{g}} \sim 400$  GeV, and light stops and sbottoms,  $m_{\tilde{t}} = 166$  GeV and  $m_{\tilde{b}} = 327$  GeV,  $\delta_{t\bar{b}}^{CP}$  can go up to  $\sim 20\%$ .

The decay rate asymmetry  $\delta_{Wh^0}^{CP}$  can be of the order of few percents if both  $m_{H^+}$  and  $\tan\beta$  are small and will be sensitive to  $\phi_\tau$  and  $\phi_1$ . The BR of  $H^+ \rightarrow W^+h^0$  can go up to 10%.

In principle, these asymmetries could be directly measured at an ILC or at CLIC if  $\sqrt{s} > 2m_{H^+}$ . But after doing a more detailed estimation, in all three cases a higher luminosity would be necessary to observe CP violation in these decays.

For LHC one must take into account CP violation in the production of  $H^\pm$  as well, see the contribution [6] within these proceedings.

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