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D_s physics from fine lattices

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ALPHA Collaboration

Georg von Hippel[*], **Rainer Sommer**
Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, 15738 Zeuthen, Germany
Email: `georg.von.hippel@desy.de`

Jochen Heitger
Westfälische Wilhelms-Universität Münster, Institut für Theoretische Physik,
Wilhelm-Klemm-Str. 9, 48149 Münster, Germany

Stefan Schaefer
Humboldt-Universität zu Berlin, Institut für Physik, Newtonstr. 15, 12489 Berlin, Germany

Nazario Tantalo
INFN sezione di Roma "Tor Vergata", I-00133 Roma, Italy

We present a preliminary analysis of the charm quark mass and the mass and decay constant f_{D_s} of the D_s meson obtained from dynamical simulations of $N_f = 2$ Wilson QCD on the large and fine lattices simulated by the CLS effort.

Based on CLS configurations

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[*] Speaker.

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1. Introduction

Leptonic decays of charmed mesons were not expected to be a channel where new physics might be found. However, new, precise experimental results by CLEO [1] show unexpectedly high rates in the decays $D_{\mathrm{s}} \to \tau\nu\,,\mu\nu$ compared to estimates from decay constants in the quenched approximation. The HPQCD collaboration found the effect of (rooted) dynamical staggered quarks to be significantly smaller than the difference between experiment and the quenched calculations [2][1]. Is this evidence for new physics [3], is it a statistical fluctuation or underestimate of systematics in the experiment, or is it a systematic effect unaccounted for by the errors quoted in [2]? We are aiming at a precise calculation of f_{D_s} as well as other observables such as the charm quark mass, using the $N_f = 2$ CLS configurations which reach small lattice spacings, where the charm quark mass in lattice units is really small. Here we describe first, encouraging, steps. In particular, we find small lattice spacing effects for O(a) improved Wilson quarks.

2. The CLS coordinated lattice simulations effort

Coordinated Lattice Simulations (CLS) is a community effort to bring together the human and computer resources of several teams in Europe interested in lattice QCD. CLS member teams are located at CERN, in Germany (Berlin, DESY/Zeuthen, Mainz), Italy (Rome) and Spain (Madrid, Valencia). All CLS simulations use M. Lüscher's implementation of the DD-HMC algorithm [4] to efficiently simulate $N_f = 2$ Wilson QCD with non-perturbative O(a) improvement on a variety of computer architectures ranging from PC clusters to the BlueGene/P at NIC/Forschungszentrum Jülich.

Table 1 shows the existing CLS ensembles. For this initial study of heavy quark physics on these ensembles we will use the D2, E6 and Q4 ensembles in order to get an idea of the size of the sea quark mass and lattice spacing effects.

3. Setting the scale

A final determination of the lattice scale, e.g. via m_Ω, is not yet available for the CLS ensembles. In [5] Del Debbio *et al.* determined the scale on the coarsest ($\beta = 5.3$) CLS ensembles to be $a = 0.0784(10)$ fm via a combination of m_K and m_{K^*}. We use this as our value for a on the D2 and E6 ensembles, and run it to $\beta = 5.7$ for the Q4 ensemble by means of the scale L^* defined in [6] via $\bar{g}^2(L^*) = 5.5$ in the Schrödinger functional scheme. Specifically, we use the linear fit $\log(L^*/a) = 2.3338 + 1.4025(\beta - 5.5) \pm 0.02$.

Since the uncertainty about the scale is an important source of error at $\beta > 5.3$, and the somewhat unphysical determination of the scale may be considered as a source of an unquantifiable systematic error even at $\beta = 5.3$, a more accurate determination of the scale is certainly a priority in order to make accurate predictions.

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[1] Note that a relatively new discretization for the charm quarks is used.

4. Measurements and Analysis

We use 6 time-localized $U(1)$ noise sources per configuration to measure the correlators C_{AA}, C_{AP}, C_{PA} and C_{PP}, where

$$C_{XY}(x_0) = -a^3 \sum_{\mathbf{x}} \langle X_{12}(x) Y_{21}(0) \rangle \,, \tag{4.1}$$

$P_{ij} = \bar{q}_i \gamma_5 q_j$ and $A_{ij} = \bar{q}_i \gamma_0 \gamma_5 q_j$, on 61 configurations of D2, 28 configurations of E6, and 31 configurations of Q4, performing a fully correlated error analysis using the Jackknife method in each case.

As in [7], we define the effective mass $M_{\text{eff}}(x_0)$ via

$$\frac{g(M_{\text{eff}}(x_0), x_0 - a)}{g(M_{\text{eff}}(x_0), x_0)} = \frac{C(x_0 - a)}{C(x_0)} \tag{4.2}$$

where $g(M,x) = e^{-Mx} + e^{-M(T-x)}$. Effective matrix elements are defined as e.g.

$$G_{PS,\text{eff}}(x_0) = \sqrt{\frac{C_{PP}(x_0) M_{\text{eff}}(x_0)}{g(M_{\text{eff}}(x_0), x_0)}} \,. \tag{4.3}$$

We also define the PCAC quark mass as

$$(m_s + m_c) = m(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*) C_{PA}(x_0) + c_A a \partial_0 \partial_0^* C_{PP}(x_0)}{C_{PP}(x_0)} \,, \tag{4.4}$$

which needs to be renormalized and O(a) improved as $m(\mu) = Z_A Z_P^{-1}(\mu)(1 + \frac{1}{2}(b_A - b_P)(m_q a))m$. In terms of these quantities, the (renormalized) pseudoscalar decay constant is defined as $F_{PS} = Z_A(1 + \frac{1}{2} b_A (m_q a)) \frac{m G_{PS}}{M_{PS}^2}$.

We use non-perturbative renormalization wherever possible, in particular for c_A [8], Z_A [9], Z_P [10], and $b_A - b_P$ [11]. Perturbative (one-loop) renormalization is used only for b_A, where no non-perturbative results are available. We translate the PCAC masses into the RGI masses through non-perturbative running in the Schrödinger functional scheme as in [10].

Id	Size	a [fm]	κ	MD τ	Id	Size	a [fm]	κ	MD τ
D1	48×24^3	0.08	0.13550	2575	M1	64×32^3	0.06	0.13620	4055
D2			0.13590	2565	M2			0.13630	3772
D3			0.13610	2520	M3			0.13640	2980
D4			0.13620	2505	M4			0.13650	3790
D5			0.13625	2510	M5			0.13660	2570
E1	64×32^3	0.08	0.13550	2672	P1	96×48^3	0.04	0.13620	1702
E2			0.13590	2512	P2			0.13630	started
E3			0.13605	2512	P3			0.13640	started
E4			0.13610	2497	**Q4**	128×64^3	0.04	0.13640	1450+
E5			0.13625	2656	Q5			0.13650	1120+
E6			0.13635	4960	Q6			0.136575	started

Table 1: The existing and running CLS ensembles; ensembles used in this study are shown in boldface. The molecular dynamics time τ is given in MD units after thermalisation; trajectory length is typically $\tau = 0.5$.

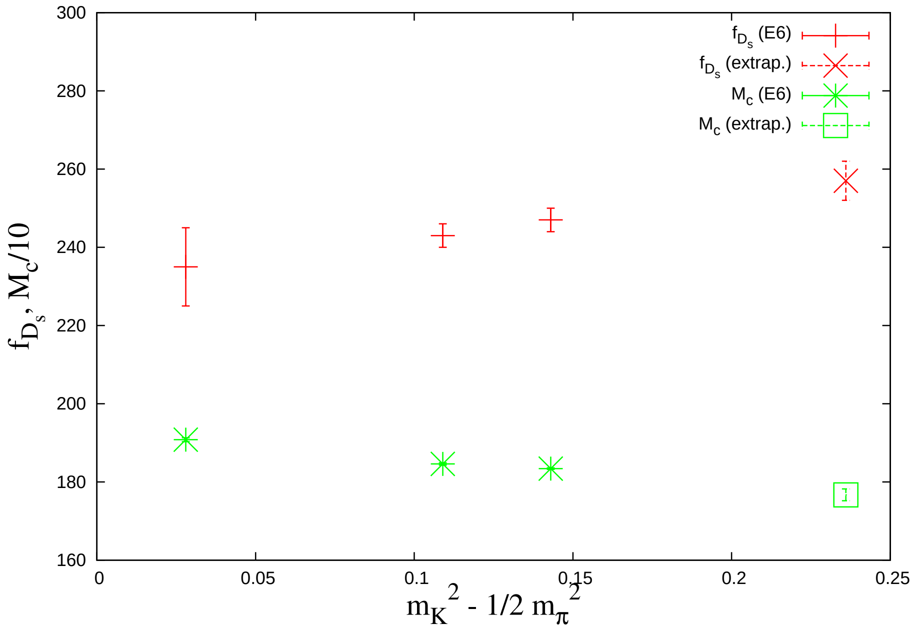

Figure 1: The quark mass M_c (divided by 10 for scale) and f_{D_s} as a function of $(m_K^2 - \frac{1}{2}m_\pi^2)$ on the E6 ensemble, together with the linearly extrapolated values at the physical point.

5. Preliminary results

5.1 D_s masses and M_c

Using two heavy quark masses on each ensemble, and two light/strange-quark masses on E6 and Q4, three light/strange quark masses on D2, and one additional light quark mass on E6, we measure the masses of all possible mass combinations of pseudoscalar mesons, as well as the corresponding PCAC quark masses. From the latter, we extract the RGI mass M_c as described in [10, 12].

To extract a physical value for M_c from these data, we first interpolate M_c linearly as a function of M_{D_s} to get M_c as a function of the light and strange quark masses. This we treat as a function of $(m_K^2 - \frac{1}{2}m_\pi^2)$, a χPT-inspired proxy of the strange quark mass, and extrapolate to the physical point $(m_K^2 - \frac{1}{2}m_\pi^2) = 0.236\ (\mathrm{GeV})^2$ as shown in fig. 1.

Our results are $M_c = 1694(3)(34)$ MeV, 1767(15)(35) MeV and 1666(1)(33) MeV on the D2, E6 and Q4 ensembles, respectively. We note that the lattice spacing dependence (the 1% difference between the D2 and Q4 ensembles) is small, but the sea quark mass dependence (the 4% difference between the E6 and D2 ensembles) is noticeable.

5.2 Lattice spacing effects in f_{D_s}

To get an estimate of what the lattice spacing effects on the decay constant of the D_s are likely to be, we define an (unphysical) reference point at $m_\pi^{\mathrm{ref}} = m_K^{\mathrm{ref}} = 618$ MeV, $m_D = 1968$ MeV, to compare results obtained at different lattice spacings. This point is realized directly on the D2 ensemble, where $L^* f_K^{\mathrm{ref}} = 0.541(16)(11)$, and $L^* f_D^{\mathrm{ref}} = 0.805(12)(16)$. On the Q4 ensemble, we

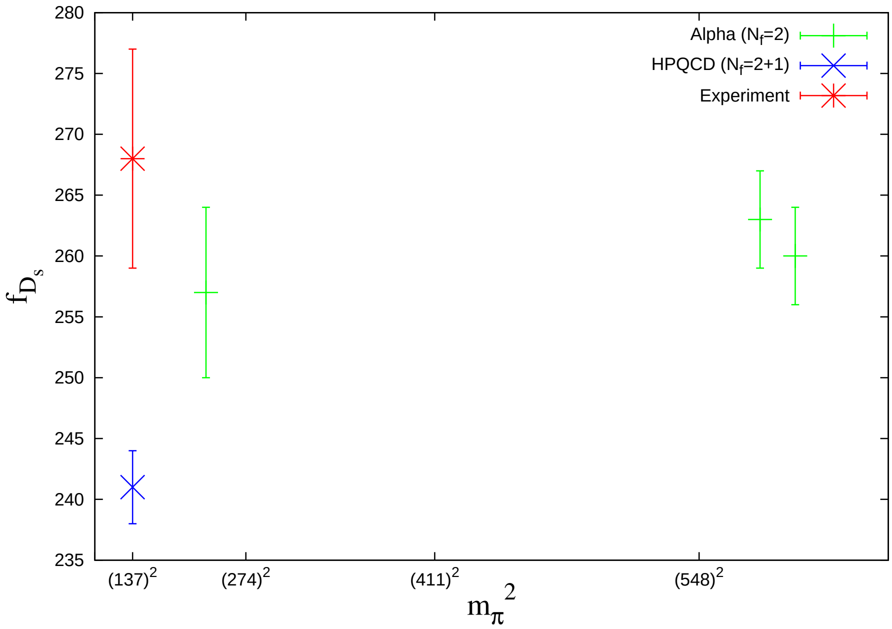

Figure 2: Summary plot showing the dependence of our results on the pion (sea quark) mass, as well as the HPQCD result [2] and the experimental value [1] for comparison.

need to interpolate in m_π to obtain $L^* f_K^{\rm ref} = 0.578(9)(12)(6)$ and $L^* f_D^{\rm ref} = 0.797(9)(16)(9)$. The errors quoted are from statistics, L^* scale setting and interpolation, respectively. We find that lattice spacing effects are about $7(5)\%$ in $f_K^{\rm ref}$, but small in $f_D^{\rm ref}$.

5.3 f_{D_s} towards the physical point

To approach the physical point, we take our lightest pion mass, which is $m_\pi = 234(10)(3)$ MeV on the E6 ensemble.

As for the quark mass, we interpolate linearly to $m_{D_s} = 1968$ MeV at fixed $(m_K^2 - \frac{1}{2}m_\pi^2)$. A plot of f_{D_s} as a function of $(m_K^2 - \frac{1}{2}m_\pi^2)$ is shown in fig. 1. Extrapolating to the physical point $(m_K^2 - \frac{1}{2}m_\pi^2) = 0.236\ (\mathrm{GeV})^2$, we obtain our preliminary estimate of $f_{D_s} = 257(3)(3)(5)(?)$, where the question mark denotes unknown systematic errors, including those coming from the overall scale ambiguity and the quenching of the strange and charm quarks.

We summarize our preliminary findings for f_{D_s} in fig. 2, which illustrates that cutoff effects are small (at least for heavy pions), and that the light-quark mass dependence is also small (at least on the coarser lattice). The chiral and continuum extrapolations therefore seem to be well possible.

6. Summary

The CLS effort is now simulating very large and fine $N_f = 2$ lattices, and lattice spacings as small as $a = 0.04$ fm have become accessible, making fully relativistic charm quarks feasible. As simulations are progressing, lighter sea quarks are also being simulated.

Our preliminary study of the D_s system indicates that cutoff effects are small and under control, but a more precise scale determination is a priority in order to eliminate an important source of systematic error.

With better statistics and more sea and valence quark masses to come, we expect to be able to perform an accurate determination of f_{D_s} in the near future.

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