Predictions of chiral perturbation theory for Compton scattering off protons

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We compute the Compton scattering off the nucleons in the framework of manifestly covariant baryon chiral perturbation theory (BχPT). The results for observables differ substantially from the corresponding calculations in heavy-baryon chiral perturbation theory (HBχPT), most appreciably in the forward kinematics. We verify that the covariant p^3 result fulfills the forward-Compton-scattering sum rules. We also explore the effect of the $\Delta(1232)$ resonance at order p^4/Δ, with $\Delta \approx 300$ MeV, the resonance excitation energy. We find that the substantial effect of the Δ-excitation on the nucleon polarizabilities can naturally be accommodated in the manifestly covariant calculation.

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1. Introduction

Compton scattering off nucleons allows to study the structure and the e.m. properties of the nucleon. At very low energies, the process depends on the static e.m. moments of nucleon, the charge and magnetic moment [1]. At larger energies (around 100 MeV and above), the effects of the nucleon structure can be detected [2, 3, 4, 5, 6, 7, 8]. For instance, these effects show up in the values of nucleon *polarizabilities* — see the PDG column of Table 1.

More insights come from chiral perturbation theory (χPT), an effective theory of the low-energy strong interaction [9, 10]. The leading-order χPT result for the nucleon polarizabilities is a prediction — the low-energy constants (LECs) start to contribute at the next order. The first calculation of polarizabilities in χPT [11] at leading order yields the values shown in the BχPT $\mathcal{O}(p^3)$ column of Table 1. This calculation was done in manifestly Lorentz-covariant baryon χPT (BχPT), to distinguish it from the heavy-baryon χPT (HBχPT), which was introduced [15] in order to cure the chiral power counting problems that BχPT had apparently had [16]. Incidentally, the HBχPT result agrees with experiment much better, see Table 1. More detailed analyses of Compton scattering in HBχPT followed [12, 17, 18]. However, it was shown more recently [19, 20] that

	HBχPT		BχPT		PDG
	$\mathcal{O}(p^3)$ [12]	$\mathcal{O}(\varepsilon^3)$ [13]	$\mathcal{O}(p^3)$ [11]	$\mathcal{O}(p^4/\Delta)$	[14]
$\alpha^{(p)}$	12.2	20.8	6.8	10.8	12.0 ± 0.6
$\beta^{(p)}$	1.22	14.7	-1.8	2.9	1.9 ± 0.5

Table 1: The electric (α) and magnetic (β) polarizabilities of the proton in units of $10^{-4}\,\mathrm{fm}^3$. The last column quotes the PDG compilation of experimental results, while the first two represent the predictions of the HBχPT and BχPT, respectively.

BχPT does not have a problem with power counting *per se*. It was also pointed out [21] that the difference between BχPT and HBχPT results can be large due to the presence of physical cuts and other non-analytic structures. Moreover, HBχPT is incompatible with the sum rules [22, 23, 24]. Finally, the effect of the Δ excitation in Compton scattering cannot be accommodated in the HB framework in a natural way [25, 26] (see also the HBχPT column of Table 1). These observations make a strong case for adopting the BχPT formalism in favor of the heavy-baryon one. Here we present the results of a calculation of Compton scattering in BχPT to orders p^3 and p^4/Δ [27].

2. Chiral loops and Lagrangians

Up to $\mathcal{O}(p^3)$, the χPT expansion for the Compton amplitude contains the Born graphs, the Wess-Zumino-Witten anomaly (see, e.g., Ref. [26]), and the loop graphs shown in Fig. 1. To calculate these loops, we consider the leading-order chiral Lagrangian for the nucleon:

$$\mathscr{L}_N^{(1)} = \overline{N}\left(i\slashed{D} - M_N + g_A \slashed{u}\gamma_5\right)N, \tag{2.1}$$

where N denotes the isodoublet Dirac field of the nucleon, M_N is the nucleon mass and g_A is the axial-coupling constant, and the chiral covariant derivative is given by $D_\mu N = \partial_\mu N + iv_\mu N$,

whereas the vector and axial-vector fields above are defined in terms of the pion field, $\pi^a(x)$, as

$$v_\mu \equiv \tfrac{1}{2}\tau^a v_\mu^a(x) = \frac{1}{2i}\left(u\,\partial_\mu u^\dagger + u^\dagger \partial_\mu u\right), \qquad a_\mu \equiv \tfrac{1}{2}\tau^a a_\mu^a(x) = \frac{1}{2i}\left(u^\dagger \partial_\mu u - u\,\partial_\mu u^\dagger\right), \quad (2.2)$$

with $u = \exp(i\pi^a \tau^a / 2f)$, and f the pion decay constant. Then, we apply a redefinition of the nucleon field, $N \to \xi N$, where ξ has the form:

$$\xi = \exp\left(\frac{ig_A \pi^a \tau^a}{2f}\gamma_5\right). \tag{2.3}$$

For the one-loop contributions to Compton scattering it is sufficient to expand up to the second order in the pion field. After the expansion, the redefined Lagrangian takes the following form:

$$\mathscr{L}_N^{\prime(1)} = \overline{N}\left(i\slashed{\partial} - M_N - i\frac{g_A}{f}M_N \tau^a \pi^a \gamma_5 + \frac{g_A^2}{2f^2}M_N \pi^2 - \frac{(g_A-1)^2}{4f^2}\tau^a \varepsilon^{abc}\pi^b \slashed{\partial}\pi^c\right)N + O(\pi^3). \tag{2.4}$$

Finally, one gets the set of diagrams in Fig. 1 with the couplings from Eq. (2.4) instead of the usual set [11]. However, the two sets of one-loop diagrams give identical expressions for the Compton amplitude. This fact also explains why the one-loop result for polarizabilities in the linear sigma model with heavy σ-meson [28] is exactly the same as in BχPT at $\mathcal{O}(p^3)$ [11].

The Δ excitation starts to contribute at order p^4/Δ, where $\Delta = M_\Delta - M_N \approx 0.3$ GeV. The relevant diagrams are shown in Fig. 2. The Δ Born contribution is calculated in the same way as in Ref. [26], except that we use the values of the $\gamma N \to \Delta$ couplings ($g_M = 2.95$ and $g_E = -1.0$) from the pion-photoproduction analyses of Refs. [31, 32], and also include the corresponding crossed graph. We use $h_A = 2.85$, corresponding to the $\Delta \to \pi N$ decay width of 115 MeV.

The one-particle-reducible graphs in Figs. 1 and 2 contribute to the nucleon mass, field, charge, and magnetic moment renormalization. We adopt the on-mass-shell renormalization scheme, and use the following values of the parameters: $e^2/4\pi = 1/137$, $g_A = 1.267$, $f = f_\pi = 0.0924$ GeV, $m_\pi = 0.139$ GeV, $M_N = 0.9383$ GeV, $\kappa_N = 1.79$ for the proton.
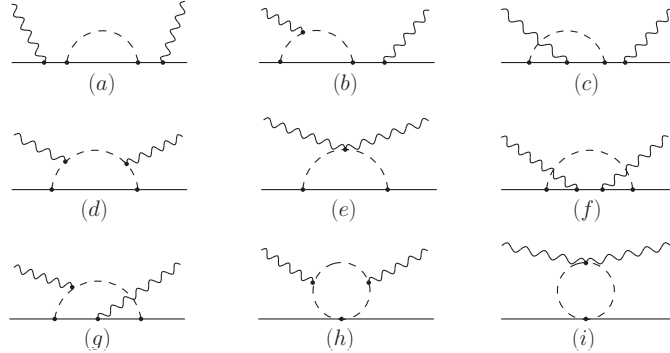

Figure 1: The loop graphs evaluated in this work. Graphs obtained from these by crossing and time-reversal are not shown, but are evaluated too.

3. Consistency with sum rules

The amplitude of forward Compton scattering can be related to an integral over energy of the photoabsorption cross-section, which in combination with the low-energy expansion yields a
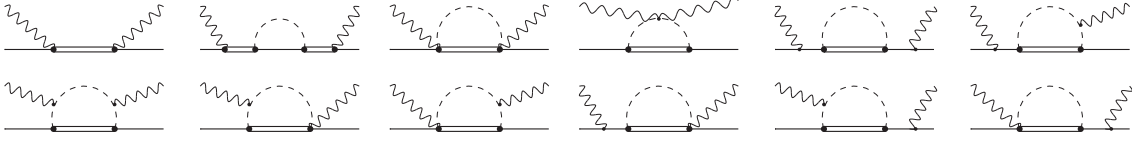
Figure 2: The graphs with $\Delta(1232)$ that contribute at order p^4/Δ. Graphs obtained from these by crossing are not shown, but are calculated as well.

number of model-independent sum rules. A famous example is the Baldin sum rule:

$$\alpha + \beta = \frac{1}{2\pi^2} \int\limits_0^\infty d\omega \frac{\sigma_T(\omega)}{\omega^2 - i0} \tag{3.1}$$

that relates the sum of polarizabilities to an integral of the total photoabsorption cross-section σ_T.

In general, the forward Compton-scattering amplitude can be decomposed into two scalar functions of single variable:

$$T_{fi}(\omega) = \vec{\varepsilon}'^* \cdot \vec{\varepsilon} f(\omega) + i\vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \vec{\varepsilon}) \, \omega \, g(\omega), \tag{3.2}$$

where $\vec{\varepsilon}'$, $\vec{\varepsilon}$ are the polarization vectors of the initial and final photons, respectively, and $\vec{\sigma}$ are the Pauli spin matrices. Using analyticity and unitarity, one can write down the following sum rules:

$$f(\omega) = f(0) + \frac{\omega^2}{2\pi^2} \int\limits_0^\infty d\omega' \frac{\sigma_T(\omega')}{\omega'^2 - \omega^2 - i0}, \quad g(\omega) = \frac{1}{4\pi^2} \int\limits_0^\infty d\omega' \, \omega' \frac{\sigma_{1/2}(\omega') - \sigma_{3/2}(\omega')}{\omega'^2 - \omega^2 - i0}, \tag{3.3}$$

where σ_λ is the doubly-polarized photoabsorption cross-section, with λ being the helicity of the initial photon-nucleon state. We showed that the loop contributions in Fig. 1 fulfill the sum rules Eq. (3.3). From the sum rules, one can see that at $\mathscr{O}(p^3)$, chiral symmetry is not relevant for the forward Compton amplitude. The graphs (h) and (i) in Fig. 1 take the role of chiral symmetry. In the forward angles these graphs vanish, but play an important role in the backward angles. Without them the value of $\alpha - \beta$ would diverge as $1/m_\pi^2$ in the chiral limit (instead of $1/m_\pi$ as it should). Thus, chiral symmetry plays a more prominent role in the backward Compton scattering.

The results for the polarizabilities are in worse agreement with experiment than the HBχPT p^3 result (see Table 1). This, in fact, opens a room for the $\Delta(1232)$ contributions. The $\Delta(1232)$ plays an important role in nucleon polarizabilities, as can be seen from the Baldin sum rule and the fact that the photoabsorption cross-section is dominated, at lower energies, by the Δ resonance. In contrast, the HBχPT p^3 value for $\alpha + \beta$ saturates the sum rule, leaving no room for other contributions.

4. Results for observables

In Fig. 3, we show the unpolarized differential cross-section of the $\gamma p \to \gamma p$ process as a function of the scattering angle in center-of-mass system, with the incident photon energy fixed at just below the pion-production threshold. The major differences between the HBχPT and BχPT p^3 calculations arise at forward angles. This is because at low energies the p^3 contribution to the cross-section at forward (and backward) angles is determined by the p^3 contribution to $\alpha + \beta$ (and

$\alpha - \beta$). The sum of polarizabilities differs between the two calculations much more than their difference, and this fact reflects itself in the cross-section.

The red solid line with an error band in Fig. 3 shows the result of adding the Δ contribution to the covariant p^3 result. The Δ contribution in BχPT is compatible with both photoproduction and Compton scattering data, which is further demonstrated in Fig. 4, where the $\gamma p \to \gamma p$ cross-section is plotted as a function of photon energies at fixed angles (in the lab system). The HBχPT result is omitted here, but can be found in Ref. [18]. The results for the proton polarizabilities, complete up to $\mathcal{O}(p^4/\Delta)$, are displayed in Table 1.
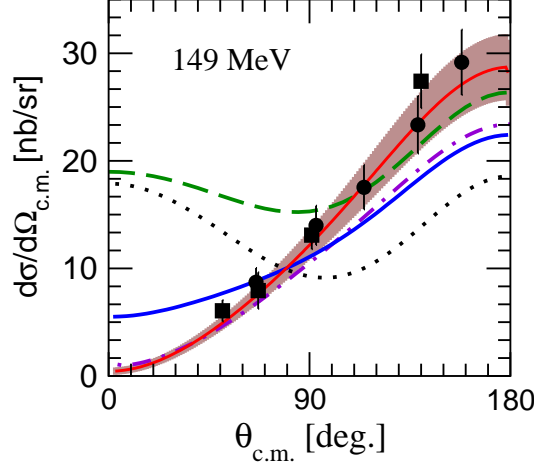

Figure 3: (Color online) Angular dependence of the $\gamma p \to \gamma p$ differential cross-section in the center-of-mass system for fixed photon-beam energy, $E_\gamma^{(lab)} = 149$ MeV. Data points are from SAL [4] — filled squares, and MAMI [6] — filled circles. The curves are: Klein-Nishina — dotted, Born graphs and WZW-anomaly — green dashed, adding the HBχPT — violet dash-dotted, adding the BχPT — blue solid. The result of adding the Δ-excitation contribution to the BχPT p^3 is shown by the red solid line with a band.
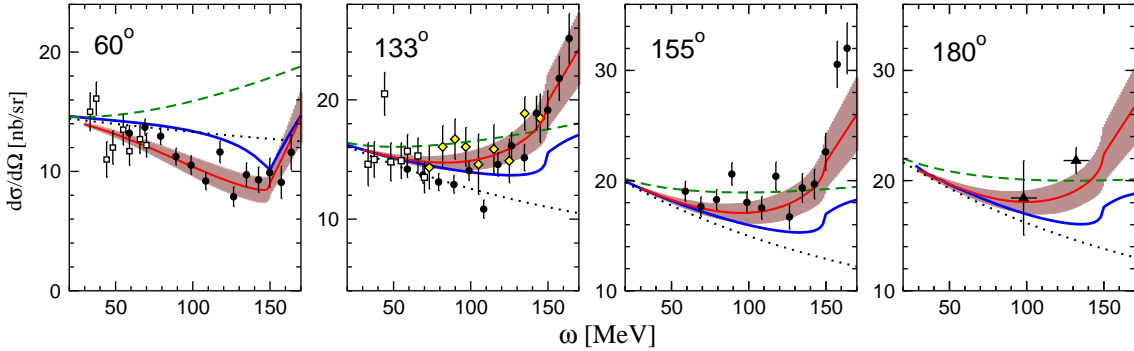

Figure 4: (Color online) Energy dependence of the $\gamma p \to \gamma p$ differential cross-section in the laboratory frame for fixed values of the scattering angle. Data: Illinois [2] — open squares, MAMI [3] — filled triangles, SAL [5] — open diamonds, and MAMI [6] — filled circles. The curves are as in Fig. 3.

5. Conclusion

We have studied the nucleon Compton scattering in the framework of BχPT at orders p^3 and

p^4/Δ. The covariant p^3 result fulfills the forward-Compton-scattering sum rules. Chiral symmetry has no effect on the forward scattering but plays an important role at the backward scattering. For the $\gamma p \to \gamma p$ cross sections we find that the difference between the HBχPT and BχPT results can indeed be unnaturally large, especially in the forward kinematics. We argue that higher-order effects of the $\Delta(1232)$ excitation can more naturally be accommodated in the BχPT calculation. This is due to partial cancellation of the relativistic and Δ-excitation effects which is explicit in the covariant calculation. In contrast to the HBχPT approach, in BχPT the effect of $\Delta(1232)$ appears to be compatible both with the Compton scattering and pion photoproduction data.

Acknowledgments

References

[1] F. E. Low, *Scattering Of Light Of Very Low Frequency By Systems Of Spin 1/2*, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, *Scattering Of Low-Energy Photons By Particles Of Spin 1/2* Phys. Rev. **96**, 1433 (1954).

[2] F. J. Federspiel *et al.*, *The Proton Compton effect: A Measurement of the electric and magnetic polarizabilities of the proton*, Phys. Rev. Lett. **67**, 1511 (1991).

[3] A. Zieger, R. Van de Vyver, D. Christmann, A. De Graeve, C. Van den Abeele and B. Ziegler, *180-degrees Compton scattering by the proton below the pion threshold*, Phys. Lett. B **278**, 34 (1992).

[4] E. L. Hallin *et al.*, *Compton scattering from the proton*, Phys. Rev. C **48**, 1497 (1993).

[5] B. E. MacGibbon, G. Garino, M. A. Lucas, A.M. Nathan, G. Feldman and B. Dolbilkin, *Measurement of the electric and magnetic polarizabilities of the proton*, Phys. Rev. C **52**, 2097 (1995). [arXiv:nucl-ex/9507001].

[6] V. Olmos de Leon *et al.*, *Low-Energy Compton Scattering And The Polarizabilities Of The Proton*, Eur. Phys. J. A **10**, 207 (2001).

[7] D. Drechsel, B. Pasquini and M. Vanderhaeghen, *Dispersion relations in real and virtual Compton scattering*, Phys. Rept. **378**, 99 (2003). [arXiv:hep-ph/0212124].

[8] M. Schumacher, *Polarizability of the nucleon and Compton scattering*, Prog. Part. Nucl. Phys. **55**, 567 (2005). [arXiv:hep-ph/0501167].

[9] S. Weinberg, *Phenomenological Lagrangians*, Physica A **96**, 327 (1979).

[10] J. Gasser and H. Leutwyler, *Chiral Perturbation Theory To One Loop*, Annals Phys. **158**, 142 (1984);

[11] V. Bernard, N. Kaiser and U.-G. Meißner, *Chiral expansion of the nucleon's electromagnetic polarizabilities*, Phys. Rev. Lett. **67**, 1515 (1991); *Nucleons with chiral loops: Electromagnetic polarizabilities*, Nucl. Phys. B **373**, 346 (1992).

[12] V. Bernard, N. Kaiser, and U.-G. Meißner, *Chiral dynamics in nucleons and nuclei*, Int. J. Mod. Phys. E **4**, 193 (1995). [arXiv:hep-ph/9501384].

[13] T. R. Hemmert, B. R. Holstein and J. Kambor, *Delta(1232) and the Polarizabilities of the Nucleon, Phys. Rev. D* **55**, 5598 (1997). [arXiv:hep-ph/9612374].

[14] W. M. Yao *et al.* [Particle Data Group], *Review of particle physics, J. Phys. G* **33**, 1 (2006).

[15] E. Jenkins and A. V. Manohar, *Baryon Chiral Perturbation Theory Using A Heavy Fermion Lagrangian, Phys. Lett. B* **255**, 558 (1991).

[16] J. Gasser, M. E. Sainio and A. Svarc, *Nucleons With Chiral Loops, Nucl. Phys. B* **307**, 779 (1988).

[17] J. A. McGovern, *Compton scattering from the proton at fourth order in the chiral expansion, Phys. Rev. C* **63**, 064608 (2001) [Erratum-ibid. **66**, 039902 (2002)]. [arXiv:nucl-th/0101057].

[18] S. R. Beane, M. Malheiro, J. A. McGovern, D. R. Phillips and U. van Kolck, *Nucleon polarizabilities from low-energy Compton scattering, Phys. Lett. B* **567**, 200 (2003) [Erratum-ibid. **607**, 320 (2005)]; [arXiv:nucl-th/0209002]. *Compton scattering on the proton, neutron, and deuteron in chiral perturbation theory to* $O(Q^4)$, *Nucl. Phys. A* **747**, 311 (2005). [arXiv:nucl-th/0403088].

[19] J. Gegelia and G. Japaridze, *Matching heavy particle approach to relativistic theory, Phys. Rev. D* **60**, 114038 (1999); J. Gegelia, G. Japaridze and X. Q. Wang, *J. Phys. G* **29**, 2303 (2003).

[20] T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, *Renormalization of relativistic baryon chiral perturbation theory and power counting, Phys. Rev. D* **68**, 056005 (2003).

[21] T. Becher and H. Leutwyler, *Baryon chiral perturbation theory in manifestly Lorentz invariant form, Eur. Phys. J. C* **9**, 643 (1999). [arXiv:hep-ph/9901384].

[22] A. I. L'vov, *A Dispersion look at the chiral perturbation theory: Nucleon electromagnetic polarizabilities, Phys. Lett. B* **304**, 29 (1993).

[23] V. Pascalutsa, B. R. Holstein and M. Vanderhaeghen, *A derivative of the Gerasimov-Drell-Hearn sum rule, Phys. Lett. B* **600**, 239 (2004). [arXiv:hep-ph/0407313].

[24] B. R. Holstein, V. Pascalutsa and M. Vanderhaeghen, *Sum rules for magnetic moments and polarizabilities in QED and chiral effective-field theory, Phys. Rev. D* **72**, 094014 (2005).

[25] R. P. Hildebrandt, H. W. Griesshammer, T. R. Hemmert and B. Pasquini, *Signatures of chiral dynamics in low energy Compton scattering off the nucleon, Eur. Phys. J. A* **20**, 293 (2004).

[26] V. Pascalutsa and D. R. Phillips, *Effective theory of the Delta(1232) in Compton scattering off the nucleon, Phys. Rev. C* **67**, 055202 (2003). [arXiv:nucl-th/0212024].

[27] V. Lensky and V. Pascalutsa, *Manifestly-covariant chiral PT calculation of nucleon Compton scattering, JETP Lett.* **89** (2009) 108 [arXiv:0803.4115 [nucl-th]].

[28] A. Metz and D. Drechsel, *Generalized polarizabilities of the nucleon studied in the linear sigma model, Z. Phys. A* **356**, 351 (1996).

[29] A. I. L'vov, *Compton Scattering On Proton At Energies Up To 400-Mev And Finite Energy Sum Rules, Sov. J. Nucl. Phys.* **34**, 597 (1981)

[30] A. I. L'vov, V. A. Petrun'kin and M. Schumacher, *Dispersion Theory Of Proton Compton Scattering In The First And Second Resonance Regions, Phys. Rev. C* **55**, 359 (1997).

[31] V. Pascalutsa and M. Vanderhaeghen, *Electromagnetic nucleon to Delta transition in chiral effective field theory, Phys. Rev. Lett.* **95**, 232001 (2005); *Chiral effective-field theory in the Delta(1232) region. I: Pion electroproduction on the nucleon, Phys. Rev. D* **73**, 034003 (2006).

[32] V. Pascalutsa, M. Vanderhaeghen and S. N. Yang, *Electromagnetic excitation of the Delta(1232) resonance, Phys. Rept.* **437**, 125 (2007). [arXiv:hep-ph/0609004].