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Large N transition in the 2D SU(N)xSU(N) nonlinear sigma model.

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We consider the characteristic polynomial associated with the smoothed two point function in two dimensional large N principal chiral model. We numerically show that it undergoes a transition at a critical distance of the order of the correlation length. The transition is in the same universality class as two dimensional large N QCD.

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1. Two dimensional SU(N) X SU(N) principal chiral model

The two dimensional SU(N) X SU(N) principal chiral model is similar to four dimensional SU(N) gauge theory in many respects[1]. The continuum action is given by

$$S = \frac{N}{T}\int d^2x Tr\partial_\mu g(x)\partial_\mu g^\dagger(x) \tag{1.1}$$

where $g(x) \in$ SU(N). The global symmetry group SU(N)$_L\times$ SU(N)$_R$ reduces down to a single SU(N) "diagonal subgroup" if we make a translation breaking "gauge choice", $g(0) = 1$. This model is asymptotically free and there are $N-1$ particle states with masses

$$M_R = M\frac{\sin(\frac{R\pi}{N})}{\sin(\frac{\pi}{N})}, \quad 1 \le R \le N-1. \tag{1.2}$$

The states corresponding to the R-th mass are a multiplet transforming as an R component antisymmetric tensor of the diagonal symmetry group.

The two point function $W = g(0)g^\dagger(x)$ plays the role of Wilson loop with the separation x playing the role of area. We expect the behavior to be perturbative for small x. On the other hand, non-perturbative effects become important for large x.

One expects

$$G_R(x) = \langle\chi_R(g(0)g^\dagger(x))\rangle \sim C_R\binom{N}{R}e^{-M_R|x|} \tag{1.3}$$

where χ_R is the trace in the R-antisymmetric representation. Comparison with the heat-kernel representation of the characteristic polynomial associated with the Wilson loop operator in two dimensional large N QCD [2] suggests the following connections:

- The two point correlator, $W(d) = g(0)g^\dagger(d)$, is analogous to the Wilson loop operator.
- $M|x|$ is analogous to the dimensionless area, t.

Based on this analogy, we hypothesize [3] that the characteristic polynomial, $\det\left(z - g(0)g^\dagger(d)\right)$, will undergo a transition at some value d_c. The universal behavior at this transition will be in the same universality class as two dimensional large N QCD.

2. Setting the scale

Numerical measurement of the correlation length using the lattice action

$$S_L = -2Nb\sum_{x,\mu}\Re Tr[g(x)g^\dagger(x+\mu)] \tag{2.1}$$

and

$$\xi_G^2 = \frac{1}{4}\frac{\sum_x x^2 G_1(x)}{\sum_x G_1(x)} \tag{2.2}$$

yields the following continuum result [4]:

$$M\xi_G = 0.991(1) \tag{2.3}$$

We use ξ_G to set the scale and it is well described by

$$\xi_G = 0.991 \left[\frac{e^{\frac{2-\pi}{4}}}{16\pi}\right] \sqrt{E} \, \exp\left(\frac{\pi}{E}\right) \tag{2.4}$$

in the range $11 \le \xi_G \le 20$ with

$$E = 1 - \frac{1}{N}\Re\langle Tr[g(0)g^\dagger(\hat{1})]\rangle = \frac{1}{8b} + \frac{1}{256b^2} + \frac{0.000545}{b^3} - \frac{0.00095}{b^4} + \frac{0.00043}{b^5} \tag{2.5}$$

The above equations will be used to find a b for a given ξ.

3. Smeared SU(N) matrices

Well defined operators are obtained using smeared matrices. We start with $g(x) \equiv g_0(x)$ and one smearing step takes us from $g_t(x)$ to $g_{t+1}(x)$ using the following procedure. Define $Z_{t+1}(x)$ by:

$$Z_{t+1}(x) = \sum_{\pm\mu} [g_t^\dagger(x) g_t(x+\mu) - 1]. \tag{3.1}$$

Construct anti-hermitian traceless $SU(N)$ matrices $A_{t+1}(x)$

$$A_{t+1}(x) = Z_{t+1}(x) - Z^\dagger_{t+1}(x) - \frac{1}{N}\mathrm{Tr}(Z_{t+1}(x) - Z^\dagger_{t+1}(x)) \equiv -A^\dagger_{t+1}(x). \tag{3.2}$$

Set

$$L_{t+1}(x) = \exp[fA_{t+1}(x)]. \tag{3.3}$$

$g_{t+1}(x)$ is defined in terms of $L_{t+1}(x)$ by:

$$g_{t+1}(x) = g_t(x) L_{t+1}(x). \tag{3.4}$$

This procedure is iterated till we reach $g_n(x)$ and the smearing parameter is defined by $\tau = nf$. For a fixed ξ_G, the parameter τ is fixed such that τ/ξ_G^2 remains unchanged. We set $n = 30$ in our numerical simulations and this was found sufficiently large to eliminate a dependence on the two factors, f and n, individually.

4. Numerical details

We need $L/\xi_G > 7$ to minimize finite volume effects. We worked in the range $11 \le \xi_G \le 20$ and therefore we chose $L = 150$. We used a combination of Metropolis and over-relaxation at each site x for our updates. The full SU(N) group was explored. 200-250 passes of the whole lattices were sufficient to thermalize starting from $g(x) \equiv 1$. 50 passes per step were enough to equilibrate if ξ_G was increased in steps of 1.

The test of the universality hypothesis proceeds in the same manner as for the three dimensional large N gauge theory. We defined the characteristic polynomial, $F(y,d)$, as

$$F(y,d) = \langle \det(e^{y/2} + e^{-y/2} W(d)) \rangle \tag{4.1}$$

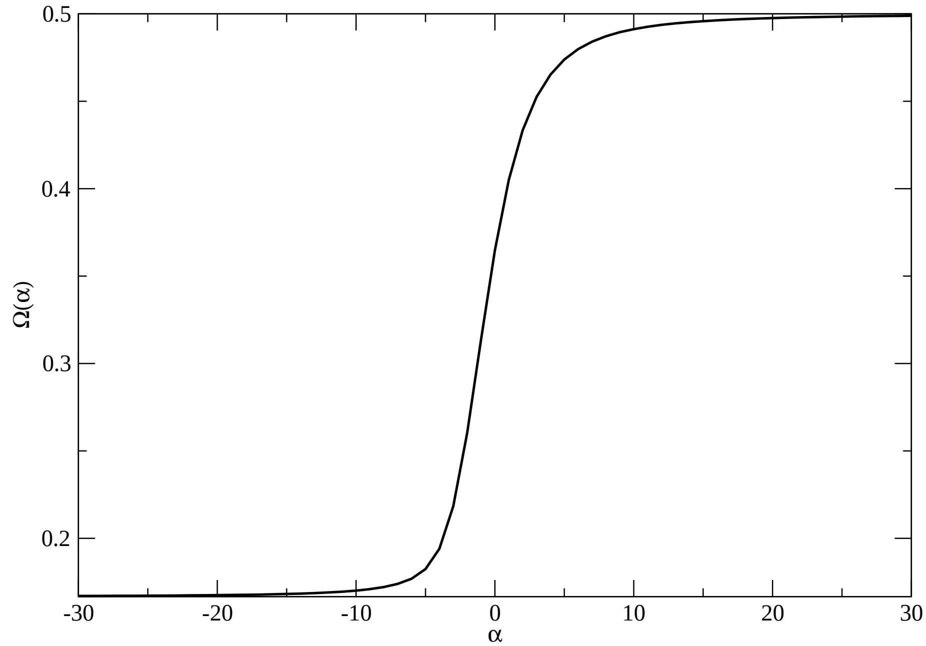

Figure 1: Behavior of Ω as a function of α in the scaling region.

We perform a Taylor expansion,

$$F(y,d,N) = C_0(d,N) + C_2(d,N)y^2 + C_4(d,N)y^4 + \ldots \tag{4.2}$$

since $F(y,d)$ is an even function of y. It is useful to define

$$\Omega(d,N) = \frac{C_0(d,N)C_4(d,N)}{C_2^2(d,N)} \tag{4.3}$$

which resembles a Binder cumulant.

As $N \to \infty$, $\Omega(d,\infty)$ is a step function with $\Omega = \frac{1}{6}$ for short distances $d < d_c$ and $\Omega = \frac{1}{2}$ for long distances, $d > d_c$. Zooming in on the step function as $N \to \infty$ in the vicinity of $d = d_c$ using the scaling variable $\alpha \propto \sqrt{N}(d - d_c)$, we obtain Fig. 1.

We use $\Omega(\alpha = 0) = 0.364739936$ to obtain the critical size d_c in the following manner. Given an N and a ξ, we find the d_c that makes the Binder cumulant $\Omega(d_c, N) = 0.364739936$ as shown in Fig. 2. We look at d_c as a function of ξ for a given N. This gives us the continuum value of d_c/ξ for that N. This extrapolation is shown in Fig. 3 for $N = 30$. We then take the large N limit as shown in Fig. 4 and it gives us

$$\frac{d_c}{\xi_G}|_{N=\infty} = 0.885(3) \tag{4.4}$$

Further substantiation of the universal behavior can be given by comparing the eigenvalues distribution in the model to the Durhuus-Olesen eigenvalue distributions in two dimensional QCD. This is shown for one example each on either side of the critical point in Fig. 5 and very close to the critical point in Fig. 6. We use $2k = t$ to match with the notation in [5].

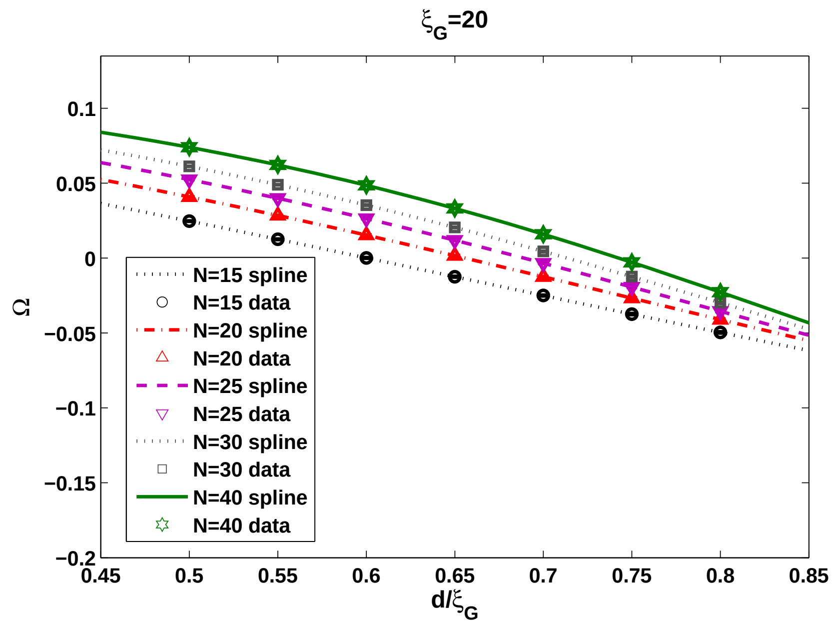

Figure 2: Plot of $\Omega(d)$ after the subtraction of $\Omega(\alpha = 0) = 0.364739936$ as a function of d/ξ_G.

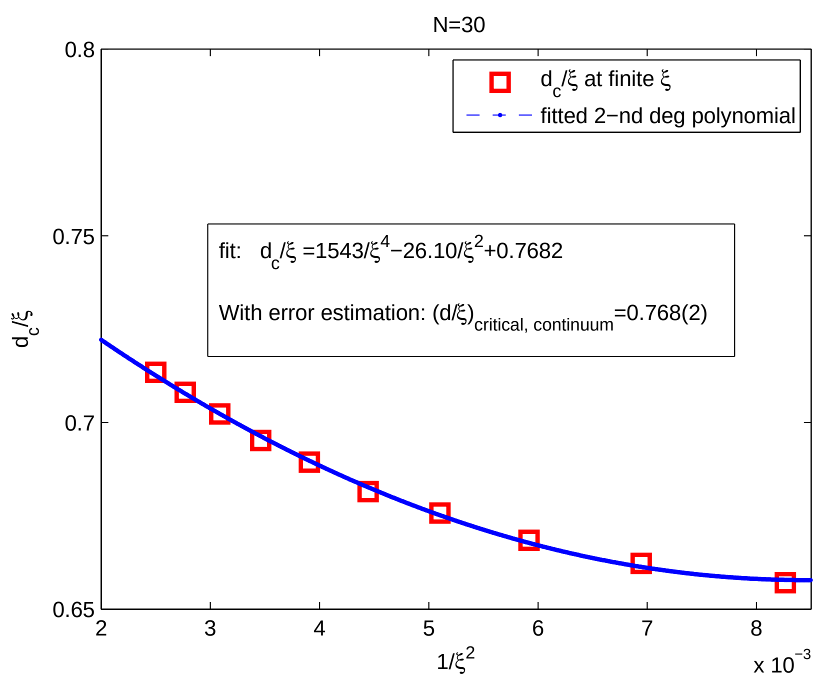

Figure 3: Extrapolation to continuum of d_c/ξ for $N = 30$.

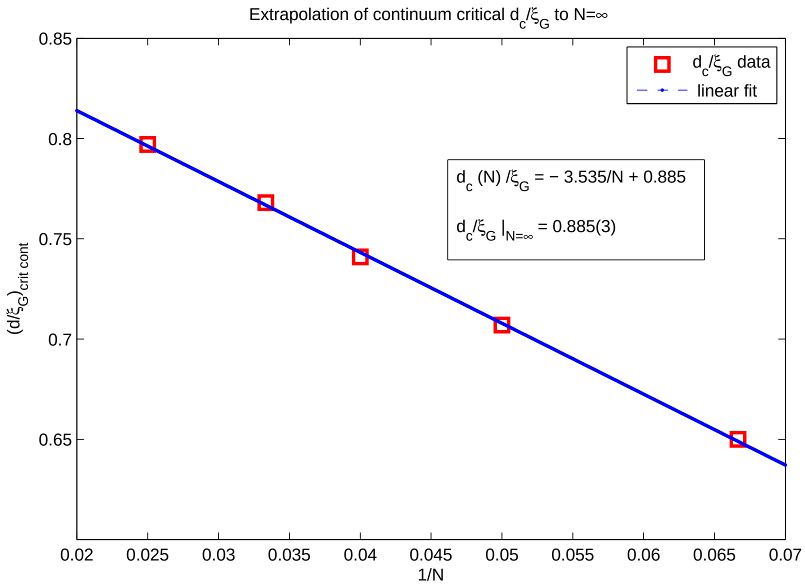

Figure 4: Extrapolation of the continuum d_c/ξ to infinite N.

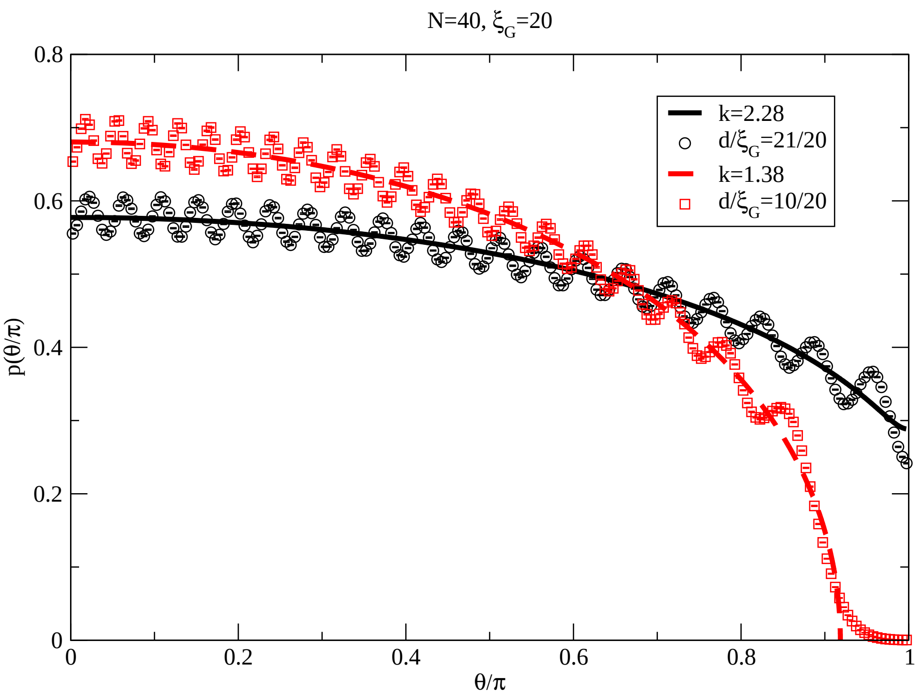

Figure 5: Examples of eigenvalue distribution for one small and one large distance.

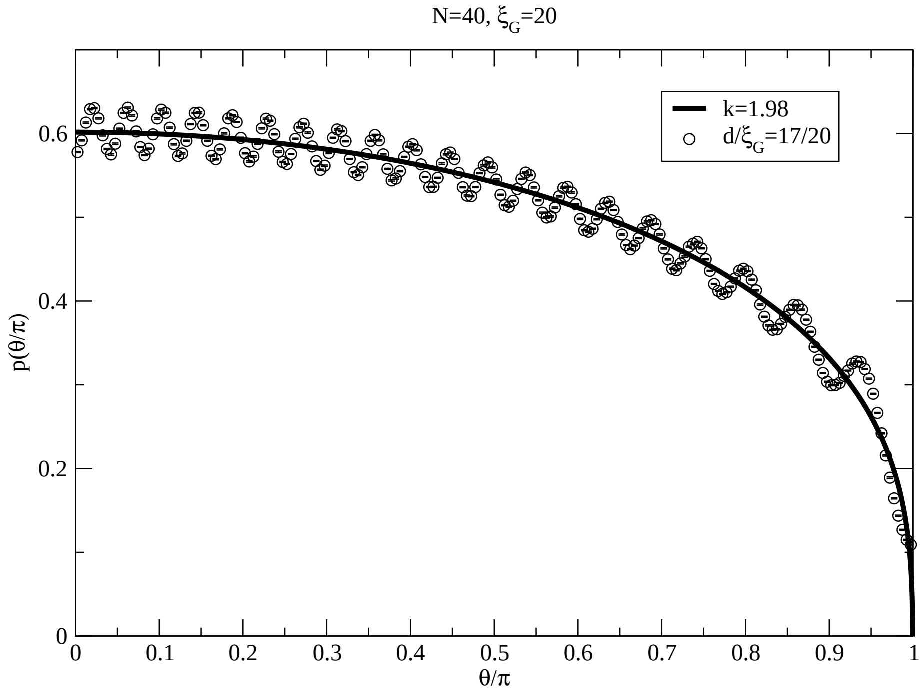

Figure 6: An example of an almost critical eigenvalue distribution.

Acknowledgments

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