

## Partially quenched study of strange baryon with $N_f = 2$ twisted mass fermions

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We present results on the mass of the baryon octet and decuplet using two flavors of light dynamical twisted mass fermions. The strange quark mass is fixed to its physical value from the kaon sector in a partially quenched set up. Calculations are performed for light quark masses corresponding to a pion mass in the range 270-500 MeV and lattice sizes of 2.1 fm and 2.7 fm. We check for cut-off effects and isospin breaking by evaluating the baryon masses at two different lattice spacings. We carry out a chiral extrapolation for the octet baryons and discuss results for the  $\Omega$ .

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## 1. Introduction

Twisted mass fermions provide a promising formulation of lattice QCD that allows for automatic  $\mathcal{O}(a)$  improvement, infrared regularization of small eigenvalues and fast dynamical simulations [1]. This work is an extension of the study on the nucleon and  $\Delta$  masses [2] to the strange baryon sector. It uses two degenerate dynamical twisted mass fermions ( $N_F = 2$ ) and a strange quark in the partially quenched approximation. The octet and decuplet baryon masses are computed at several pion masses.

We use the tree-level Symanzik improved gauge action and work at maximal twist to realize  $\mathcal{O}(a)$ -improvement. The fermionic action for two degenerate flavors of quarks in twisted mass QCD is given by

$$S_F = a^4 \sum_x \bar{\psi}(x) (D_W[U] + m_0 + i\mu\gamma_5\tau^3) \psi(x) \quad (1.1)$$

with  $D_W[U]$  the massless Wilson Dirac operator. The parameter  $m_0$  is adjusted such that  $\mu$  represents the bare quark mass [3]. The twisted mass term in the fermion action of Eq. (1.1) breaks isospin symmetry since the u- and d- quarks differ by having opposite signs for the  $\mu$ -term. This isospin breaking is a cutoff effect of  $\mathcal{O}(a^2)$ .

In all the results presented here, the lattice spacing  $a$  as well as the strange quark mass has been fixed in the meson sector ( $f_\pi$  [4] and  $m_K$  [5]). Our work will confirm the consistency between meson and baryon description in the partially quenched approximation. It is important to note that no new parameter has been tuned in this study.

## 2. Lattice techniques

The input parameters of the calculation ( $L$ ,  $\beta$  and  $\mu$ ) are collected in Table 1. The corresponding lattice spacing  $a$  and the pion mass values are taken from [4]. They span a pion mass range from 270 to 500 MeV. At  $m_\pi \approx 300$  MeV we have simulations for lattices of spatial size,  $L_s = 2.1$  fm and  $L_s = 2.7$  fm at  $\beta = 3.9$  allowing to investigate finite size effects. We provide a preliminary check of finite lattice spacing effects by comparing results at  $\beta = 3.9$  and  $\beta = 4.05$ . The masses of the octet and decuplet are extracted from two-point correlators using the standard interpolating fields (see e.g. [6]) and errors are estimated with the jackknife method.

Local interpolating fields are not optimal for suppressing excited state contributions. We instead apply Gaussian smearing to each quark field,  $q(\mathbf{x}, t)$ :  $q^{\text{smear}}(\mathbf{x}, t) = \sum_{\mathbf{y}} F(\mathbf{x}, \mathbf{y}; U(t)) q(\mathbf{y}, t)$  using the gauge invariant smearing function

$$F(\mathbf{x}, \mathbf{y}; U(t)) = (1 + \alpha H)^n(\mathbf{x}, \mathbf{y}; U(t)), \quad (2.1)$$

constructed from the hopping matrix,  $H(\mathbf{x}, \mathbf{y}; U(t)) = \sum_{i=1}^3 \left( U_i(\mathbf{x}, t) \delta_{\mathbf{x}, \mathbf{y}-i} + U_i^\dagger(\mathbf{x}-i, t) \delta_{\mathbf{x}, \mathbf{y}+i} \right)$ . Furthermore we apply APE smearing to the spatial links that enter the hopping matrix. The parameters for the Gaussian and APE smearing are the same as those used in our previous work devoted to the nucleon [2].

For a partially quenched strange quark we use a Osterwalder-Seiller fermion, defined by the action :

$$S_{OS} = a^4 \sum_x \bar{s}(x) (D_W[U] + i\mu_s \gamma_5) s(x) \quad (2.2)$$

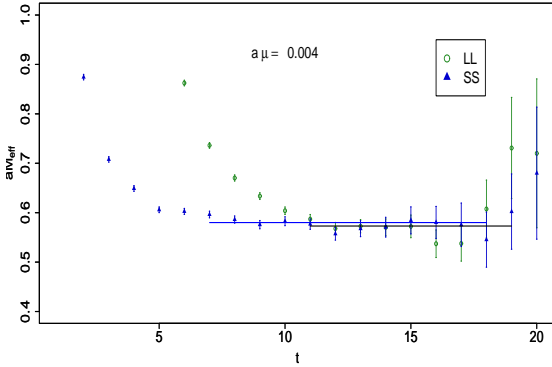
The bare mass  $\mu_s$  is tuned using the mass of the kaon at the physical point [5].

$\beta = 3.9, a = 0.0855(6)$ fm from $f_\pi$ [4]						
$24^3 \times 48, L_s = 2.1$ fm	$a\mu$	0.0030	0.0040	0.0064	0.0085	0.010
	Stat.	-	795	547	348	477
	$m_\pi$ (GeV)	-	0.3131(16)	0.3903(9)	0.4470(12)	0.4839(12)
$32^3 \times 64, L_s = 2.7$ fm	$a\mu$	0.003	0.004			
	Stat.	133	101			
	$m_\pi$ (GeV)	0.2696(9)	0.3082(6)			
$\beta = 4.05, a = 0.0666(6)$ fm from $f_\pi$ [4]						
$32^3 \times 64, L_s = 2.1$ fm	$a\mu$	0.0030	0.0060	0.0080	0.012	
	Stat.	138	126	113	182	
	$m_\pi$ (GeV)	0.3070(18)	0.4236(18)	0.4884(15)	0.6881(18)	

**Table 1:** The parameters of our calculation.

### 3. Octet baryon masses

We illustrate the quality of the plateaus that we obtain for the case of  $\Lambda$  in Fig. 1, where we compare the effective masses computed using local source-local sink (LL) and smeared source-smeared sink (SS) correlators.



**Figure 1:** Plateaus for  $\Lambda$  for  $\beta = 3.9$  ( $a \approx 0.085$  fm) on a  $24^3 \times 48$  lattice for local-local (LL) and smeared-smeared (SS) correlators at a pion mass of 313 MeV.

#### 3.1 Chiral extrapolation

As we will demonstrate in the next section, lattice artifacts are small. This allows to perform the chiral extrapolation of our data to the physical point at fixed lattice spacing. For the current

discussion we do not distinguish between the different isospin components of  $\Sigma$  and  $\Xi$  and present results averaging over the corresponding correlators. Unless we mention otherwise we use the notation  $\Sigma$  and  $\Xi$  to denote the average of the  $(\Sigma^+, \Sigma^0, \Sigma^-)$  and  $(\Xi^0, \Xi^-)$  multiplet mass. We will discuss isospin breaking in a separate section.

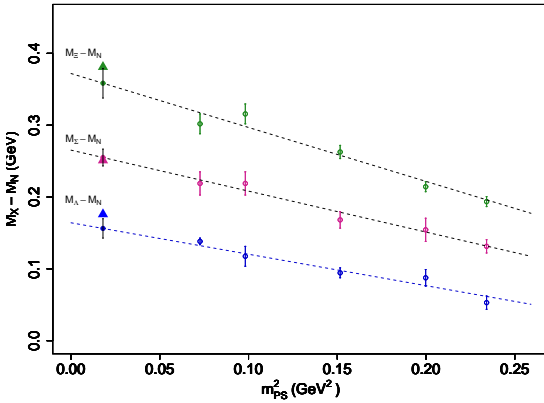
We take the chiral expansion of the baryon mass ( $M_X$ ) in a partially quenched setup to leading order to be [7] :

$$M_X = M_0 + a_X m_\pi^2 + b_X m_\pi^3 \quad (3.1)$$

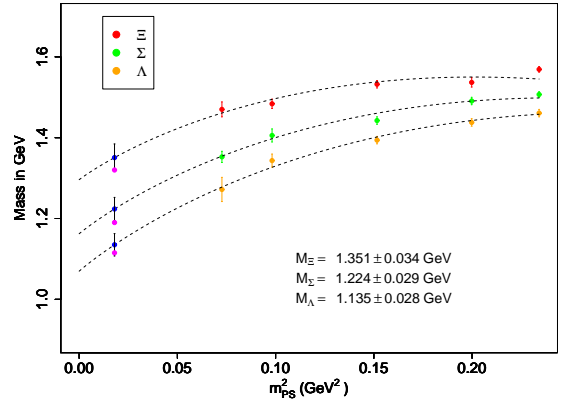
Performing a three-parameter fit of our data is not stable. The method followed in the present analysis relies on the observation displayed in Fig. 2 that the mass difference between any member of the octet and the nucleon is, within errors, linear in  $m_\pi^2$ . This suggests that the coefficient of the cubic term in Eq. (3.1) for the  $\Lambda, \Sigma$  and  $\Xi$  baryons is compatible with the nucleon one [8], i.e :

$$b_N = -\frac{3g_A^2}{32\pi f_\pi^2}. \quad (3.2)$$

Note that the SU(6) quark model, as well as the SU(3) phenomenological analysis of the semi-leptonic decays and hyperon-nucleon interaction [9, 10] predict different values for this coefficient. In Fig. 3, we fix the cubic term to the nucleon one and fit the parameters  $M_0$  and  $a_X$  for the other states of the octet. The mass values we find extrapolationa at the physical point are in good agreement with the experimental results. A careful analysis of the cubic term contribution, including systematics, is however required and will be detailed in a coming paper [11].



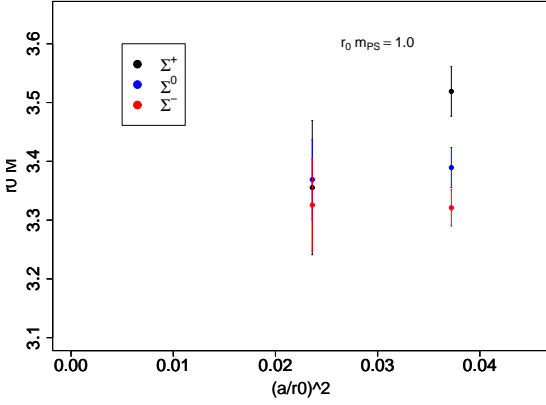
**Figure 2:** Mass difference between members of the octet and the Nucleon extracted from ratio of correlators as a function of  $m_\pi^2$ , at fixed lattice spacing ( $\beta = 3.9$ ,  $a = 0.085$  fm). Physical points are represented by triangles.



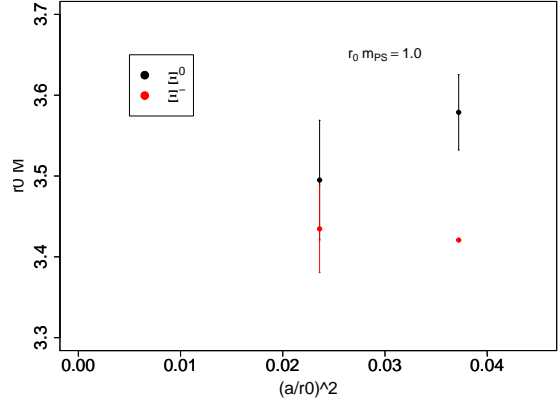
**Figure 3:** Chiral fits of the  $\Lambda, \Sigma$  and  $\Xi$  baryon masses. The experimental values for the masses (shown in magenta) are not included in the fits. The masses obtained from extrapolating lattice data keeping the cubic term fixed are shown in blue with fixed.

### 3.2 Lattice artifacts and isospin breaking

An important issue one needs to check in the twisted fermions formulation is the isospin symmetry breaking at finite lattice spacing. In the case of the  $\Delta$  baryon it was shown in Ref. [2] that the isospin breaking is compatible with zero. In the strange baryon sector we can study the isospin splitting of the three  $\Sigma$  states and the two  $\Xi$  states. In Figs. 4 and 5 we show the mass splitting for the  $\Sigma$ 's and  $\Xi$ 's respectively as a function of the lattice spacing. Using the value of the Sommer scale  $r_0(a)$  at the chiral limit, we compare results obtained at different lattice spacings, for a fixed pion mass of reference ( $r_0 m_\pi = 1.0$ ). We plot the behavior of  $r_0 M_\Sigma$  and  $r_0 M_\Xi$  as a function of  $(a/r_0)^2$ . Results show a decrease of the isospin splitting with  $a$ . Furthermore, averaging over the different charge states of the  $\Sigma$  and  $\Xi$ , results in a weaker lattice spacing dependence and justifies the use of the average for the chiral extrapolation.



**Figure 4:**  $r_0 M_\Sigma$  as a function of  $(a/r_0)^2$  for  $\Sigma^+$ ,  $\Sigma^0$  and  $\Sigma^-$ .



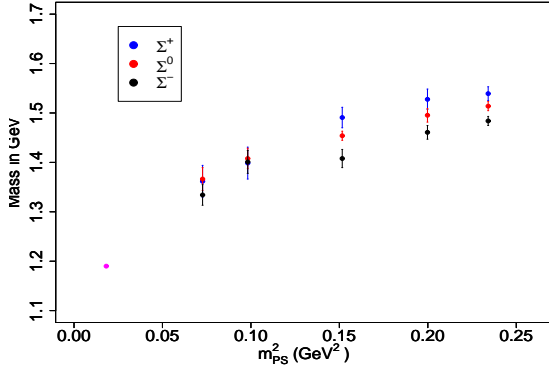
**Figure 5:**  $r_0 M_\Xi$  as a function of  $(a/r_0)^2$  for  $\Xi^0$  and  $\Xi^-$ .

In Figs. 6 and 7 we display the mass of  $(\Sigma^+, \Sigma^0, \Sigma^-)$  and  $(\Xi^0, \Xi^-)$  multiplets as a function of  $m_\pi^2$  at fixed lattice spacing. We observe that the splitting decreases with the pion mass. The naive argument saying that for small quark mass, the u and d propagators are equivalent seems to apply in this case.

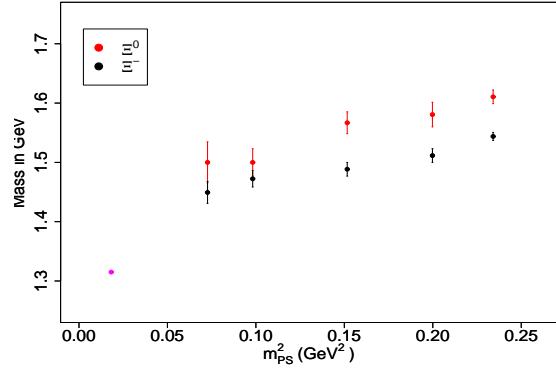
### 4. $\Omega$ baryon

In this section we study the mass dependence of the  $\Omega$ ,  $\Xi$  and  $\Lambda$  baryons on the bare strange quark mass. In Fig. 8, we show that their mass is, as expected, linear as a function of  $a\mu_s$ .

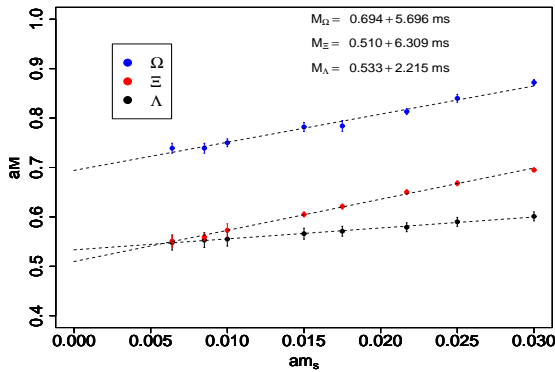
The chiral extrapolation of the  $\Omega$  mass is very interesting because its dependence on  $m_\pi$  is a purely sea quark effect. Our data (see Fig. 9) confirm that there is indeed a dependence on the u and d quark mass, which has to be understood in order to extrapolate the  $\Omega$ -mass to the physical point. As can be seen in Fig. 9, the  $\Omega$  cubic term is expected to be smaller than the nucleon and  $\Delta$  one. A more extended discussion of the chiral extrapolation of the  $\Omega$ -mass will be detailed in a forthcoming publication [11].



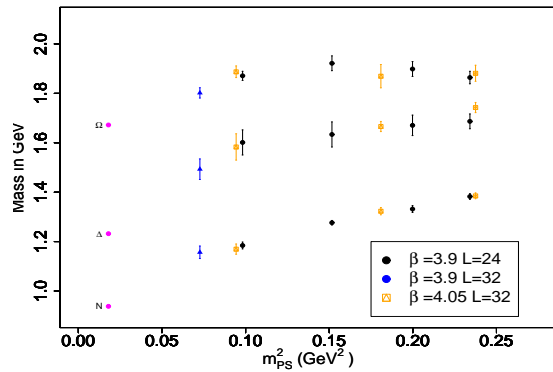
**Figure 6:**  $\Sigma^+, \Sigma^0, \Sigma^-$  for  $\beta = 3.9$  ( $a = 0.0085$  fm) for smeared-smeared correlators as a function of the pion mass squared ( $GeV^2$ ).



**Figure 7:**  $\Xi^0, \Xi^-$  for  $\beta = 3.9$  ( $a = 0.0085$  fm) for smeared correlators as a function of the pion mass squared ( $GeV^2$ ).



**Figure 8:** Mass of  $\Lambda$ ,  $\Sigma$  and  $\Omega$  at fixed lattice spacing for a pion mass of  $\approx 310$  MeV as a function of the bare strange quark mass.



**Figure 9:** Comparison of the pion mass dependence of  $N$ ,  $\Delta$  and  $\Omega$ , at fixed lattice spacing ( $a = 0.085$  fm). The experimental value is shown magenta.

## 5. Conclusion

We have shown, in this contribution, that the use of twisted mass fermions yields promising and accurate results in the spectroscopy of strange baryons. In the chiral extrapolation, the cubic term of the octet members can be constrained by studying their mass difference with the nucleon. This leads to the conclusion that the  $m_\pi^3$  term in the octet chiral expansion is compatible with the nucleon one. Contrary to the case of the  $\Delta$ , the strange baryon sector shows an isospin breaking, which however decreases with the pion mass and becomes compatible with zero at  $m_\pi \approx 310$  MeV. A first study of lattice artifacts show small finite size effects. Strange quark mass dependence is, as expected, linear in  $a\mu_s$ . The  $\Omega$  exhibits a sea quark dependence, which, even if it is small, shows the importance of dynamical quarks in this sector.

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