Electromagnetic splittings of hadrons from improved staggered quarks in full QCD

S. Basak[*],[a][†] **A. Bazavov,** [b] **C. Bernard,** [c] **C. DeTar,** [d] **W. Freeman,**[b] **Steven Gottlieb,**[a]
U.M. Heller,[e] **J.E. Hetrick,**[f] **J. Laiho,**[c] **L. Levkova,**[d] **J. Osborn,**[g] **R. Sugar,**[h] **and**
D. Toussaint[b]   **(MILC Collaboration)**

a *Department of Physics, Indiana University*
 Bloomington, IN 47405, USA
b *Physics Department, University of Arizona*
 Tucson, AZ 85721, USA
c *Physics Department, Washington University*
 St. Louis, MO 63130, USA
d *Physics Department, University of Utah*
 Salt Lake City, UT 84112, USA
e *American Physical Society*
 One Research Road, Box 9000, Ridge, NY 11961-9000, USA
f *Physics Department, University of the Pacific*
 Stockton, CA 95211, USA
g *Argonne Leadership Computing Facility, Argonne National Laboratory*
 Argonne, IL 60439, USA
h *Physics Department, University of California*
 Santa Barbara, CA 93106, USA
 E-mail: sbasak@indiana.edu

We present our initial study of the electromagnetic splittings of charged and neutral mesons, and the violation of Dashen's theorem. Hadron masses are calculated on MILC $N_f = 2+1$ QCD ensembles at lattice spacing ≈ 0.15 fm, together with quenched non-compact $U(1)$ configurations. The $\mathscr{O}(a^2)$ tadpole improved staggered quark (asqtad) action is used both for the sea quarks and for six different valence quark masses. Chiral extrapolations are performed using partially quenched chiral perturbation theory incorporating electromagnetic corrections.

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[*]Speaker.

[†]Presently at NISER, Bhubaneswar, Orissa 751005, India

1. Introduction

Lattice QCD has long been used to determine nonperturbative hadronic properties; the spectrum, in particular, has been extensively investigated. However, electromagnetic and isospin-breaking effects are usually not included in those investigations. Although the strong interaction is blind to the electromagnetic charges, we do in fact have mass splittings of mesons and baryons in the real world which depend on both isospin-breaking and electromagnetic interactions. Further, it has been pointed out that the current evaluation of the light quark masses, particularly the ratio m_u/m_d, suffers significant uncertainties coming from the electromagnetic contributions to the masses of π and especially K mesons [1].

It is therefore important to include electromagnetic effects in the lattice QCD simulations for more realistic spectrum calculations. The pioneering attempt to calculate charged and neutral pion splittings and the light quark masses from lattice QCD with electromagnetic interactions was by Duncan, Eichten and Thacker [2]. The photon fields were introduced in non-compact form and were treated in the quenched approximation. The pseudoscalar meson masses were calculated with the Wilson action and at different values of electric charge. Recently, Blum *et al.* [3] have calculated electromagnetic contributions to π and K mesons mass splittings and determined the light quark masses with domain wall quarks and $N_f = 2$ QCD configurations at the physical value of electric charge. They have also estimated the correction to the Dashen's theorem [4] at $\mathcal{O}(\alpha m_q)$, and found $\Delta_{\mathrm{em}} = 0.337(40)$ or $0.264(43)$ depending on their fitting range. Electromagnetic splittings of π and ρ mesons have also been calculated using a RG-improved gauge action and a mean-field improved clover quark action at two different volumes and three lattice spacings in Ref. [5]. Electromagnetic effects on baryons have been discussed in Refs. [6, 7]; and calculation with unquenched photon fields in Ref. [8].

In this work, we study the electromagnetic mass splittings of pseudoscalar mesons in the presence of a quenched electromagnetic field and calculate the correction to the Dashen's theorem, parametrized following the chiral perturbation theory calculation of Urech [9]. The lattice data is fitted using partially quenched chiral perturbation theory formulas at $\mathcal{O}(p^4, e^2 m_q)$ given by Bijnens *et al.* [10]. We have used a non-compact action for the photon fields following [2]. The valence quarks are asqtad [11] staggered quarks, and the configurations are the MILC $N_f = 2+1$ QCD configurations at lattice spacing 0.15 fm, with varying sea quark masses [12].

2. Lattice QCD with electromagnetic interaction

The mass differences among the members of hadron isomultiplets arise from two sources: (*a*) strong isospin-breaking contributions due to the difference in masses of the light quarks and (*b*) different electromagnetic charges of the quarks. Apart from a small isospin-breaking contribution of order $(m_d - m_u)^2$, the $\pi^{\pm} - \pi^0$ mass splitting is almost purely electromagnetic in origin; whereas it is the isospin-breaking contributions that dominate for $K^0 - K^{\pm}$. Dashen's theorem [4] summarizes the electromagnetic effects on meson masses: in the chiral limit to $\mathcal{O}(e^2)$, electromagnetic interactions modify the leading order $M_{\pi^{\pm}}^2$ and $M_{K^{\pm}}^2$ masses while the π^0 and K^0 masses remain

unaffected,

$$M_{\pi^\pm}^2 = 2\hat{m}B_0 + A^{(1)}e^2(q_u - q_d)^2, \qquad M_{K^\pm}^2 = (\hat{m} + m_s)B_0 + A^{(1)}e^2(q_u - q_s)^2$$
$$\left(M_{\pi^\pm}^2 - M_{\pi^0}^2\right)_{\text{em}} = \left(M_{K^\pm}^2 - M_{K^0}^2\right)_{\text{em}}, \tag{2.1}$$

where q_u, q_d, and q_s are the u, d, and s quark charges in units of e. At $\mathscr{O}(e^2 m)$, however, $M_{\pi^{\pm,0}}^2$ and $M_{K^{\pm,0}}^2$ can receive large and different contributions, and the correction to Dashen's theorem can parametrized as,

$$\Delta M_D^2 = \Delta M_K^2 - \Delta M_\pi^2 = \left(M_{K^\pm}^2 - M_{K^0}^2\right)_{\text{em}} - \left(M_{\pi^\pm}^2 - M_{\pi^0}^2\right)_{\text{em}}, \tag{2.2}$$

or as Δ_E, defined by $\Delta M_K^2 = (1 + \Delta_E)\Delta M_\pi^2$. The partially quenched chiral perturbation theory relevant for QCD + QED with 2+1 dynamical flavors at NLO has recently been worked out by Bijnens *et al.* [10]; it can be used to perform fits to the lattice $M_{\pi,K}^2$ data in order to determine the electromagnetic low energy constants. The pure electromagnetic correction, relevant for calculating the correction to Dashen's theorem, can be computed from the expression,

$$\Delta M^2|_{\text{em}} = M_{\text{ps}}^2(\chi_1, \chi_3, q_1, q_3) - M_{\text{ps}}^2(\chi_1, \chi_3, q_3, q_3)$$
$$- M_{\text{ps}}^2(\chi_1, \chi_1, q_1, q_3) + M_{\text{ps}}^2(\chi_1, \chi_1, q_3, q_3). \tag{2.3}$$

In this notation [10], the normalized quark mass is $\chi_i = 2B_0 m_i$, where B_0 is related to the quark-antiquark vacuum expectation value in the chiral limit, and $M_{\text{ps}}^2(\chi_1, \chi_3, q_1, q_3)$ denotes the squared pseudoscalar meson mass having valence quark masses χ_1, χ_3 and valence charges q_1, q_3 in units of electron charge e. In the isospin limit $m_u = m_d$, when $\chi_1 = m_u$, $\chi_3 = m_s$ and the quark charges $q_1 = q_u$, $q_3 = q_s$, we have $\Delta M_D^2 = \Delta M^2|_{\text{em}}$. We fit the lattice data for ΔM_D^2 according to

$$\Delta M_D^2 = \mathscr{A}_1(\chi_{13} - \chi_{11}) + \mathscr{A}_2 \left[\chi_{13}\log\left(\frac{\chi_{13}}{\mu^2}\right) - \chi_{11}\log\left(\frac{\chi_{11}}{\mu^2}\right)\right]$$
$$+ \mathscr{A}_3 \left[\chi_{1f}\log\left(\frac{\chi_{1f}}{\mu^2}\right) - \chi_{3f}\log\left(\frac{\chi_{3f}}{\mu^2}\right)\right]$$
$$+ \mathscr{A}_4 \left[\chi_{13}\left\{1 - \frac{1}{2}\log\left(\frac{\chi_{13}}{\mu^2}\right)\right\} - \chi_{11}\left\{1 - \frac{1}{2}\log\left(\frac{\chi_{11}}{\mu^2}\right)\right\}\right], \tag{2.4}$$

where, $\chi_{ij} \equiv (\chi_i + \chi_j)/2$ and $\mathscr{A}_i$ are the constants to be determined from the fit. For the scale μ we have used 1000MeV. The sea-quark dependence of the log-terms appears through χ_{if}, where the sea-quark index f is summed over all the sea quarks. Terms with the sea-quark charges contributing to ΔM_D^2 do not involve unknown low energy constants (LECs) at this order and hence are computable without lattice simulation [10]. In our expression we have included the parameters $\mathscr{A}_{2,3,4}$, which are not present in Ref. [10], in front of the logarithm terms. This is because we expect the continuum expression to be modified by discretization effects on the quite coarse lattices we are using. Finite volume effects, due to the masslessness of the photon, may also be rather significant. At this stage, therefore, we can obtain at best a rough estimate of the quantity ΔM_D^2; a precise determination will require lattice data at finer lattice spacings and larger volumes.

3. Numerical results

Phenomenologically relevant quantities, such as the mass-squared differences of charged and neutral mesons and the correction to Dashen's theorem, can be calculated from partially quenched lattice QCD where photons are treated in the quenched approximation. Therefore, we can use the existing $SU(3)$ configurations generated without dynamical photons. For unquenched QCD gluon configurations, we have used four ensembles of existing $a \approx 0.15$ fm MILC lattices with 2+1 dynamical flavors, the details of which are provided in Table 1. The lattice spacing is set by the static quark potential [12].

Size	$\beta = 10/g^2$	$a\hat{m}'/am'_s$	a (fm)	# Cfgs
$20^3 \times 48$	6.566	0.00484/0.0484	≈ 0.15	200
$16^3 \times 48$	6.572	0.0097/0.0484	≈ 0.15	400
$16^3 \times 48$	6.586	0.0194/0.0484	≈ 0.15	400
$16^3 \times 48$	6.600	0.0290/0.0484	≈ 0.15	400

Table 1: $N_f = 2 + 1$ MILC lattices used in this project.

The quenched photon configurations $\{A_\mu(n)\}$ are generated from the non-compact $U(1)$ action,

$$S_{\text{em}} = \frac{1}{4} \sum_{n\mu\nu} \left(\partial_\mu^r A_\nu(n) - \partial_\nu^r A_\mu(n) \right)^2, \tag{3.1}$$

subjected to Coulomb gauge fixing $\partial_i^l A_i(n) = 0$. Additional global gauge fixing is done to ensure that Gauss's law is satisfied. In momentum space, the action is Gaussian distributed, and the scalar and vector potentials $[A_0(p), \vec{A}(p)]$ can be generated independently of each other from Gaussian distributed random numbers. The coordinate-space Coulomb-gauge photon configurations are recovered by FFT. The necessary valence quark propagators are calculated with wall sources in these $SU(3) \times U(1)$ background fields. We calculate the pseudoscalar meson propagator with nine different valence quark masses, $0.1m'_s \leq m_q \leq 1.0m'_s$. All the fits we report here are obtained by correlated χ^2-fit; the errors are obtained from a jackknife analysis. The meson masses are extracted from the exponential fall-off of the meson propagators in the time range $9 - 24$, taking into account the correlations among the time slices. We have ignored the contribution from the disconnected diagram that effects the neutral pion mass.

We first try to estimate the $\mathcal{O}(e^2)$ and $\mathcal{O}(e^2m)$ contribution to pseudoscalar masses at three different (physical and non-physical) values of electric charge,

$$M_\pi^2(e \neq 0) - M_\pi^2(e = 0) = \mathscr{A}_0 e^2 (q_u - q_d)^2 + \mathcal{O}(e^2m). \tag{3.2}$$

In Fig. 1, we plot the dependence of π mass-squared splittings corresponding to Eq. (3.2). A straight line fit describes the $\mathcal{O}(e^2)$ behavior fairly well over the full range of electric charges that are examined. In a further test to ascertain the $\mathcal{O}(e^2m)$ or higher order effects, in Fig. 2, we plot the variation of the same pion mass-squared splittings against quark masses. The slope of each of line, corresponding to different electric charges, nicely matches to its e^2 value.
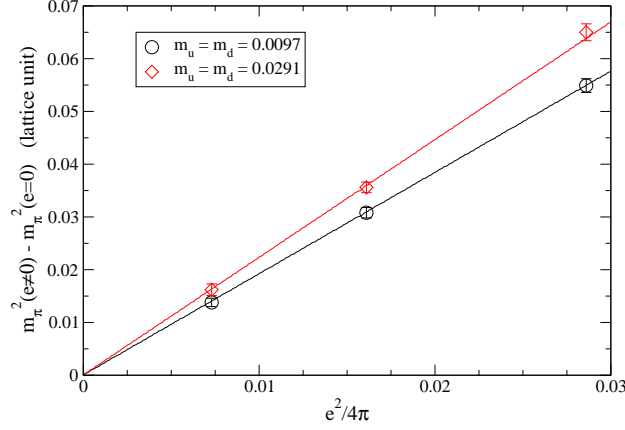
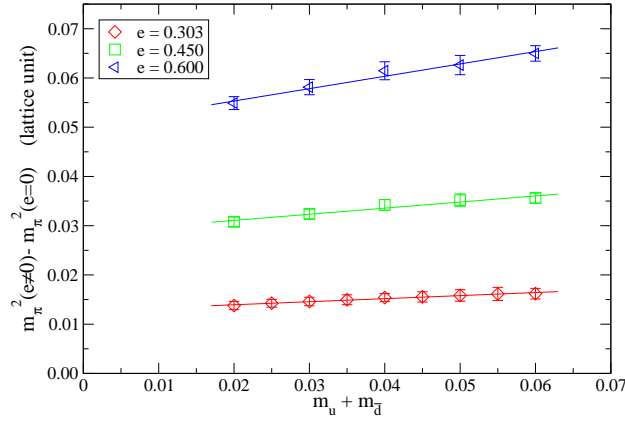**Figure 1:** The e^2 dependence of mass-squared electromagnetic splittings of the pion.

Figure 2: The $\mathcal{O}(e^2 m)$ behavior of mass-squared electromagnetic splittings of the pion.

In a series of four plots in Fig. 3, we show the meson mass splittings between mesons computed with physical electric charge $e^2 = 4\pi\alpha_{\rm em}$ and with $e^2 = 0$. The labels $q_1\bar{q}_2$ in the plots denote quark charges, *i.e.*, $u\bar{d}$ indicates quarks with charge $q_u = 2e/3$ and $q_d = -1e/3$. The points using the same symbol and color are obtained by varying the q_2 quark mass. The lines are merely to guide the eye. We have not yet fit the squared meson masses to the Bijnens form based on partially quenched chiral perturbation theory (and hence not yet extracted the electromagnetic LECs) [10]. These LECs will be important in determining the physical isomultiplet splittings in mesons and therefore in calculating quark masses.

The electromagnetic LECs can be used to obtain the correction to Dashen's theorem, but we have estimated this correction directly by chiral extrapolation of Eq. (2.3) using Eq. (2.4). We put $q_1 = q_u, q_3 = q_s$ in Eq. (2.3) and extrapolate/interpolate to $\chi_1 = 2B_0 m_u$, $\chi_3 = 2B_0 m_s$. Figure 4 is our preliminary result for the violation of Dashen's theorem, where we have plotted the ΔM_D^2 for two ensembles, $\beta = 6.566$ and 6.572, as a function of M_π^2, obtained with $e^2 = 0$, in physical units. The data from the remaining two QCD ensembles at heavier quark mass are still being analyzed. We have determined the coefficients $\mathscr{A}_i$ in Eq. (2.4) using our lattice data for pseudoscalar masses
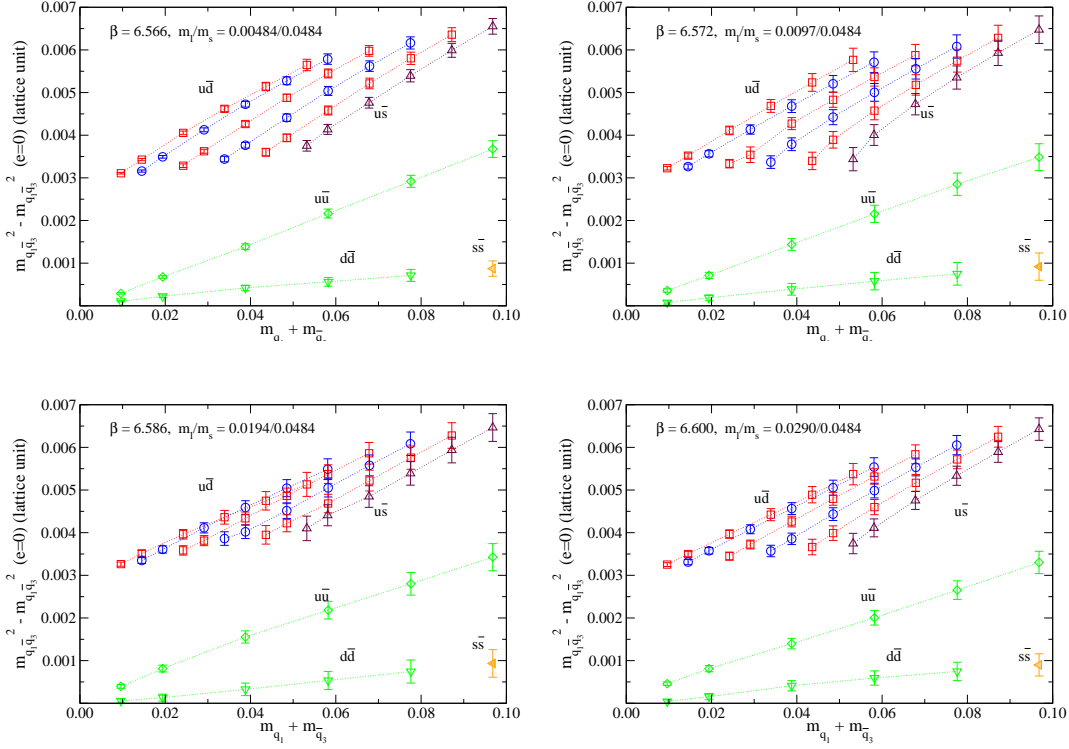
Figure 3: The mass-squared splittings between pseudoscalar mesons with physical value of electric charge $e^2 = 4\pi\alpha_{\mathrm{em}}$ and $e^2 = 0$. Each panel corresponds to different β and light sea-quark mass am'_l for fixed sea-strange quark mass $am'_s = 0.0484$. The labels $q_1\bar{q}_2$ denote the quark charges. We write $\bar{s}$ instead of $\bar{d}$ whenever m_{q_2} is equal to m'_s, the strange sea-quark mass in the simulation.

χ_{ij}, after which chiral extrapolation is performed at the experimental masses for π and K. Our preliminary result for the deviation from Dashen's theorem is $7.0 \times 10^{-4} < \Delta M_D^2/(\mathrm{GeV}^2) < 1.8 \times 10^{-3}$. This may be compared with the estimate in Ref [10]: 1.07×10^{-3}. Although we find that our value is consistent with the phenomenological estimate, the error in our result is large and does not yet include the systematic errors due to discretization and finite volume effects. We hope to reduce the error by increasing the statistics and extending the calculation to finer lattices and larger volumes. Fitting the meson splittings in Fig. 3 to determine the electromagnetic LECs directly may also provide better control over our errors.

4. Conclusions and outlook

In this work we have calculated the electromagnetic mass splittings of pseudoscalar masses with $\mathcal{O}(a^2)$ improved staggered fermions on $a \approx 0.015\,\mathrm{fm}$ $N_f = 2+1$ MILC $SU(3)$ ensembles and Coulomb-gauge $U(1)$ configurations. We have determined the correction to Dashen's theorem at the physical point (in the isospin limit), but with relatively large errors, *i.e.*, a wide range for ΔM_D^2. Our determination of meson and baryon isomultiplet splittings and quark masses is in progress. Our immediate goal is using finer lattices to reduce the error in the correction to Dashen's theorem enough to bring down the present error in m_u/m_d.
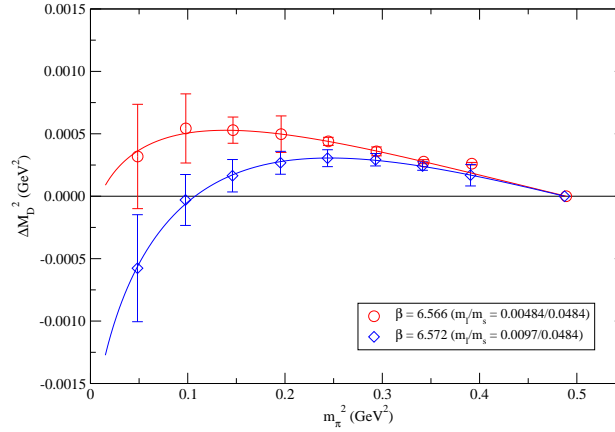
Figure 4: Correction to Dashen's theorem, the difference of electromagnetic contributions between *K* and π masses, as a function of the LO π mass squared (equivalent to the π mass squared with $e^2 = 0$).

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