

## Nucleon Structure with Domain Wall Fermions at $a = 0.084$ fm

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We present initial calculations of nucleon matrix elements of twist-two operators with 2+1 flavors of domain wall fermions at a lattice spacing  $a = 0.084$  fm for pion masses down to 300 MeV. We also compare the results with the domain wall calculations on a coarser lattice.

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## 1. Introduction

The calculation of nucleon generalized form factors has been performed recently using a mixed action that combines the computationally economical Asqtad fermion action for sea quarks with the chirally symmetric domain wall action for valence quarks [1, 2].

With the advance of algorithms [3] and computational facilities, use of the domain wall (DW) action for light sea quarks on large, fine lattices has now become possible. Hence, using gauge configurations generated by the RBC and UKQCD collaborations [4], we investigate nucleon structure in fully unitary, chirally symmetric lattice QCD. Currently, lattices with two different lattice spacings are available,  $a = 0.114$  fm and  $a = 0.084$  fm, which we will refer to as *coarse* and *fine*, respectively. The lowest pseudoscalar meson mass is  $m_\pi \approx 300$  MeV.

The next section describes the details of our calculation. In the Sect. 3, we study the pion correlation functions to determine the renormalization constant for the axial vector quark current, the pion decay constant and the pion mass, and use the pion decay constant and mass to set the scale of the fine lattices. The results for nucleon form factors are given in Sect. 4, where we focus on the isovector flavor combination  $u - d$  since it has no contributions from disconnected diagrams. We also show the  $Q^2$  dependence of the generalized form factors, although we still need to determine their overall renormalization. Conclusions follow in Sect. 5.

## 2. Calculation details

The gauge configurations were generated by the RBC and UKQCD collaborations using the Iwasaki gauge action with  $N_f = 2 + 1$  flavors of domain wall fermions, as described in Ref. [4] and references therein. The extent of the fifth dimension is  $L_s = 16$ , which is large enough to keep the residual quark mass below the bare quark mass as shown in Tab. 1. The spatial volume is  $\approx (2.7 \text{ fm})^3$  for both lattice spacings.

Before analyzing the gauge configurations, we undertook a systematic search for the optimal source parameters that provide the best overlap with the nucleon ground state. As in Ref. [1], we use a gauge-invariant Gaussian-smearred quark source that minimizes the excited state contamination to the nucleon two-point correlation function. In addition, we apply APE smearing to the gauge field used to construct the sources to reduce the large variation of the norm of the smeared sources due to the gauge field noise. The optimization of the source overlap with the nucleon is shown in Fig. 1a. With the optimized source, the plateau for the effective nucleon mass starts as early as  $t = 6$  for the fine lattice (see Fig. 1b) and  $t = 5$  for the coarse lattice. This justifies our choice of the source-sink separation  $T = 9$  and  $T = 12$  for coarse and fine lattices, respectively, which both correspond to physical separations 1.0 fm, for the calculation of the nucleon three-point correlators.

As described in Ref. [2], to increase the statistics, we use four nucleon sources separated by  $T = 16$  and calculate the forward quark propagators. The backward propagators are calculated for the sum of four nucleon sinks on each lattice. The cross-contributions between different sources and sinks average to zero due to the gauge invariance, provided there is no temporal link gauge fixing. Similarly, four antinucleon sinks are also treated analogously to obtain a total of eight measurements per lattice, which have been verified to be independent by jackknife binning [2].

For each source, we construct sinks with momenta  $\vec{P}' = (0, 0, 0)$  and  $\vec{P}' = (-1, 0, 0)$ . Since the three-point correlators quickly become noisy with growing initial and final state momenta, we have limited the source momenta to  $\vec{P}'^2 \leq 4$  for the non-zero sink momentum  $\vec{P}'$ .

**Table 1:** Dynamic DW fermion gauge configurations calculated by the RBC/UKQCD collaboration. The total number of nucleon correlator measurements, #, includes eight measurements per gauge field.

	$a$ [fm]	#	$am_l/am_h$	$am_{res} \times 10^3$	$m_\pi$ [MeV]
$24^3 \cdot 64$	0.114	3208	0.005/0.04	3.15(1)	329(5)
$32^3 \cdot 64$	0.084	1568	0.008/0.03	0.668(3)	406(7)
		4208	0.006/0.03	0.663(2)	356(6)
		2392	0.004/0.03	0.665(3)	298(5)

### 3. Pion correlation functions and the current renormalization

The operators calculated on a lattice must be renormalized in order to compare the results with other lattice studies and phenomenology. In the case of the vector quark current, the renormalization constant,  $Z_V = 1/g_V$ , is determined by the total charge measured as  $g_V = F_1(0)$ . The axial vector quark current renormalization constant  $Z_A$  can be determined from the relation between the local axial vector current  $A_0$  and the true (partially conserved) axial vector current  $\mathcal{A}_0$  associated with the axial transformation of the DW fermion integral [5]:

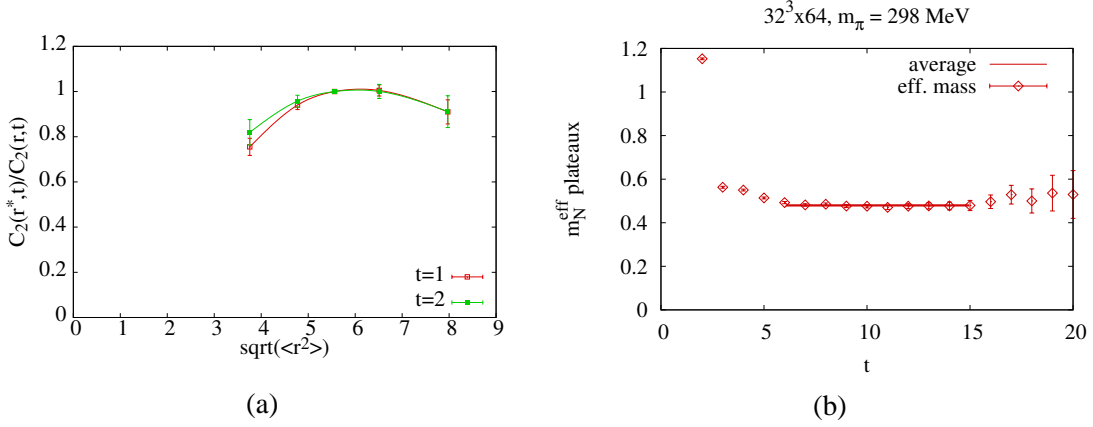
$$\langle \pi | \mathcal{A}_0 | 0 \rangle = Z_A \langle \pi | A_0 | 0 \rangle, \quad \frac{\langle \mathcal{A}_0(t) \tilde{J}_5(0) \rangle}{\langle A_0(t) \tilde{J}_5(0) \rangle} \rightarrow Z_A, \quad t \rightarrow \infty, \quad (3.1)$$

where  $\tilde{J}_5$  is the pseudoscalar density operator  $\bar{q}\gamma_5 q$ , with the tilde signifying that the quark field is smeared. Averaging the ratio (3.1) over the plateau region  $10 \leq t \leq 54$ , we extract  $Z_A$  with high precision as shown in Tab. 2. We determine the pion mass, the residual mass  $m_{res}$  and the pion decay constant from the simultaneous fit of the following correlators of local operators:

$$\begin{aligned} \langle A_0(t) \tilde{J}_5(0) \rangle &= c_{smear} A_5 \left( e^{-m_\pi t} - e^{-m_\pi(L-t)} \right), \\ \langle J_5(t) \tilde{J}_5(0) \rangle &= c_{smear} B_5 \left( e^{-m_\pi t} + e^{-m_\pi(L-t)} \right), \\ \langle J_{5q}(t) \tilde{J}_5(0) \rangle &= c_{smear} m_{res} B_5 \left( e^{-m_\pi t} + e^{-m_\pi(L-t)} \right), \end{aligned}$$

where  $J_{5q}$  is the DW mid-point contribution to the divergence of the axial vector current and  $c_{smear}$  is the factor due to the source smearing, which is evaluated separately. Constants  $A_5 = f_\pi^2 m_\pi^2 / 4Z_A(m_q + m_{res})$  and  $B_5 = f_\pi^2 m_\pi^3 / 8(m_q + m_{res})^2$  provide us with two ways to extract the pion decay constant,  $f_\pi$ . The obtained values agree within errors. Results are summarized in Tab. 2.

At the time of the talk, the lattice scale had only been set for the coarse lattice, using the the combination of the physical  $m_\pi$ ,  $m_K$  and  $\Omega^{-1}$  baryon mass, see Ref. [4]. Hence, we set the fine lattice scale by comparing the lattice values of the pion decay constant  $af_\pi$  and the nucleon mass



**Figure 1:** Source is optimized studying the overlap with the ground state (a). The effective mass plateau starts at  $t = 6$  (b).

$am_N$  on the coarse lattice to that on the fine lattices, linearly interpolated in  $(m_\pi/f_\pi)^2$  to point  $x^* = (m_\pi/f_\pi)^2|_{\text{coarse}}$ :

$$(af_\pi)^*/(af_\pi)^{\text{coarse}} = 0.7369(15), \quad (am_N)^*/(am_N)^{\text{coarse}} = 0.7530(54).$$

Since the discrepancy between these ratios is smaller than the uncertainty in  $a^{\text{coarse}}$ , we set  $a^{\text{fine}} = 0.0841(14)$ .

**Table 2:** Pseudoscalar meson quantities. The residual mass is shown in Tab. 1. The fact that  $Z_{AgV}$  is so close to unity shows the close agreement between the vector and axial current renormalization constants.

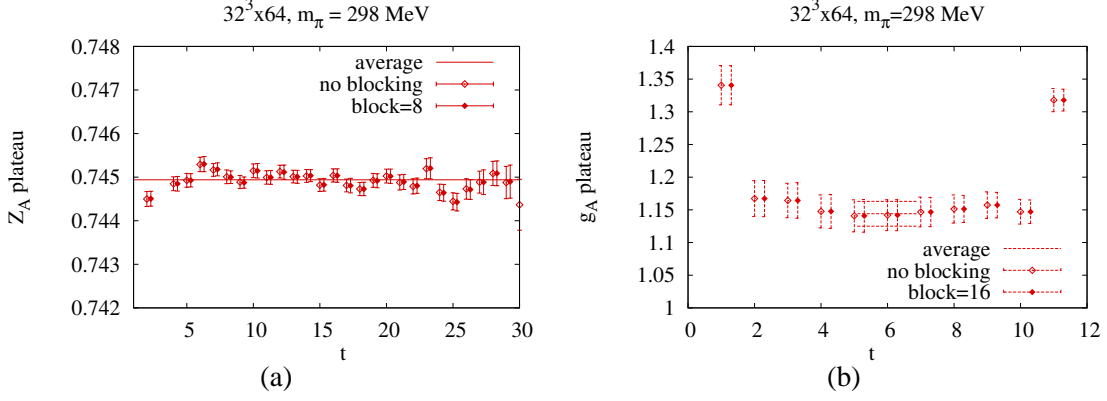
$a[\text{fm}]$	$am_l/am_h$	$am_\pi$	$af_\pi$	$Z_A$	$Z_A/Z_V$	$am_N$
0.114	0.005/0.04	0.1900(1)	0.08615(13)	0.71722(4)		
0.086	0.008/0.03	0.1729(1)	0.06707(11)	0.74530(4)	0.988(4)	0.5338(25)
	0.006/0.03	0.1516(1)	0.06460(8)	0.74523(3)	0.999(4)	0.5048(24)
	0.004/0.03	0.1269(1)	0.06229(11)	0.74494(4)	1.000(4)	0.4758(12)

#### 4. Nucleon form factors

To extract the quark current matrix elements between nucleon states, we use the standard ratio of the momentum projected correlation functions [1]:

$$R^\mathcal{O}(T, \tau; P', P) = C_{3pt}^\mathcal{O} \cdot \left( C_{2pt} \text{ combination at } T, \tau, T - \tau \right) \rightarrow \langle P' | \mathcal{O} | P \rangle, \quad \text{with } T \rightarrow \infty,$$

where the quantity in brackets represents the appropriate combination of two-point functions to cancel out normalization factors at the source and sink. We calculate the following operators,



**Figure 2:** Ratio determining the renormalization constant  $Z_A$  (a) and the plateau for the axial charge  $g_A$  (b) for  $m_\pi = 298$  MeV. Both graphs show the average and the points with/without Jackknife binning.

which we use to extract the corresponding (generalized) form factors

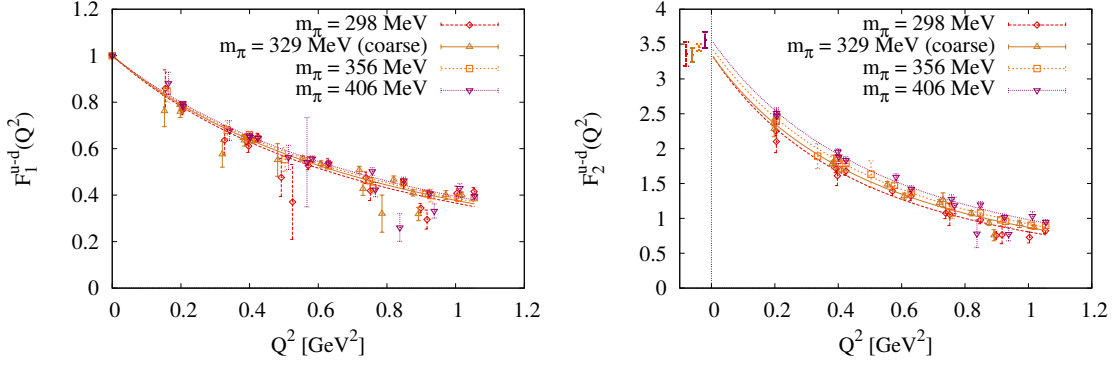
$$\begin{aligned} \langle P' | \bar{q} \gamma^\mu q | P \rangle &= \bar{U}(P') \left[ F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{\sigma^{\mu\nu} q_\nu}{2m_N} \right] U(P), \\ \langle P' | \bar{q} \gamma^\mu \gamma^5 q | P \rangle &= \bar{U}(P') \left[ G_A(Q^2) \gamma^\mu \gamma^5 + G_P(Q^2) \frac{q^\mu}{2m_N} \right] U(P), \\ \langle P' | \mathcal{O}_{[\gamma^5]}^{\mu_1 \dots \mu_n} | P \rangle &= \bar{U}(P') \left[ A_{n0}^{(\sim)}(Q^2) \gamma^{\{\mu_1} [\gamma^5] \bar{P}^{\mu_2} \dots \bar{P}^{\mu_n\}} + \dots \right] U(P), \end{aligned}$$

$$\text{where } \mathcal{O}_{[\gamma^5]}^{\mu_1 \dots \mu_n} = \bar{q} \gamma^{\{\mu_1} [\gamma^5] i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} q, \quad q^\mu = P'^\mu - P^\mu, \quad \bar{P}^\mu = \frac{1}{2} (P'^\mu + P^\mu).$$

On Fig. 3, we show the results for the isovector form factors of the the vector current  $F_1^{u-d}(Q^2)$  and  $F_2^{u-d}(Q^2)$ , where both form factors are fitted with the dipole formula. The form factors  $F_1(Q^2)$  and  $F_2(Q^2)$  are renormalized with  $g_V = F_1^{\text{bare}}(0)$ , so that  $F_1(0) \equiv 1$ .

These initial results for both form factors, using only a small fraction of the full set of planned domain wall ensembles, are already of high quality and consistent on coarse and fine lattices. As expected, decreasing the pion mass leads to a larger Dirac mean squared radius and correspondingly to a steeper form factor  $F_1(Q^2)$ . Since the intermediate pion mass is nearly halfway between the light and heavy masses, and the form factors differ only by a very small amount, expanding it to leading order one would expect, in the absence of any lattice spacing dependence, that the intermediate curve would also be halfway between the upper and lower curve. The fact that the coarse lattice result at the intermediate mass indeed lies halfway between the two fine lattice results is a clear signature that lattice artifacts associated with the lattice spacing are very small for these form factors.

The only generalized form factors whose dependency on the transferred momentum  $Q^2$  can be extracted reliably with the present statistics are the leading ones,  $A_{n0}$  and  $\tilde{A}_{n0}$ . In Fig. 4 we show the results for these form factors, normalized to unity at the zero momentum transfer  $Q^2 = 0$ . As one goes to higher moments, involving correspondingly more derivatives in the twist-two operators, the statistical errors increase as expected. However, when the statistics is eventually increased by up to an order of magnitude, these higher generalized form factors will also be well determined.



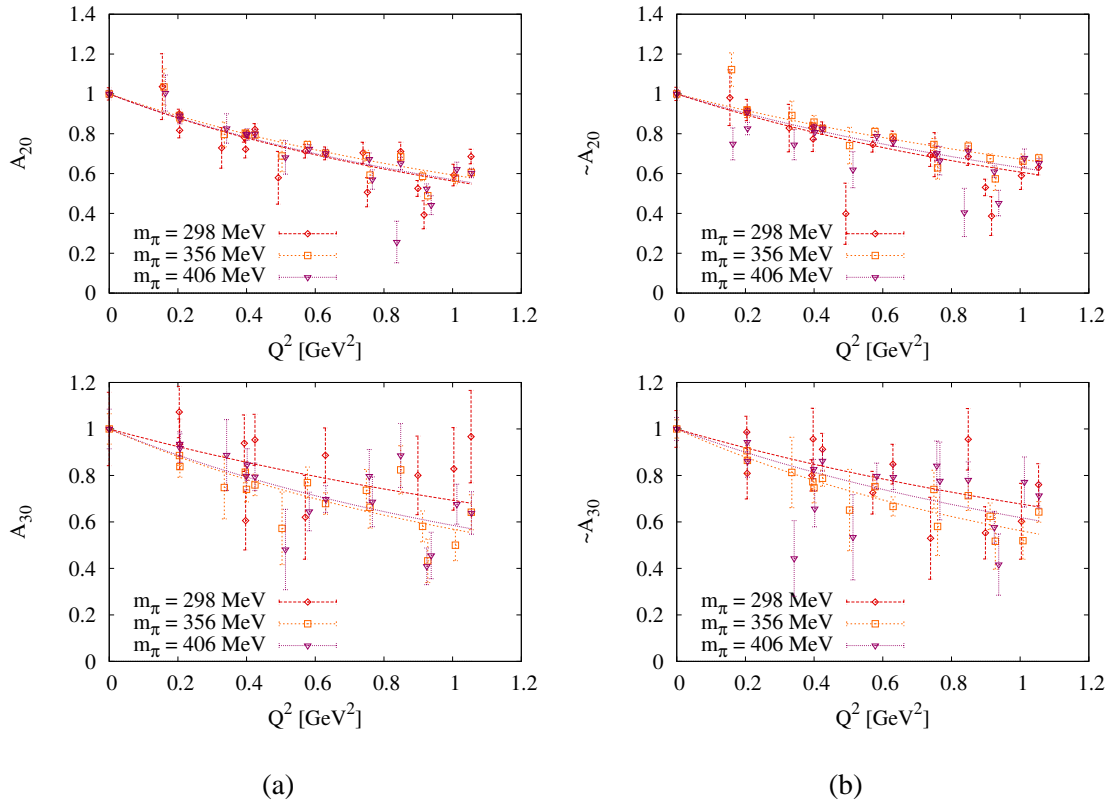
**Figure 3:** The vector form factors  $F_1^{u-d}(Q^2)$  and  $F_2^{u-d}(Q^2)$  for three different  $m_\pi$ . The curves correspond to the dipole fit in range  $0 \leq Q^2 \leq 0.4 \text{ GeV}^2$  and range  $0.2 \leq Q^2 \leq 0.8 \text{ GeV}^2$ , respectively. On the  $F_2^{u-d}(Q^2)$  plot, the errorbars to the left of  $Q^2 = 0$  line show the uncertainty in the determination of  $F_2^{u-d}(0)$  from the dipole fits.

## 5. Conclusions

By virtue of performing eight measurements per fine domain wall lattice configuration, our initial calculations on ensembles of roughly 300 to 500 configurations show a good statistical precision for nucleon generalized form factors. Comparison of results with coarse and fine lattices indicates that the errors in the form factors arising from lattice spacing artifacts are quite small for domain wall fermions. The study of other generalized form factors, such as those related to the quark angular momentum, will require calculation of renormalization constants for the corresponding operators, which is in progress.

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**Figure 4:** Helicity-even (a) and helicity-odd (b) generalized form factors on fine lattices. All the form factors are normalized to one at  $Q^2 = 0$ . Lines represent one-parameter dipole fits.

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