PROCEEDINGS OF SCIENCE

Review on the weak chiral lagrangian

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We discuss the weak chiral lagrangian. After a discussion of the nice cases, like $K_S \to \gamma\gamma$, where only the weak $\mathcal{O}(p^2)$ is needed, we address the issue to determine the coefficients of the weak $\mathcal{O}(p^4)$; we discuss also the related issue to reduce the number of CT's by theoretical models like VMD or factorization; the decays $K^+ \to \pi^+\gamma\gamma$ and $K^+ \to \pi^+\pi^0\gamma$ are particulaly useful to this purpose. We investigate also the issue of CP violation in $K_L \to \pi^0 e^+ e^-$ and the background process CP conserving $K_L \to \pi^0\gamma\gamma \to \pi^0 e^+ e^-$. We mention other weak kaon decays close to observation.

International Workshop on Effective Field Theories: from the pion to the upsilon
February 2-6 2009
Valencia, Spain

PoS(EFT09)061

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[†]This work was supported in part by the EU Contract No. MRTN-CT-2006-035482, "FLAVIAnet".

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1. The weak chiral lagrangian

Chiral Perturbation theory is the appropriate framework to describe QCD at low energies, since relies completeley on QCD symmetries and that there is a low energy expansion of the of physical amplitudes: this was extensively discussed at the workshop [1, 2] . As an example of precision physics there are the measurements of the $\pi\pi$ scattering lenghts, determined with an accuracy of 1.5% [1, 3]. This also thanks to the theoretical understanding and experimental determination of the Gasser-Leutwyler coefficients. I think it has been very fruitful to explain theoretically the value of the $\mathscr{O}(p^4)$ Gasser-Leutwyler coefficients, L_i: in fact the study of Vector Meson Dominance (VMD) models has given a relevant breakthrough to this research has been given [4]. An initial problem that had to be overcome was that the traditional formulation of the vectors "V^μ" was contributing only at $\mathscr{O}(p^6)$; however phenomenology and a better matching of the pion form factors with known UV QCD behaviour was demanding for a $\mathscr{O}(p^4)$ contribution, which could be obtained with the antisymmetric formulation of the vectors, "$V^{\mu\nu}$". With the help of the KSFR relations these remarkable successful predictions can be compactly written as

$$L_1^{(V)} = L_2^{(V)} = \frac{-L_3^{(V)}}{6} = \frac{L_9^{(V)}}{8} = -L_{10}^{(V+A)} = \frac{F_\pi^2}{16\pi^2} \tag{1.1}$$

This picture has been substantially confirmed by Large N, chiral quark model, Nambu Jona Lasinio model [3] and even lately by ADS/CFT models [5]. Thus should be inspiring if we want to study weak interactions . The basic $\Delta S = 1$ chiral lagrangian up to $\mathscr{O}(p^4)$ can be written as $\mathscr{L}_{\Delta S=1} = \mathscr{L}^2_{\Delta S=1} + \mathscr{L}^4_{\Delta S=1}$

$$G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi,\gamma\gamma} + \underbrace{G_8 F^2 \sum N_i W_i}_{K^+\to\pi^+\gamma\gamma,\ K\to\pi\pi\gamma} , \tag{1.2}$$

G_8 is fixed by the $K \to \pi\pi$ amplitudes, while the second term represents the weak $\mathscr{O}(p^4)$ [6, 7] lagrangian: There are 37 coefficients, N_i's, and operators, W_i's. Unfortunately the N_i's are both theoretically and phenomenologically very poorly known [7]. Still predictions are available like the same counterterm combination in Table 1, $N_{14} - N_{15} - N_{16} - N_{17}$ for the electric E1 contributions to $K_S \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$. The N_i's requires the evaluation of integrals of appropriate QCD Green functions over all loop momenta. However we need extra assumptions to have predictive power: two interesting ideas are factorization and VMD. At scales larger than the QCD scale is reasonable to assume a Fermi lagrangian; we can test this working idea at low energies: the currents, $\delta S/\delta \ell_\mu$, can be obtained from the general bosonized hadronic action, S, being ℓ_μ, the left-handed hadronic current; then we can write the current $\times$ current structure as

$$\mathscr{L}_{FM} \; = \; 4 k_F \, G_8 \, \langle \lambda \, \frac{\delta S}{\delta \ell_\mu} \, \frac{\delta S}{\delta \ell^\mu} \, \rangle \; + \; h.c. \, , \tag{1.3}$$

Another hypothesis to test is VMD; there are two main reasons to test this hypothesis: first of all it has been shown in the strong sector how relevant has been the QCD matching, secondly is phenomenologically at work, as we shall see, in many instances. Either there is evidence for poles or VMD predictions for the local terms, either $\mathscr{O}(p^4)$ or $\mathscr{O}(p^6)$, are phenomenologically at work: the weak VMD picture is just more complicated since the two and three point Green's functions require integrals of form factor over all momenta. Nevertheless, VMD MUST work to improve

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the matching with QCD in the UV region; indeed there are examples $m_{\pi^+} - m_{\pi^0}$ -electroweak contribution [8], $K_L \to \mu\bar{\mu}$ [9] and as we shall see in $K_L \to \pi^+\pi^-\gamma$. An intriguing feature of weak VMD is that most of the results, contrary to what it happened in the strong sector, seem not to depend of what kind of formulation one is using, "$V^{\mu\nu}$" or "V^{μ}", and not even the FM in eq. (1.3) seem relevant [7] . However we are very far from a VMD relation in the weak sector. We mention some cases where we can get some info on various N_i's soon:

Table 1

Decay	$\mathscr{L}^4_{\Delta S=1}$ counterterms
$K^+ \to \pi^+ l^+ l^-$	$N^r_{14} - N^r_{15}$
$K_S \to \pi^0 l^+ l^-$	$2N^r_{14} + N^r_{15}$
$K^\pm \to \pi^\pm \gamma\gamma$	$N_{14} - N_{15} - 2N_{18}$
$K_S \to \pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
$K^\pm \to \pi^\pm\pi^0\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
$K_L \to \pi^+\pi^-\ e^+e^-$	$N^r_{14} + 2N^r_{15} - 3(N^r_{16} - N_{17})$
$K^+ \to \pi^+\pi^0 e^+e^-$	$N^r_{14} + 2N^r_{15} - 3(N^r_{16} - N_{17})$
$K_S \to \pi^+\pi^- e^+e^-$	$N^r_{14} - N^r_{15} - 3(N^r_{16} + N_{17})$

2. $K_S \to \gamma\gamma$ / $K_L \to \pi^0\gamma\gamma$

$K_S \to \gamma\gamma$ has vanishing short-distance contributions and starts at $\mathscr{O}(p^4)$, $A^{(4)}$ in Fig. 1, but with no counterterm structures. This implies that i) we have only a loop contribution and ii) this contribution is scale-independent [10]. The predictions for $B(K_S \to \gamma\gamma)$ is unambigous, depending only from $\mathscr{L}^2_{\Delta S=1}$ in (1.2). This is the *ideal* χPT test (and in general of effective field theories) at the *quantum level*; the experimental and theoretical picture is

$$\begin{array}{ll} \mathrm{TH} \quad (p^4) \quad 2.1\times 10^{-6} & \\ \mathrm{NA48} \quad (2.78\pm 0.072)\times 10^{-6}. & \Rightarrow \dfrac{A^{(6)}}{A^{(4)}} \leq 15\% \\ \mathrm{KLOE} \quad (2.26\pm 0.13)\times 10^{-6} & \end{array}$$

The experimental results [11, 12] show a disagreement that must be clarified, maybe by KLOE2, and also that this χPT predictions works better than the naïve dimensional analysis: $A^{(6)}/A^{(4)} \sim m_K^2/(4\pi F_\pi)^2$.

$A(K_L \to \pi^0\gamma\gamma)$ shares at $\mathscr{O}(p^4)$ the same finiteness properties of $K_S \to \gamma\gamma$ and the same helicity amplitude, A, proportional to $F^{\mu\nu}F_{\mu\nu}$ and relative angular momentum $J_{\gamma\gamma} = 0$ for the diphoton system [13] . At $\mathscr{O}(p^6)$, a new helicity amplitude, B, where the diphoton system is in a $J_{\gamma\gamma}$ =2 state, adds to the A-type amplitude. Defining $y = p(q_1 - q_2)/m_K^2$ and $z = (q_1+q_2)^2/m_K^2$, then the double differential rate is given by

$$\frac{\partial^2\Gamma}{\partial y\,\partial z} \sim \left[z^2\,|A+B|^2 + \left(y^2 - \frac{\lambda(1,r_\pi^2,z)}{4}\right)^2 |B|^2 \right] , \tag{2.1}$$

As we see, for $z \to 0$, we can disentangle the size of B-type amplitude and this is crucial to establish the CP conserving contribution to $K_L \to \pi^0\ell^+\ell^-$, due to $K_L \to \pi^0$"$\gamma\gamma$" $\to \pi^0\ell^+\ell^-$ that is

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m_ℓ-suppressed for the A-type amplitude and unsuppressed for the B-type amplitude. The situation has been confused for some time since data, while they showed, consistentely with $\mathcal{O}(p^4)$, small or negligible contributions at low diphoton invariant mass, z, they strongly disagreed in the rate, by a factor $2 \sim 3$ larger. Then it was realized that large $\mathcal{O}(p^6)$ unitarity contributions in Fig. 2 and VMD contributions [14], parametrized by a_V, enhance the amplitude A and produce a $B-$type amplitude. An initial disagreement between NA48 and KTeV experiments for the spectrum at low z has been solved lately: in fact KTeV [16] has reanalyzed the data finding agreement in the width and in the spectrum with NA48. Now the PDG average [18]

$$\begin{aligned} B(K_L \to \pi^0\gamma\gamma) &= 1.273 \pm 0.034 \quad \text{PDG average}, \\ a_V &= -0.43 \pm 0.06 \quad \text{PDG average}, \end{aligned} \tag{2.2}$$

The value of a_V leads to suppressed CP conserving contribution to $B(K_L \to \pi^0 e^+ e^-)$ and incidentally it is exactely the sign and the size of FM in eq. (1.3) [19]. Actually experiments in eq.(2.2) show that the local, a_V, and non-local contributions ($\pi\pi$- unitarity loop) conspire to give a vanishing contribution for the B-type amplitude; and consequentely the CP conserving contribution to $B(K_L \to \pi^0 e^+ e^{-)}$ is suppressed . Also as a result the recent PDG average [18] the value of a_V is consistent with the theoretical prediction in Ref. [15] . All this is good news for the search of CP violation and New Physics in this channel.

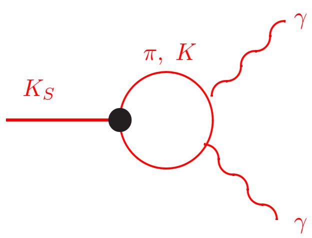

Figure 1: $K_S \to \gamma\gamma$ [10]

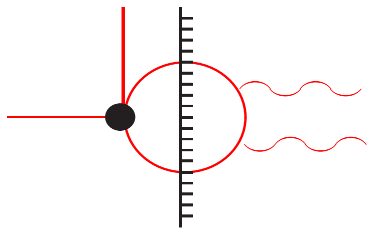

Figure 2: Unitarity contributions to $K \to \pi\gamma\gamma$

3. $K^+ \to \pi^+\gamma\gamma$ and $K^+ \to \pi^+$“γ”$\gamma \to \pi^+ e^+ e^- \gamma$

These channels start at $\mathcal{O}(p^4)$, with pion (and kaon) loops and a local term $\hat{c}$: the external charged particles allow a non-vanishing $\mathcal{O}(p^4)$ CT. Due to the presence of the pion pole, a new amplitude, C, proportional to $F^{\mu\nu}\widetilde{F}_{\mu\nu}$ [20] ; in this case at $\mathcal{O}(p^6)$ the unitarity contributions in Fig.2 enhance the amplitude A 30%-40% , along with the generation of B-type amplitude, while the VMD term plays a minor role [21]

$$\frac{d^2\Gamma}{dydz} \sim \left[z^2(|A+B|^2+|C|^2) + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2 \right) \right)^2 |B|^2 \right] \tag{3.1}$$

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A precise determination of the rate and the spectrum would fix the constant $\hat{c}$, predicted to have contributions from the axial spin-1 contributions

$$\hat{c} = \frac{128\pi^2}{3}\left[3(L_9 + L_{10}) + N_{14} - N_{15} - 2N_{18})\right] = 2.3\;(1 - 2\;k_f)\;,$$

with k_f is the factorization factor in the FM model of eq. (1.3 or the weak axial vector coupling of Ref. [7] . As shown in Fig. 3 a careful investigation of the diphoton spectrum and the rate will allow the $\hat{c}$ determination [21].

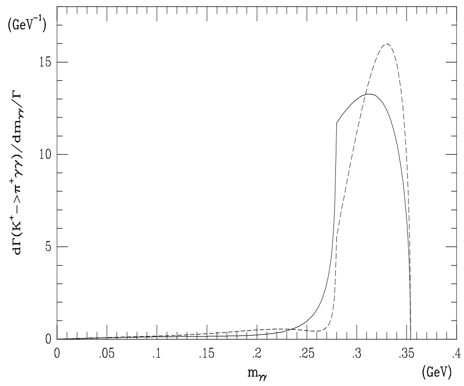

Figure 3: $K^+ \to \pi^+\gamma\gamma$: $\hat{c} = 0$, full line, $\hat{c} = -2.3$, dashed line, [7]

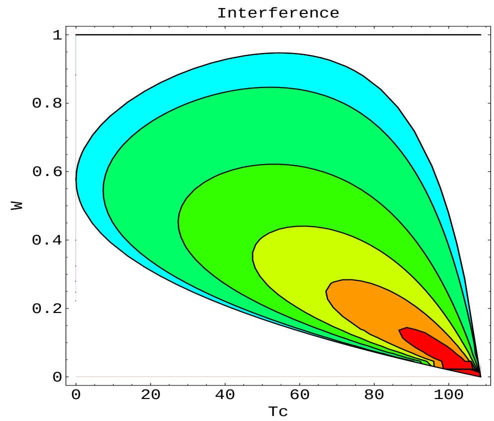

Figure 4: $T_c^* - W$-Dalitz plot. In this contour plot of the interference Branching the red area corresponds to more dense and thus larger contribution

Actually BNL 787 got 31 events leading to $B(K^+ \to \pi^+\gamma\gamma) \sim (6 \pm 1.6) \cdot 10^{-7}$ [22] and a value of $\hat{c} = 1.8 \pm 0.6$. Recentely NA48 has presented some preliminary results with 40% of their statistics, leading to 1164 events and normalization channel $K^+ \to \pi^+\pi^0$; their result is $B(K^+ \to \pi^+\gamma\gamma) = (1.07 \pm 0.04 \pm 0.08) \cdot 10^{-6}$ assuming $\hat{c} = 2$ [23].

The same physics has been investigated by NA48 in $K^+ \to \pi^+$"γ"$\gamma \to \pi^+ e^+ e^- \gamma$ with the theoretical evaluation in Ref. [24]: form this channel the value $\hat{c} = 0.90 \pm 0.45$ is found [25]. Rcentely also a caution warning on some sizable pole contamination to $C(z)$ have been arisen [26].

4. $K_S \to \pi^0 \ell^+ \ell^-$

The CP-conserving decays $K^\pm(K_S) \to \pi^\pm(\pi^0)\ell^+\ell^-$ are dominated by the long-distance process $K \to \pi\gamma^* \to \pi\ell^+\ell^-$ [27, 28]. The decay amplitudes can in general be written in terms of one form factor $W_i(z)$ $(i = \pm, S)$ $z = q^2/M_K^2$; $W_i(z)$, which can be decomposed as the sum of a polynomial piece plus a non-analytic term, $W_i^{\pi\pi}(z)$, generated by the $\pi\pi$ loop, is completely determined in terms of the physical $K \to 3\pi$ amplitude [28]. Keeping the polynomial terms up to $\mathscr{O}(p^6)$ we can write

$$W_i(z) \;=\; G_F M_K^2\,(a_i \,+\, b_i z) + W_i^{\pi\pi}(z)\;, \tag{4.1}$$

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where the parameters a_i and b_i parametrize local contributions starting respectively at $\mathscr{O}(p^4)$ and $\mathscr{O}(p^6)$. The most accurate determination come from BNL-E865 [29] and NA48 [30]

$$\text{E865} \;\; a_+ = -0.587 \pm 0.010, \qquad b_+ = -0.655 \pm 0.044 \tag{4.2}$$

$$\text{NA48} \;\; a_+ = -0.578 \pm 0.016, \qquad b_+ = -0.779 \pm 0.066. \tag{4.3}$$

The experimental size of the ratio b_+/a_+ exceeds the naive dimensional analysis estimate $b_+/a_+ \sim \mathscr{O}[M_K^2/(4\pi F_\pi)^2] \sim 0.2$, but can be explained by a large VMD contribution. Chiral symmetry alone does not allow us to determine the unknown couplings a_S and b_S in terms of a_+ and b_+ [27, 28]; then approximately $B(K_S \to \pi^0 l^+ l^-)$

$$B(K_S \to \pi^0 e^+ e^-) \approx 5 \times 10^{-9} \cdot a_S^2 \qquad B(K_S \to \pi^0 \mu^+ \mu^-) \approx 1.2 \times 10^{-9} \cdot a_S^2 \tag{4.4}$$

NA48, assuming a VMD form factor, finds respectively [31] [32]

$$|a_S|_{ee} = 1.06^{+0.26}_{-0.21} \pm 0.07 \qquad |a_S|_{\mu\mu} = 1.54^{+0.40}_{-0.32} \pm 0.06 \tag{4.5}$$

KLOE hopefully may assess the value of this branching and establish the amplification of the CP violating branching

$$B(K_L \to \pi^0 e^+ e^-)_{CPV} = \left[15.3\, a_S^2 - 6.8 \frac{\Im \lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\Im \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12} \,, \tag{4.6}$$

The sign of the interference term is model-dependent but there are good theoretical motivations that predict it negative and good strategies to fix it experimentally [17].

5. $K \to \pi\pi\gamma \,/\, K \to \pi\pi ee$

We can decompose $K(p) \to \pi(p_1)\pi(p_2)\gamma(q)$ decays, according to gauge and Lorentz invariance, in electric (E) and magnetic (M) terms [33] In the electric transitions one generally separates the bremsstrahlung amplitude E_B, firmly predicted theoretically by the Low theorem in terms of the non-radiative amplitude and enhanced by the $1/E_\gamma$ behaviour, from the direct emission amplitudes (DE). Summing over photon helicities, there is no interference among electric and magnetic terms: $\mathrm{d}^2\Gamma/(\mathrm{d}z_1 \mathrm{d}z_2) \sim |E(z_i)|^2 + |M(z_i)|^2$. At the lowest order, (p^2), one obtains only E_B. Magnetic and electric direct emission amplitudes, appearing at $\mathscr{O}(p^4)$, can be decomposed in a multipole expansion [33]. In Table 2 we show the present experimental status of the DE amplitudes and the leading multipoles.

Table 2 DE_{exp}

$K_S \to \pi^+\pi^-\gamma$	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	$(0.44 \pm 0.07)10^{-5}$	$M1, E1$
$K_L \to \pi^+\pi^-\gamma$	$(2.92 \pm 0.07)10^{-5}$	$M1$,VMD

Particularly interesting are the recent interesting NA48 data regarding $K^+ \to \pi^+\pi^0\gamma$ decays [23]. Due to the $\Delta I = 3/2$ suppression of the bremsstrahlung, interference between E_B and $E1$ and magnetic transitions can be measured. Defining $z_i = p_i \cdot q/m_K^2 \quad z_3 = p_{_K} \cdot q/m_K^2$ and $z_3 z_+ = \frac{m_{\pi^+}^2}{m_K^2} W^2$ we

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can study of the Dalitz plot distribution in Figure 1

$$\frac{\partial^2\Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^* \partial W^2}\left[1+\frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E_{DE}}{eA}\right)W^2+\frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E_{DE}}{eA}\right|^2+\left|\frac{M_{DE}}{eA}\right|^2\right)W^4\right],$$

where $A = A(K^+ \to \pi^+\pi^0)$; an accurate study of the Dalitz plot in Figure 1 has lead NA48 these preliminary results [23]

Table 3

	NA48	$T_c^* \in [0,80]$MeV
Frac(DE)	=	$(3.35 \pm 0.35 \pm 0.25) \times 10^{-2}$
Frac(INT)	=	$(-2.67 \pm 0.81 \pm 0.73) \times 10^{-2}$

This is the first evidence of non-vanishing interference [23] and gives a determination of the counterterm coefficient in Table 1, contributing to $E1$ [6, 7]. The magnetic contributions, is by now well established; there are two contributions i) an indirect contributions generated by a pion pole mediating a Wess Zumino Witten (WZW) term and a vertex from $\mathscr{L}^2_{\Delta S=1}$ from (1.2) and ii) genuine $\mathscr{O}(p^4)$ anomalous-like contributions from $\mathscr{L}^4_{\Delta S=1}$ in Table 1 $N_{28},..,N_{31}$. These last terms can be obtained from factorization in eq. (1.3) [34], where we consider also the *anomalous current (from the WZW term)*. Then we generate $\mathscr{L}^4_{\Delta S=1}$ in (1.2) with coefficients

$$a_1 = 8\pi^2 N_{28} \quad , \quad a_2 = 32\pi^2 N_{29} \, ,$$
$$a_3 = \frac{16}{3}\pi^2 N_{30} \quad , \quad a_4 = 16\pi^2 N_{31} \, .$$

The a_i are positive parameters of $\mathscr{O}(1)$. Once we have proven that these terms are there, several dynamical mechanism can generate them. In fact theory [7] and data from $K_L \to \pi^+\pi^-\gamma$ [35, 36] point towards a large VMD contributions in these decays. In terms of the counterterms we can write

$$M_L^{(4)} = \frac{eG_8 m_K^3}{2\pi^2 F}(a_2+2a_4), \tag{5.1}$$

$$M_+^{(4)} = -\frac{eG_8 m_K^3}{4\pi^2 F}[2+3(2a_3-a_2)]. \tag{5.2}$$

It is interesting that the direct emission branching in the Table 3, can be obtained by $M_+^{(4)}$ in (5.2) neglecting the contributions of the a_i's:

The interpretation of $B(K^+ \to \pi^+\pi^0\gamma)_{DE}$ dominated by WZW is challenged by the fact that the observed $B(K_L \to \pi^+\pi^-\gamma)_{DE}$ shows i) $a_i \sim O(1)$ and ii) large VMD [18, 36]: this calls for a more accurate theoretical investigation. In fact since the presence of a form factor affects the Dalitz plot distribution we think that a thorough analysis is required to disentangle as much as possible the interference from possible competing effects in the magnetic amplitudes [37].

Other interesting channels are $K \to \pi\pi e^+e^-$-decays: particularly appealing at KLOE, are $K_S \to \pi^+\pi^-e^+e^-$-decays , where we can perform the CHPT tests described in Table 1. Also New Physics and CP violation is principle interesting to investigate. Actually so far NA48, with 676 evts. has measured $B(K_S \to \pi^+\pi^-e^+e^-) = 4.69 \pm 0.30$.

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6. Acknowledgement

I thank the organizers of EFT09 for the nice workshop, in particular to Jorge Portolés.

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