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Decay constants from twisted mass QCD.

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We present results for chiral extrapolations of the mass and decay constants of the rho meson. The data sets used are the n_f=2 unquenched gauge configurations generated with twisted mass fermions by the European Twisted Mass Collaboration. We describe a calculation of three decay constants in charmonium and explain why they are required.

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1. Introduction

The twisted mass formalism [1, 2] has proven itself to be a powerful tool to extract precision physics in the pseudo-scalar [3, 4] and baryon sector [5]. In this work we look at one of the simplest resonances the ρ meson. The additional complication for the ρ meson is that it decays via the strong interaction to two pions. In this project we aim to study the effect of the decay of the ρ mass and decay constant via the chiral extrapolation formulae. As an extension of the study of the decay constants of the light vector meson we also studied the decay constants of the J/ψ and η_c mesons.

The twisted Wilson action with the tree level Symanzik gauge action was used (see [4] for details). In this analysis we use the n_f = 2 unquenched data sets at β=3.9 24^3 48 and $\beta = 4.05$ 32^3 64 with lattice spacings determined from f_π to be 0.0855 fm and 0.0667 fm respectively [6]. These lattice spacings were consistent with those from the mass of the nucleon [5].

2. Mass of the ρ meson

The masses and decay constants for the light vector meson were obtained by fitting an order 4 smearing matrix to a factorising fit model [4]. The correlators were generated with the "one-end" trick to improve the signal to noise ratio. We only consider unitary data. Vladikas et al. [7] studies an independent set of vector meson correlators with partial quenching on the same set of gauge configurations.

In this section we discuss the chiral extrapolation of the mass of the ρ meson. Bruns and Meißner [8] have published a "new" chiral extrapolation formulae for the ρ meson, based on a modified $\overline{MS}$ regulator.

$$M_\rho = M_\rho^0 + c_1 M_\pi^2 + c_2 M_\pi^3 + c_3 M_\pi^4 \ln(\frac{M_\pi^2}{M_\rho^2}) \tag{2.1}$$

It is important to check that size of c_i from the fits to the lattice data is consistent with other estimates from phenomenology. Bruns and Meißner [8] claim that phenomenology prefers $\mid c_i \mid <$ 3. Leinweber et al. [9] claim to know sign of c_2 (negative) and magnitude from $\omega\rho\pi$ coupling. The original effective field theory calculation Jenkins, et al. [10] had c_3=0. There are too many parameters to determine from our data, so we use an augmented χ^2 with the above constraints built in [11]

$$\chi^2_{aug} = \chi^2 + \sum_{j=2}^{3} \frac{(c_i - 0)^2}{3^2}$$

We also tried the fit model suggested by Leinweber et al. [9]. The formulae doesn't obey the power counting of effective field theory, but does include an explicit term for the decay of the ρ to two mesons. Also some coefficients are fixed from experiment in the fit model.

Although we have data at two lattice spacings we only fit the β = 3.9 24^3 48 data sets. Unfortunately the ρ correlators at $\beta = 4.05$ and the 32^3 64 $\beta = 3.9$ data were too noisy to be useful. See [12] for a comparison between our results for the mass of the ρ meson and other lattice collaborations. We are investigating various measurements techniques, such as colour diluted stochastic sources, to reduce the statistical errors.

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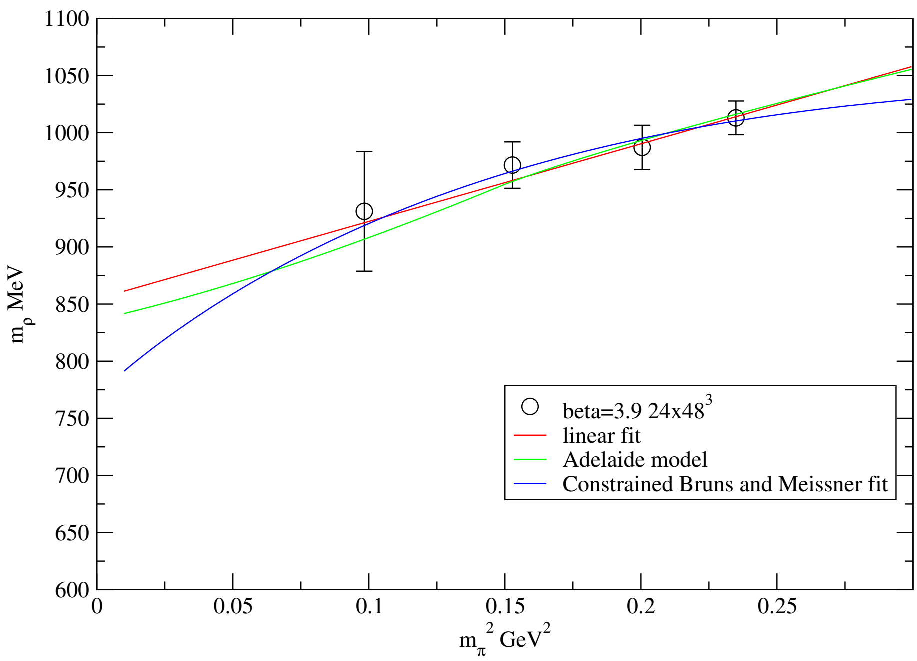

Figure 1: Chiral extrapolation of the mass of the ρ meson

The lattice data are shown in figure 1. A fit linear in the square of the pion mass gives m_ρ = 867(29) MeV ($\beta = 3.9$ data), compared to the experimental value of 770 MeV. Naively the ρ data prefers a different lattice spacing to that from f_π, but we think that the error is due to missing chiral corrections. We fit the Bruns-Meißner model to the data with constraints on the c_2 and c_3 coefficients and obtain the preliminary result is m_ρ = 807(8) MeV. For the Adelaide fit [9] method for the ρ meson, we obtain m_ρ = 847(72) MeV. The statistical errors need to be reduced for a detailed comparison between the lattice data and the results of effective field theory.

3. Decay constants of the ρ meson

Light cone sum rules require the transverse decay constant of the ρ meson [13, 14] for the extraction of $\frac{|V_{td}|}{|V_{ts}|}$ from the $B \rightarrow \rho\gamma$ and $B \rightarrow K^\star\gamma$ decays, and other semileptonic decays of the B meson [15].

In the continuum the decay constant of the ρ meson is defined via

$$\langle 0 \mid \overline{\psi}(x)\gamma_\mu \psi(x) \mid \rho \rangle = m_\rho f_\rho \varepsilon_\mu \tag{3.1}$$

The transverse decay constant ($f_V^T(\mu)$) of the ρ meson is defined by

$$\langle 0 \mid \overline{\psi}\sigma_{\mu\nu}\psi \mid \rho \rangle = i f_V^T(\mu)(p_\mu \varepsilon_\nu - p_\nu \varepsilon_\mu) \tag{3.2}$$

β	Z_A	$Z_T(\mu = 1/a)$
3.90	0.771(4)	0.769(4)
4.05	0.785(6)	0.787(7)

Table 1: Non-perturbative renormalisation factors used in this analysis

Group	Method	f_ρ^T(2 GeV) MeV	f_ρ MeV	$\frac{f_\rho^T}{f_\rho}$
Becirevic et al. [15]	quenched	150(5)	-	$0.72(2)_0^{+2}$
Braun et al. [20]	quenched	154(5)	-	0.74(1)
QCDSF 2005 [21]	unquenched	168(3)	256(9)	0.66(3)
RBC-UKQCD [22]	unquenched	143(6)	-	0.69(3)
This work (preliminary)	unquenched	177(26)	229(35)	0.76(14)

Table 2: Summary of results for transverse and leptonic decay constants of the ρ meson

where $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$. The $f_V^T(\mu)$ decay constant can not be determined from experiment. For the charmonium part of this analysis we studied the decay constant of the pseudoscalar meson.

$$f_{PS} = (2\mu)\frac{|\langle 0 \mid P^1(0) \mid P\rangle|}{M_{PS}^2} \tag{3.3}$$

To renormalise the currents we use the results from the Rome-Southampton method, slightly updated from those reported by Dimopoulos et al. [16] last year. We remind the reader that in twisted mass QCD the charged vector current renormalises with Z_A. The renormalisation factor of the tensor current depends on the scale, so we run to 2 GeV using the method described by Becirevic et al. [15]. We will include the results from Gracey's three loop calculation at a later time [17].

The chiral perturbation theory calculations for the chiral extrapolations of the leptonic vector decay constants have been calculated [18]. The corrections due loops start at $m_q \log m_q$ and $m_q^{3/2}$ [18]. Unfortunately our light decay constant data is too noisy to look for chiral corrections, so we use simple linear fits in the quark mass. The chiral perturbation theory for tensor sources has been developed, but no loop calculations are available [19]. Although by taking ratios of correlators the ratio of $\frac{f_\rho^T}{f_\rho}$ can be extracted directly [15], we prefer to separately compute f_ρ^T and f_ρ, because f_ρ is known from experiment (207 MeV) is thus a good validation test that the ρ to 2π decay is under control. In table 2 we present our preliminary results for the light vector decay constants, and compare to previous results in the literature.

4. Decay constants in charmonium

One way to understand charmonium production and decay is to use the NRQCD formalism, where non-perturbative information is encoded in a few matrix elements. NRQCD mostly produces a good description of experimental data. See [23] for a review and a discussion of the convergence of the velocity expansion in the charm region. However the results from Belle [24] for double

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charmonium production

$$\sigma[e^+e^- \to J/\psi + \eta_c]\mathscr{B} = 25.6 \pm 2.8 \pm 3.4\, fb \tag{4.1}$$

where $\mathscr{B} < 1$, are much larger than the prediction from leading order NRQCD by Bodwin et al. [25].

$$\sigma[e^+e^- \to J/\psi + \eta_c]_{(NRQCD;LO)} = 3.78 \pm 1.26\, fb \tag{4.2}$$

BaBar has a similar result to Belle's for this process [26]. Although further NRQCD calculations that include relativistic corrections and higher order perturbative corrections have increased the leading order NRQCD results, the final result is still lower than the experimental result [27].

It is claimed that calculations that use light cone wave functions of charm mesons agree with the Belle result [28]. However, Bodwin [29] remarks that it is not clear that the model light cone wave functions used are close to true quarkonium wave functions. Lattice QCD calculations should be able to constrain some of the parameters of the light cone wave function of the charmonium mesons. In particular the decay constants are parameters of the light cone wave functions. We refer the reader to figure 6 in [30] to see the effect of the decay constants on the production cross-section. See [31] for a recent review of developments in this field.

As stressed by the FNAL and HPQCD collaborations [32], the computation of the decay constant of the J/ψ meson, that is known from experiment, is an important validation test for calculations that compute decay constants that contain heavy quarks.

The lattice calculation is essentially the same as for the light quarks in section 3. As normal the charmonium correlators are much less noisy than for the equivalent light quark correlators. We used charged pseudoscalar interpolating operators, and we do not include any disconnected diagrams in the analysis. We computed the leptonic and transverse decay constant of the J/ψ meson and the decay constant of the η_c meson for the β values 3.9 and 4.05. We used three heavy quark masses that interpolated the mass of the charm quark. We tuned the charm mass by looking at the J/ψ meson. Only local-local correlators were used.

The results for the three decay constants are plotted in figure 2. We quote preliminary results: $f_{J/\psi}$ = 413(40) MeV, $f_{J/\psi;trans}$ =396(35) MeV, and f_{η_c} =379(29) MeV, for the decay constants in the continuum limit. The errors on the final results are inflated because we only had data at two lattice spacings.

The experimental value of $f_{J/\psi}$= 411(7) MeV (see [33] for example). There is a "sort of experiment result" f_{η_c} = 335(75) MeV from $B \to \eta_c K$ with factorisation assumption from CLEO [34]. We now compare to other lattice QCD calculations. The Jlab lattice group obtained [33] $f_{J/\psi}$ = 399(4) MeV and f_{η_c} = 429(4)(28) MeV from quenched QCD, at a single lattice spacing. Chiu and Hsieh obtained f_{η_c} = 438(5)(6) MeV from a quenched QCD calculation using the overlap fermion action [35], at a single lattice spacing.

5. Conclusions

We have described our lattice QCD calculations to compute the mass and decay constants of the rho meson. We are trying to carefully study the chiral extrapolation. Similar to the chiral pertubation theory studies with light pseudoscalar mesons, the errors on the masses and decay constants have to be very precise to see the effects of loop diagrams.

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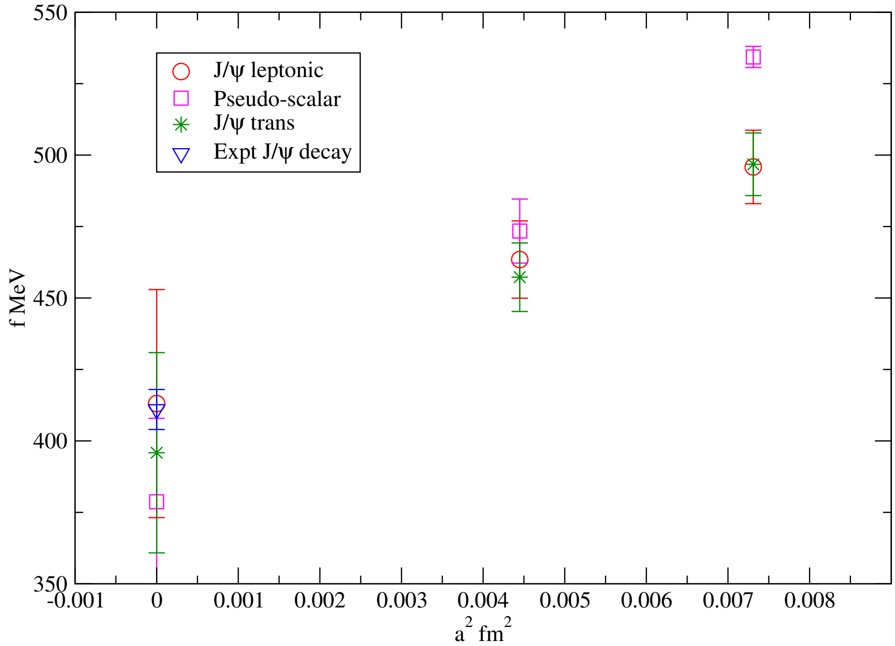

Figure 2: Decay constants as a function of square of lattice spacing

We reviewed why all three decay constants of the charmonium system are important for the phenomenology of double charmonium production and we presented preliminary results for these decay constants. We note that the moments of the light cone wave functions could also be determined in a similar manner to the calculation in the light quark sector [36].

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