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Effective Field Theories in Heavy Quarkonium

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We review some recent developments in heavy quarkonium physics from the point of view of effective field theories of QCD. We concentrate on potential nonrelativistic QCD. The main goal is to obtain a well founded connection between QCD and descriptions of the heavy quarkonium dynamics in terms of Schroedinger-like equations. Finally, we review a selected set of applications, which include spectroscopy, inclusive decays, and electromagnetic threshold production.

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1. Introduction

The study of the heavy quark–antiquark system is an old topic (see [1] for earlier references). Here we will concentrate on recent developments based on effective field theories (EFTs). For large enough masses, these systems can be considered to be non-relativistic (NR) and are then characterized by, at least, three widely separated scales: hard (the mass m, of the heavy quarks), soft (the relative momentum of the heavy-quark–antiquark pair in the center of mass frame $|\mathbf{p}| \sim mv$, $v \ll 1$), and ultrasoft (the typical kinetic energy $E \sim mv^2$ of the heavy quark in the bound state system). In 1986, NRQED [2], an EFT for NR leptons, was presented. NRQED is obtained from QED by integrating out the hard scale m. NRQCD [3] was born soon afterwards. NRQCD has proved to be extremely successful in studying Q-$\bar{Q}$ systems near threshold. The Lagrangian of NRQCD can be organized in powers of $1/m$, thus making explicit the NR nature of the physical systems, yet its connection with a NR quantum mechanical formulation of the problem was still obscure. For instance, in QED, in a first approximation, the dynamics of the Hydrogen atom can be described by the solution of the Schrödinger equation with a Coulomb potential. However, it is not always clear how to derive this equation from the more fundamental quantum field theory, QED, much less how to get corrections in a systematic way. A similar problem is faced in heavy quarkonium systems. One efficient solution to this problem comes from the use of effective field theories (EFTs) and in particular of potential NRQCD (pNRQCD) [4][1]. This EFT takes full advantage of the hierarchy of scales that appear in the system:

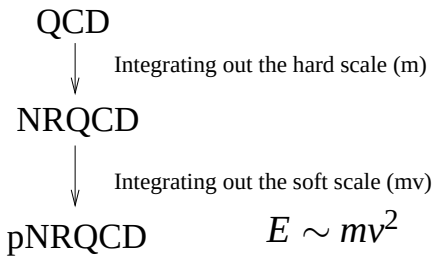
$$m \gg mv \gg mv^2 \cdots \tag{1.1}$$

and makes systematic and natural the connection of the Quantum Field Theory with the Schrödinger equation. Roughly speaking the EFT turns out to be something like:

$$\left.\begin{array}{l} \left(i\partial_0 - \dfrac{\mathbf{p}^2}{m} - V_s^{(0)}(r)\right)\Phi(\mathbf{r}) = 0 \\ + \text{ corrections to the potential} \\ + \text{interaction with other low} - \text{energy degrees of freedom} \end{array}\right\} \text{pNRQCD}$$

where $V_s^{(0)}(r) \simeq -C_f\alpha_s/r$ in the perturbative case and $\Phi(\mathbf{r})$ is the $\bar{Q}$–Q wave-function.

The key point in the construction of the EFT is to determine the kinematic situation we want to describe. This fixes the (energy of the) degrees of freedom that appear as physical states (and not only as loop fluctuations). In our case the degrees of freedom in pNRQCD are kept to have $E \sim mv^2$. In order to derive pNRQCD we sequentially integrate out the larger scales:



[1] For a comprehensive review of pNRQCD see [5].

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In order to be more specific in what follows we distinguish between the situation with $mv \gg \Lambda_{QCD}$ (weak coupling) and with $mv \simeq \Lambda_{QCD}$ (strong coupling).

2. pNRQCD AT WEAK COUPLING

In this section, we highlight the main techniques needed in order to efficiently perform high-precision perturbative computations in weakly coupled NR bound state systems. They can be summarized in four points:

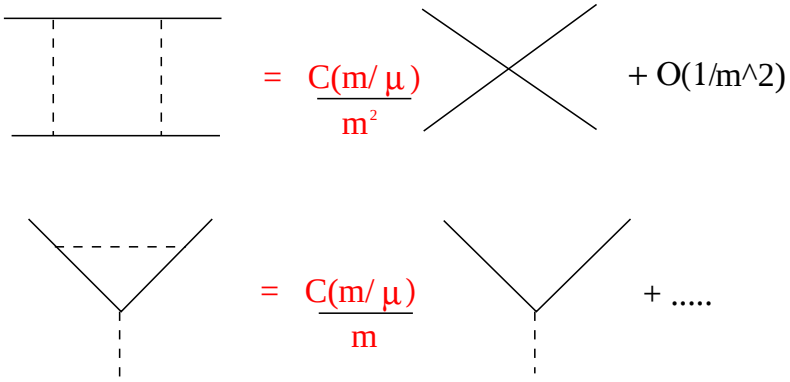
1. Matching QCD to NRQCD: Relativistic Feynman diagrams
2. Matching NRQCD to pNRQCD (getting the potential): NR (HQET-like) Feynman diagrams
3. Observable: Quantum mechanics perturbation theory
4. Observable: Ultrasoft loops

The first two points explain the techniques needed to obtain pNRQCD from QCD, whereas the last two explain the kind of computations faced in the EFT when computing observables. All the computations can be performed in dimensional regularization and only one scale appears in each type of integral, which becomes homogeneous. This is a very strong simplification of the problem. In practice this is implemented in the following way:

Point 1). One analytically expands over the three-momentum and residual energy in the integrand before the integration is made in both the full and the effective theory [6, 7].

$$\begin{aligned} \textbf{QCD} \qquad & \int d^4q f(q,m,|\mathbf{p}|,E) = \int d^4q f(q,m,0,0) + \mathcal{O}\left(\frac{E}{m},\frac{|\mathbf{p}|}{m}\right) \sim C(\frac{\mu}{m})(\text{tree level})|_{NRQCD} \\ \textbf{NRQCD} \qquad & \int d^4q f(q,|\mathbf{p}|,E) = \int d^4q f(q,0,0) = 0\,!! \end{aligned} \tag{2.1}$$

Therefore, the computation of loops in the effective theory just gives *zero* and *one matches loops in QCD with only one scale (the mass) to tree level diagrams in NRQCD*, which we schematically draw in the following figure:



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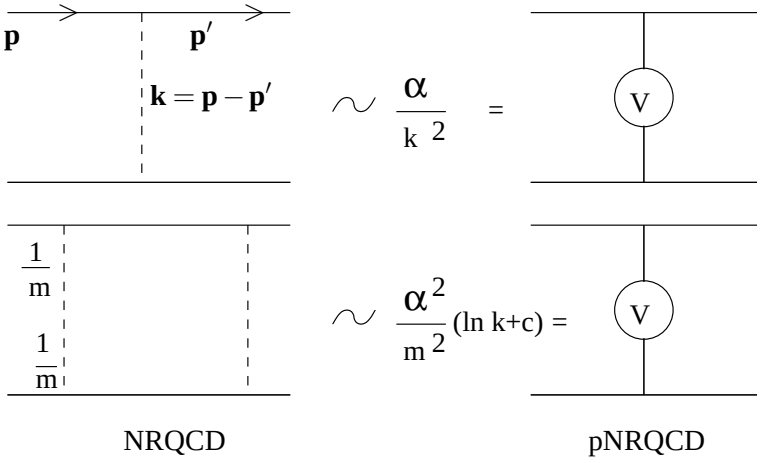
Point 2) works analogously [8]. One expands in the scales that are left in the effective theory. We integrate out the scale **k** (transfer momentum between the quark and antiquark) or its Fourier transform variable **r**. Again loops in the EFT are zero and only tree-level diagrams have to be computed in the EFT:

$$\textbf{NRQCD} \qquad \int d^4q f(q,k,|\mathbf{p}|,E) = \int d^4q f(q,k,0,0) + \mathscr{O}\left(\frac{E}{k},\frac{|\mathbf{p}|}{k}\right) \sim \delta h_s(\text{potential}) \tag{2.2}$$

$$\textbf{pNRQCD} \qquad \int d^4q f(q,|\mathbf{p}|,E) = \int d^4q f(q,0,0) = 0\,!! \tag{2.3}$$

We illustrate the matching in the figure below. Formally the one-loop diagram is equal to the QCD diagram shown above. The difference is that it has to be computed with the HQET quark propagator $(1/(q^0+i\varepsilon))$ and the vertices are also different.



Once we have obtained the potentials we have all the ingredients of the pNRQCD Lagrangian. In order to write it in a more compact form, with gauge invariance and the multipole expansion explicit, is convenient to project to the quark-antiquark sector and to express the quark-antiquark state in terms of a single bilinear field, which, by means of field redefinitions, is decomposed in S and O, two fields that transform as a singlet and octet under ultrasoft gauge transformations. Finally,

$$\mathscr{L} = S\left(i\partial^0 - h_s^{(0)} - \delta h_s\right)S + O\left(iD^0 - h_o\right)O + V_A S\mathbf{r}\cdot\mathbf{E}O + \cdots \tag{2.4}$$

where $h_s^{(0)} \simeq \frac{\mathbf{p}^2}{m} + V_s^{(0)}(r)$ and δh_s schematically represents the corrections to the potential.

Observables. Once the Lagrangian of pNRQCD has been obtained one can compute observables. A key quantity in this respect is the Green function. In order to go beyond the leading order description of the bound state one has to compute corrections to the Green Function ($H_I \sim \mathbf{x}\cdot\mathbf{E}$ schematically represents the interaction with ultrasoft gluons of the singlet and octet field):

$$G_s(E) = \frac{1}{h_s^{(0)} + \delta h_s - H_I - E} = G_s^{(0)} + \delta G_s \qquad G_s^{(0)}(E) = \frac{1}{h_s^{(0)} - E}\,.$$

These corrections can be organized as an expansion in $1/m$, α_s and the multipole expansion. Two type of integrals appear then, which correspond to points 3) and 4) above.

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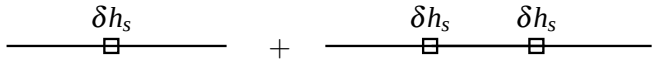
Point 3). For example, if we were interested in computing the spectrum at $O(m\alpha_s^6)$ (for QED see [9]), one should consider the iteration of subleading potentials (δh_s) in the propagator:

$\delta G_s^{pot.} =$  $+\cdots$

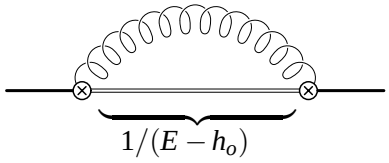
$$\sim \frac{1}{h_s^{(0)}-E}\delta h_s \frac{1}{h_s^{(0)}-E} + \frac{1}{h_s^{(0)}-E}\delta h_s \frac{1}{h_s^{(0)}-E}\delta h_s \frac{1}{h_s^{(0)}-E} + \cdots .$$

At some point, these corrections produce divergences. For example, a correction of the type: $\delta(r)G_s^{(0)}(C_f\alpha_s/r)G_s^{(0)}\delta(r)$, would produce the following divergence

$$\langle \mathbf{r}=0|\frac{1}{E-\mathbf{p}^2/m}C_f\frac{\alpha_s}{r}\frac{1}{E-\mathbf{p}^2/m}|\mathbf{r}=0\rangle \tag{2.5}$$
$$\sim \int \frac{\mathrm{d}^d p'}{(2\pi)^d}\int \frac{\mathrm{d}^d p}{(2\pi)^d}\frac{m}{\mathbf{p}'^2-mE}C_f\frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{p}')^2}\frac{m}{\mathbf{p}^2-mE} \sim -C_f\frac{m^2\alpha_s}{16\pi}\left(\frac{1}{\varepsilon}+2\ln(\frac{mE}{\mu_p})+\cdots\right).$$

Nevertheless, the existence of divergences in the effective theory is not a problem, since they get absorbed in the potentials (δh_s).

Point 4). The same happens with ultrasoft gluons, [10, 11, 12]:

$\delta G_s^{\rm us} =$ 

$\sim G_s^{(0)}(E)\int \frac{d^d\mathbf{k}}{(2\pi)^d}\mathbf{r}\frac{k}{k+h_o-E}\mathbf{r}\,G_s^{(0)}(E)$

$$\sim G_s^{(0)}(E)\,\mathbf{r}\,(h_o-E)^3\left\{\frac{1}{\varepsilon}+\gamma+\ln\frac{(h_o-E)^2}{\nu_{us}^2}+C\right\}\mathbf{r}\,G_s^{(0)}(E)\,, \tag{2.6}$$

which also produces divergences that get absorbed in h_s. Overall, we get a consistent EFT.

By obtaining the poles of the Green function one obtains the spectroscopy of the bound state. From the normalization of the Green function one can obtain inclusive electromagnetic decays, NR sum rules, and, in general, describe heavy quarkonium production near threshold. All these observables can be obtained from the vacuum polarization

$$(q_\mu q_\nu - g_{\mu\nu})\Pi(q^2) = i\int d^4x e^{iqx}\langle \mathrm{vac}|T\{J_\mu(x)J_\nu(0)\}|\mathrm{vac}\rangle\,,$$

which in the NR limit (c_1 has been computed up to $\mathcal{O}(\alpha_s^2)$ in Ref. [13] for QED and in Refs. [14, 15] for QCD, there are also some partial results at $\mathcal{O}(\alpha_s^3)$ [16])

$$J^\mu = \bar{Q}\gamma^\mu Q = c_1\psi^\dagger\sigma\chi+\cdots\,,\qquad c_1 = 1+a_1\alpha_s+a_2\alpha_s^2+\cdots\,,$$

schematically reads

$$\Pi(q^2)\sim c_1^2\langle \mathbf{r}=\mathbf{0}|G_s(E)|\mathbf{r}=\mathbf{0}\rangle$$
$$G_s(0,0,E) = \sum_{m=0}^{\infty}\frac{|\phi_{0m}(0)|^2}{E_{0m}-E+i\varepsilon-i\Gamma_t}+\frac{1}{\pi}\int_0^\infty dE'\frac{|\phi_{0E'}(0)|^2}{E_{0E'}-E+i\varepsilon-i\Gamma_t}\,.$$

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For instance, for inclusive electromagnetic decays we would have

$$\Gamma(V \to e^+e^-) \sim \frac{1}{m^2} c_1^2 |\phi(\mathbf{0})|^2 \tag{2.7}$$

$$|\phi_n(\mathbf{0})|^2 = \left|\phi_n^{(0)}(\mathbf{0})\right|^2 (1+\delta\phi_n) = \underset{E=E_n}{\mathrm{Res}} G_s(0,0,E)\,. \tag{2.8}$$

Note that $|\phi_n(\mathbf{0})|^2$ is **scheme** and **scale** dependent.

For heavy quarkonium production we would have

$$\sigma_{t-\bar{t}}(s) \sim c_1(\nu)^2 \mathrm{Im} G_s(0,0,\sqrt{s}) + \cdots \tag{2.9}$$

and for NR sum rules

$$M_n \equiv \frac{12\pi^2 e_b^2}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)\big|_{q^2=0} \simeq 48\pi e_b^2 N_c \int_{-\infty}^{\infty} \frac{dE}{(E+2m_b)^{2n+3}}\left(c_1^2 - c_1 d_1 \frac{E}{3m_b}\right) \mathrm{Im} G_s(0,0,E) \tag{2.10}$$

There is and has been an ongoing effort in obtaining the Green functions (including the potentials) and the mathing coefficients c_s with higher degree of accuracy (either at finite order or with renormalization group improvement). For the Green functions/potential one aim is obtaining expressions with NNNLO precision, for which there are some partial results [17, 18, 19, 20, 21, 22, 23, 24]. Note as well that it is possible to perform the resumation of large logarithms by using renormalization group equations in pNRQCD, see for instance [25, 26]. These results were confirmed in Ref. [27] within the vNRQCD framework.

3. pNRQCD AT STRONG COUPLING

So far we have restricted our considerations to the situation $\Lambda_{QCD} \lesssim mv^2$. It is doubtful whether we can consider most of the charmonium and bottomonium spectrum to be in this situation but rather in the (generic) non-perturbative case with $mv \sim \Lambda_{QCD}$. Then, it is not clear, a priori, what is the power counting that should be used for these systems. In particular, it is less clear how to obtain a rigorous connection between NRQCD and potential models (if it exists), although naively one would expect that, to some extent, the same philosophy as used previously to obtain pNRQCD could also be followed here. Heavy quarkonium (b-$\bar{b}$, c-$\bar{c}$) systems have been traditionally described by potential models in the past, being their inverse size assumed to be of $O(\Lambda_{QCD})$ (and that $\Lambda_{QCD} \ll m$). Potential models are characterized by the introduction of a, more or less, phenomenological potential in a Schroedinger equation. By assuming some functionality in r and by fitting the free parameters of the potential, a relatively good description of the heavy quarkonium spectrum was obtained. Nevertheless, there were two issues: 1) under which circumstances, and how, a pure Schroedinger formulation will emerge from QCD in the non-perturbative regime and, if so, 2) how to obtain the potentials from QCD, or, at least, how to relate them with objects eventually computable in QCD (then any potential model should, at least, be consistent with points 1) and 2)). The use of EFTs has helped to clarify when point 1) is satisfied and how it can be derived from QCD (see [28, 29]). The procedure is similar to the one at weak coupling:

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1. Matching QCD to NRQCD: Relativistic Feynman diagrams[2]
2. Matching NRQCD to pNRQCD (getting the potential): Potential=Wilson loops
3. Observable: Quantum mechanics perturbation theory
4. Observable: Ultrasoft loops[3]

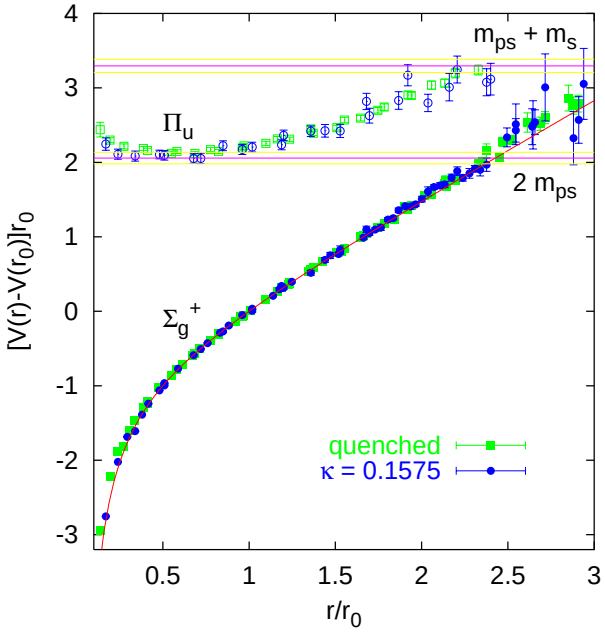

Figure 1: $r_0 \simeq 0.5$ *fm. From SESAM [30].*

Point 2. The matching scale is $\nu_{us} \ll \Lambda_{QCD}$. Therefore, coloured-like degrees of freedom decouple, since the mass gap of hybrids and glueballs is of $O(\Lambda_{QCD} \sim mv) \gg mv^2$ (see fig. 1). This means that at strong coupling the octet and (soft) gluons fields can be integrated out. Since we also assume that there are not ultrasoft gluons, our interpolating field is just **S** (we restrict ourselves to pure QCD with not light fermions) and the pNRQCD Lagrangian reads

$$\mathscr{L}_{\rm pNRQCD} = S^\dagger \left(i\partial_0 - h_s \right) S, \tag{3.1}$$

where h_s is the Hamiltonian of the singlet, i.e. of the heavy quarkonium. Schematically h_s is only a function of $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{p} = -i\nabla_{\mathbf{x}}$. It is analytic in $\mathbf{p}$ but contains non-analyticities in $\mathbf{r}$. h can be written as an expansion in $1/m$:

$$h_s = \frac{\mathbf{p}^2}{m} + V_s^{(0)} + \frac{V_s^{(1)}}{m} + \frac{V_s^{(2)}}{m^2} + \cdots. \tag{3.2}$$

Now the whole issue is to obtain the potential in terms of Wilson loops. The first attempts to answer this question started more than twenty five years ago. The expression for the leading

[2]The procedure is similar to the one of the previous section so we will not consider it further.

[3]In this case it refers to effects due to light particles (pions), which we will not consider here.

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spin-independent potential, of $O(1/m^0)$, corresponds to the static Wilson loop and was derived by Wilson and Susskind [31]:

$$V^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \ln\langle W_\Box\rangle = -C_f \frac{\alpha_s}{r} + \mathscr{O}(\alpha_s^2). \tag{3.3}$$

Some expressions for the leading spin-dependent potentials in the $1/m$ expansion, of $O(1/m^2)$, were given in Refs. [32]. The procedure followed in these works proved to be very difficult to extend beyond these leading-order potentials. In Ref. [33], a new method to calculate the potentials was proposed, where new spin-independent (some of them momentum-dependent) potentials at $O(1/m^2)$ were obtained. In [34], expressions for the spin-dependent potentials were obtained in terms of eigenstates of the static limit of the NRQCD Hamiltonian in the Coulomb gauge. In these works, the potentials did not correctly reproduce the ultraviolet behaviour expected from perturbative QCD (the hard logs $\sim \log m$). This was the first signal that a controlled derivation of the potentials from QCD was needed. The solution to this problem needs of NRQCD, where the ultraviolet behavior is encoded in the matching coefficients of the NRQCD operators. It is then possible to incorporate them to the potentials as done in [35, 36]. At that point, the obtained set of potentials at $O(1/m^2)$ seemed to be complete. Nevertheless, this view was challenged in Refs. [28, 29], where a systematic study of the potential has been done within an EFT framework: pNRQCD. The main improvements achieved in Refs. [28, 29] with respect these previous computations can be summarized as follows:

A) A general procedure to compute the potential by equating green functions in NRQCD and pNRQCD order by order in $1/m$ has been developed [28]. For illustration, within this framework, the leading order potential corresponds to Eq. (3.3) and is obtained by computing

$$\langle 0|Q_2^\dagger(x_2)\phi(x_2,x_1)Q_1(x_1)Q_1^\dagger(y_1)\phi(y_1,y_2)Q_2(y_2)|0\rangle,$$

both in NRQCD

$$\delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)\langle W_\Box\rangle,$$

and in pNRQCD

$$Z_s(\mathbf{r})\delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)e^{-iTV_s^{(0)}(\mathbf{r})}.$$

B) The general method has been developed, and formal recursive equations have been provided, to obtain the potential at any order in $1/m$ in terms of matrix elements and energies of the states solution of the static limit [28, 29]. These expressions can then be rewritten in terms of Wilson loops.

Points A) and B) solve, in alternative ways, question 2) and, thus, finally settle this issue, opened since more than twenty five years ago. Once the formalism has been developed, the *complete* potential (up to field redefinitions) in pure gluedynamics up to $O(1/m)$ in [28] and up to $O(1/m^2)$ in [29] have been obtained for the first time.

Let us stress that, to date, A) and B) are the only available methods in the literature to compute the potential in terms of Wilson loops within a *systematic* expansion in $1/m$. The attempts to implement the method of Eichten and Feinberg beyond their leading-order results were not able to obtain

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finite expressions [37]. Indeed, in a way, the procedure A) can be seen as the generalization of the Eichten and Feinberg method. In order to obtain this generalization it was crucial to understand the computation within an EFT ideology where equalities between Green functions were imposed and interpolating fields with arbitrary normalizations used. The method advocated in Ref. [33] does not appear to be correct, at least in its current formulation, since, for instance, it is not able to obtain the $1/m$ potential. The computations in Ref. [34] essentially provide the correct expressions for the spin-dependent potentials (once one takes the NRQCD matching coefficients to tree level and neglects the tree-level annihilation contribution in the equal mass case). Nevertheless, their methodology needs to be generalized (along the lines of [28, 29]) to take into account the fact that one is dealing with operators instead that with numbers in these type of computations.

Point 3. Once h_s has been obtained, we can obtain the energies of the bound states as we did in the weak coupling case by looking at the poles of the Green function. At the order of interest, one can take the energies from the real part of the Schroedinger equation

$$(\mathrm{Re}\, h_s)\, \langle \mathbf{r}|n,l,s,j\rangle = E_{njls}\, \langle \mathbf{r}|n,l,s,j\rangle, \tag{3.4}$$

with quantum numbers n, j, l and s.

From the imaginary piece of h_s, one can obtain the inclusive decay widths (to light hadrons, leptons or photons) by using the relation

$$\Gamma = -2\, \langle n,l,s,j|\mathrm{Im}\, h_s|n,l,s,j\rangle. \tag{3.5}$$

This has been done in Refs. [38, 39] at strong coupling. Actually note that Eqs. (2.8,2.9,2.10) are valid in the strong coupling limit as well, with the qualification that one has to compute the Green function with the nonperturbative potentials. For instance, a nice example between the matching of a potential model (with the right short distance structure and hence consistent with pNRQCD) and QCD within dimensional regularization can be found in Ref. [40] for the inclusive electromagnetic decay ratio of the charmonium ground state.

4. PHENOMENOLOGICAL ANALYSIS

pNRQCD should allow for the phenomenological description of the heavy quarkonium states (except, maybe, those very close to threshold). Then the first natural question is to determine **which states belong to the weak/strong coupling regime**. The cleanest place to address this question is the static potential, by checking up to which scale it can be described by a convergent perturbative series. The outcome is that, once the renormalon cancellation is achieved, the convergence of the perturbative series greatly improves and, in the cases when the comparison is possible, it agrees with lattice simulations (at least up to around 1 GeV) [41, 42, 43]. See Fig. 2 for illustration.

Spectroscopy at weak coupling

These results encourage the use of the weak coupling version of pNRQCD for spectroscopy. Its use for the $M_{\Upsilon(1S)}$ has lead to competitive determinations of the bottom mass $m_b(m_b) \sim 4.2$ with relative good convergence [45, 46, 47, 48]. See Fig. 3 for illustration.

If the bottomonium ground state can be described with the weak coupling version of pNRQCD it should also be possible to describe its pseudoscalar partner, the η_b. Nevertheless the predicted

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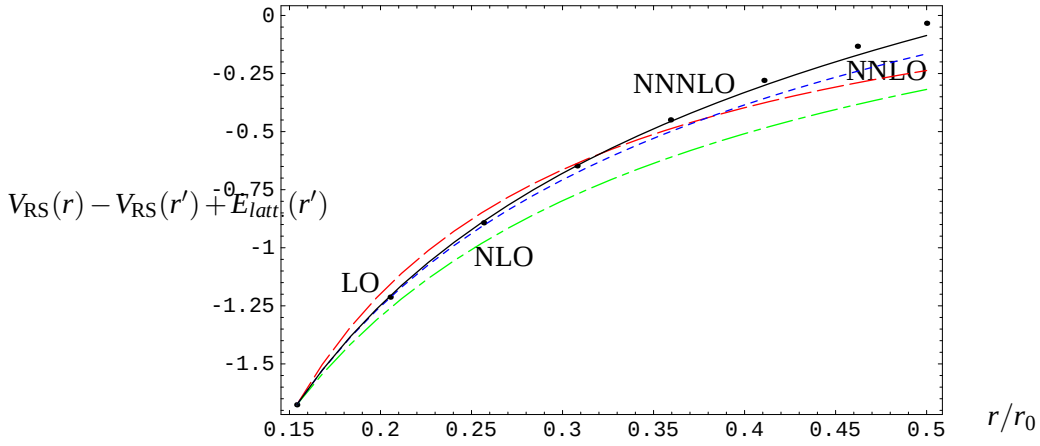

Figure 2: Static potential in the RS scheme at different orders in perturbation theory plus its comparison with lattice simulations [44] in the quenched approximation. Taken from Ref. [42].

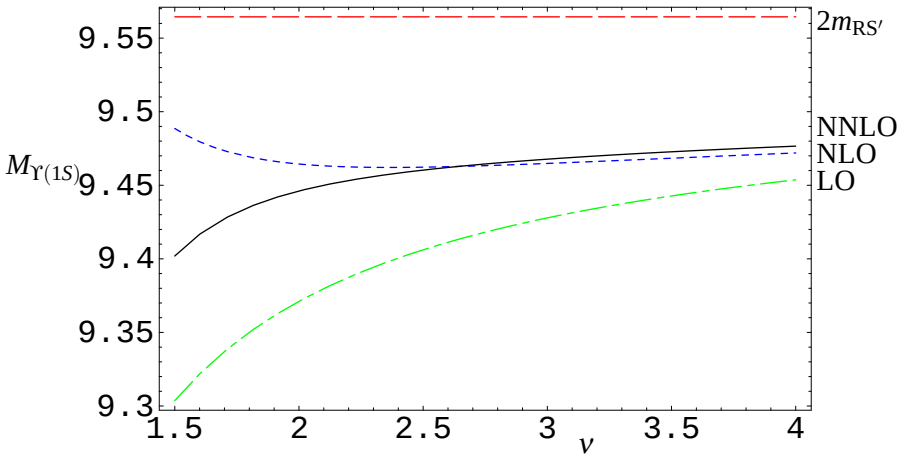

Figure 3: $M_{\Upsilon(1S)}$ at different orders in perturbation theory in the RS scheme. Taken from Ref. [46].

value ~ 40 Mev [49, 50] does not agree very well with the recent experimental determination ~ 70 MeV [51].

With respect to other quarkonium states, the $B_c(1^1S_0)$ system has been studied in Refs. [52, 53, 47] obtaining reasonable results: $M_{B_c}(1S) = 6307 \pm 17$ MeV. Actually, this figure was a prediction of the theory prior that the experimental number was obtained: $6287 \pm 4.8 \pm 1.1$ MeV [54, 55].

For higher excitations of bottomonium and charmonium the situation is not conclusive. There are different claims, whereas in Refs. [21, 56, 57] it is claimed that it is not possible to describe bottomonium higher excitations in perturbation theory, an opposite stand is taken in Refs. [53, 47, 58, 49]. At this respect we can not avoid mention that Ref. [49] produced a number for the $\eta_c(2S)$ mass before, and consistent with, the last experimental figures by Babar [59] and Cleo III [60] (before there were two excluding experimental numbers between Bell [61] and Crystal Ball [62]).

Spectroscopy at strong coupling

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The situation at strong coupling is not as developed as at weak coupling. The main reason, obviously, is that the potentials have to be computed non-perturbatively, which, nowadays means using lattice simulations. Once they have been obtained one can plug them into the Schroedinger equation and obtain the spectrum. This program has already been performed in Ref. [36]. Unfortunately, these early computations were performed in the quenched approximation, also the whole set of relativistic potentials were not included (some of them were not known at that time). Quite remarkable the $1/m$ potential was not known at that time. This has changed now by the lattice evaluation of Ref. [63], They have also provided with new simulations for some of the $1/m^2$ potentials [64], yet those simulations are still quenched. It would be interesting to try to obtain some of those potentials with dynamical fermions. At present those are only existing for the static potential. Another point is that some of those potentials are ultraviolet divergent. Therefore, they are scale and scheme dependent. This produces some errors unless the hard matching coefficients are included and computed in the very same scheme. This is not trivial, since the matching coefficients are typically computed in dimensional regularization whereas the potentials are computed in the lattice scheme. Therefore, some theoretical effort is still needed before using the full power of those nonperturbative lattice simulations.

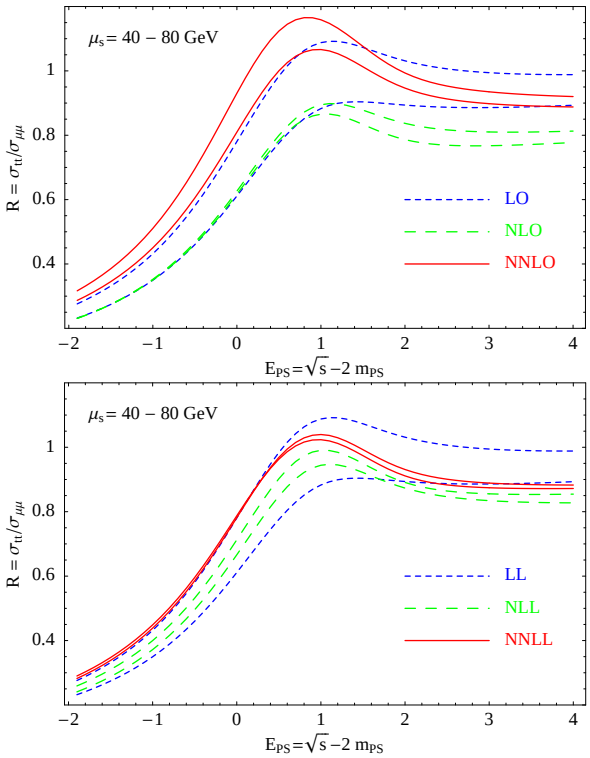

Figure 4: Threshold scan for $t\bar{t}$. The upper figure shows the fixed order results, LO, NLO and NNLO, whereas the figure belows the RGI results LL, NLL and NNLL are displayed. The soft scale is varied from μ_s=40 GeV to μ_s=80 GeV. From Ref. [65].

Coupling with hard photons

One can study the decays for the bottomonium ground state, in particular the inclusive electromagnetic ones, which are the cleanest theoretically. In this case the convergence is not very good

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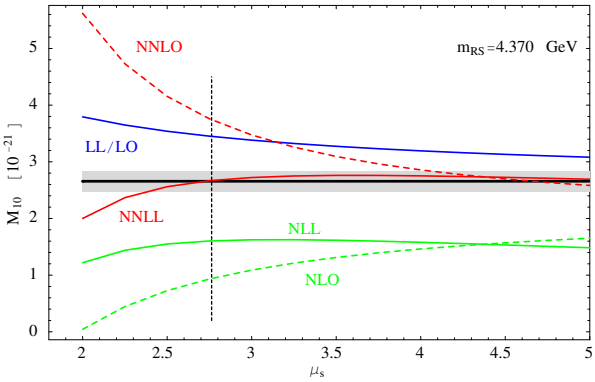

Figure 5: *The moment M_{10} as a function of μ_s at LO/LL, NLO, NLL, NNLO and NNLL for $m_{b\mathrm{RS}}$(2 GeV) = 4.370 GeV in the RS scheme. The experimental moment with its error is also shown (grey band). From Ref. [68].*

[65]. Those objects are specially sensitive to the shape of the wave function and its behavior at the origin (the hyperfine splitting is also quite sensitive to the wave function). It may well be that the present precision of finite order calculations is not enough to properly reproduce the shape of the wave function (in the same way that one has to go to high orders in perturbation theory in order to properly reproduce the static potential). This problem could be solved by performing even higher order computations or numerical analysis that include these higher order effects, preliminary computations suggest that this is indeed the case [66]. Actually, it is claimed in Ref. [21] that numerical solutions of the Schroedinger equation with the exact Coulomb potential may lead to more convergent and stable results with respect the renormalization scale variation. This indeed happens with the implementation of the renormalization group, as it has been shown in t-$\bar{t}$ production near threshold [67, 65] (see Fig. 4) or sum rules [68] (see Fig. 5). In this last case it has also lead to a more accurate determination of the bottom mass:

$$\left.\begin{array}{l} m_{b,\mathrm{PS}}(2\mathrm{GeV}) = 4.52 \pm 0.06\ \mathrm{GeV} \\ m_{b,\mathrm{RS}}(2\mathrm{GeV}) = 4.37 \pm 0.07\ \mathrm{GeV} \end{array}\right\} \rightarrow \overline{m}_b(\overline{m}_b) = 4.19 \pm 0.06\,\mathrm{GeV},$$

where the perturbative series is sign-alternating. This is the opposite than for electromagnetic decays. The convergence of the perturbative series in sum rules is also better than in electromagnetic decays. This should be compared with finite order determinations of the bottom mass from NR sum rules, which suffer from very huge theoretical uncertainties (which are not always incorporated in the errors): bad scale dependence and bad convergence of the perturbative series. Therefore, they can not provide with precise determinations of the bottom mass.

Finally, we would like to mention semi-inclusive radiative decays of the $\Upsilon(1S)$, which have been studied in Ref. [69], where relative good agreement with experiment has been obtained. See Fig. 6. Related with this work there has been a determination of α_s [71] using

$$R_\gamma \equiv \frac{\Gamma[\Upsilon(1S) \rightarrow \gamma X]}{\Gamma[\Upsilon(1S) \rightarrow X]} \rightarrow \alpha_s(M_z) = 0.120^{+0.005}_{-0.006}$$

with claimed accuracy of order $\mathcal{O}(\alpha_s, v^2)$.

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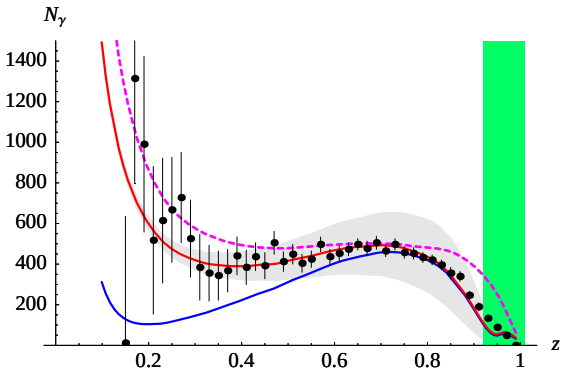

Figure 6: Photon spectrum from CLEO data. The solid lines are the NLO merging plus the fragmentation contributions: the red and blue line are obtained using different estimates for $\langle\Upsilon(1S)|O_8(^3S_1)|\Upsilon(1S)\rangle$. The grey shaded region is obtained by varying μ_c by $\sqrt{2^{\pm 1}}\mu_c$. The green shaded region shows the zone where the calculation of the shape functions is not reliable. The pink dashed line is the result from Fleming et al. [70], where only color singlet contributions were included. From Ref. [69].

5. CONCLUSIONS

We have at our disposal of an EFT from QCD that describes Heavy Quarkonium: pNRQCD. It provides with an smooth connection with potential models. The problem can be formulated in a NR quantum mechanical fashion in terms of Schroedinger equations. Every computation can be performed in dimensional regularization. We have two versions of this effective theory depending on whether the potentials can be computed within perturbation theory:

- Weak coupling regime (more predictive).
- Strong coupling regime (less predictive).

Obviously one of the major issues is to distinguish which bound states (i.e. range of energies) belong to which regime. This may provide with a much better understanding of the QCD dynamics. In any case the study of heavy quarkonium provides with good determinations of some of the parameters of the standard model. For instance:
b-$\bar{b}$ NR sum rules and/or $\Upsilon(1S)$ mass $\rightarrow$ m_b mass.
t-$\bar{t}$ production near threshold $\rightarrow$ m_t mass.
Semiinclusive radiative decays of $\Upsilon(1S)$ $\rightarrow$ $\alpha_s(M_z)$.

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