PROCEEDINGS OF SCIENCE

Lognormal γ-ray flux variations in the extreme BL Lac object PKS 2155-304

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The High Energy Stereoscopic System (H.E.S.S) observed the BL Lac object PKS 2155-304 from 2004 to 2007. We investigate the nature of the light curve of PKS 2155-304, observed during the exceptional flaring periods of July 28th (MJD 53944) to 31th 2006 (MJD 53947), through the large variations in the γ-ray fluxes which allow us to study the excess rms-flux relation. We show for the first time in this energy domain that the light curve can be considered as a random stationnary process where the logarithm of the fluxes is the relevant Gaussian variable, bearing a striking similarity with XRBs and Seyfert-type AGN variability.

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1. Introduction

H.E.S.S. is a system of four Imaging Atmospheric Cherenkov Telescopes (ACT) for γ-ray astronomy in the 100 GeV - 50 TeV energy range. Observations and monitoring of active galactic nuclei (AGN) are a key part of the scientific observation program of the instrument. PKS 2155-304 has been regularly observed and detected by H.E.S.S. since 2002. A flaring period of PKS 2155-304 was detected by H.E.S.S in July 2006 with fluxes on average 7 times the Crab nebula flux, allowing an unprecedented temporal resolution of a few minutes. For the first time in this energy domain, we have the possibility to characterize the γ-ray variability with statistical methods used in other energy bands. The light curve corresponding to the flaring period of July 28th (MJD 53944) to 31th 2006 (MJD 53947) is shown in Fig.1 with a 4 minute sampling interval. Observations and analysis of the July 28th data have been discussed in [1].

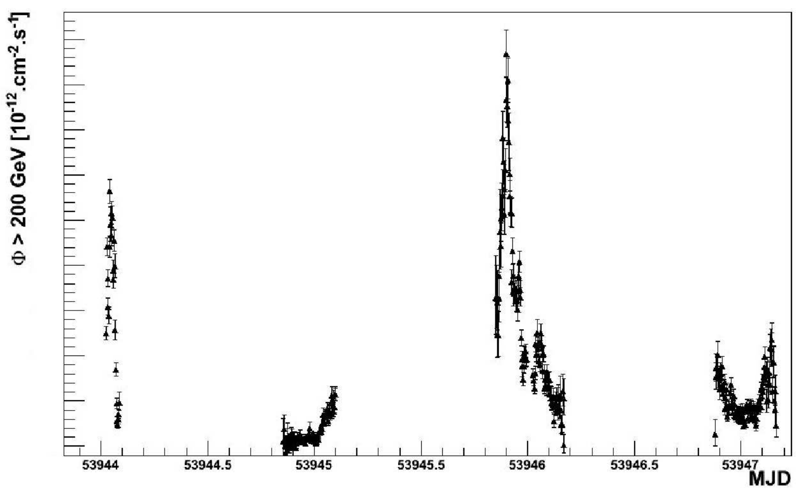

Figure 1: *PKS 2155-304 light curve (MJD 53944 to 53947) with a 4 minute sampling interval.*

We investigate whether the light curve observed by H.E.S.S. can be considered as a realization of a random stationary Gaussian process. In this context, the γ-ray flux from the object is considered as a random variable whose probability distribution function depends on time. A random stationary Gaussian process is characterized on the basis of the Fourier transform of the light curve, as follows: the Fourier components at different frequencies have random phases and their amplitudes are independent variables, normally distributed with 0 mean; the dependence of their variances upon the frequency, namely the power density spectrum (PDS), completely defines the process; the latter is called "stationary" in the sense that the PDS is independent of time. Such PDS obtained from the aperiodic X-ray light curves of binary systems have been used, together with the corresponding energy spectra, to define different "states" of these objects [2]. In large frequency intervals, the power spectra of XRBs, Seyfert-type AGNs and blazars in X-rays take the form of a power law of the frequency ν, namely $C(\nu_{\rm ref}/\nu)^{\alpha}$; α is the variability spectral index and C gives the "power" (i.e. the variance) at a reference frequency $\nu_{\rm ref}$ chosen for convenience.

We want to investigate if a similar parametrization of the PDS holds in the VHE domain. We are thus facing the following questions:

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- Can a random stationary Gaussian process account for the observed flaring periods of PKS 2155-304 in 2006 ?
- What is the relevant Gaussian variable ? Is it the γ-ray flux (as expected if it results from an additive process, i.e. from the contributions of several zones) or is it its logarithm (as expected if it results from a multiplicative process, as in a cascade)?

2. RMS-flux relation

The excess rms of the quantity X is the root mean square of the intrinsic variance and is defined as $\sigma_{xs} = \sqrt{var(X) - \overline{\sigma_{err}^2}}$, where $\overline{\sigma_{err}^2}$ represents the contribution of the measurements errors on the total variance. It gives an indication on how much the flux varies about the mean and helps understand the mechanisms at the origin of the observed variability. If the process at the origin of the variability is additive, the fluxes are normally distributed and one expects no correlation between the excess rms and the mean of the flux, because all Fourier coefficients are statistically independent. On the contrary, if the process is multiplicative, the logarithm of the flux is the relevant variable and there is a strong correlation between the two quantities [4].
Mean values and standard deviations have been calculated over intervals of 1200 and 4800 seconds extracted from PKS 2155-304 four nights light curve, with samplings of 1 and 4 minutes. Fig.2 and Fig.3 show the distribution of the measurements in the excess rms versus flux diagram for the two samplings. Only those intervals with a significant variability (i.e. positive excess variance) are taken into account.

Both figures show a proportionality between the excess rms and the flux. With intervals of 1200 seconds we find a correlation coefficient $\rho = 0.64^{+0.17}_{-0.28}$ where the confidence interval at the 95% confidence level is derived following the Fisher transform. Thus, a Gaussian process that predicts a zero correlation can be excluded at 4σ. For the 4800 seconds duration intervals, the confidence interval on the correlation coefficient $0.94^{+0.04}_{-0.10}$ excludes a Gaussian process at a 7σ level. The excess rms is strongly correlated with the mean flux level, showing that the normally distributed variable is the logarithm of the flux.

3. Characterization of the lognormal process

In order to relate the observed light curve to a lognormal process, simulations are necessary. For practical reasons, a discrete frequency spectrum with a very low fundamental frequency $\nu_0 = \frac{1}{T_0}$ is used in simulations. Since the logarithm of the flux is the normally distributed variable, one can write:

$$\ln(\Phi(t)) = \frac{u_0}{2} + \sum_{n=1}^{N} u_n \cos\left(\frac{2n\pi t}{T_0} + \phi_n\right) \tag{3.1}$$

where the Fourier coefficients follow:

$$\langle u_n^2 \rangle = \sigma_n^2 = K\nu_0 (\frac{n_{ref}}{n})^\alpha \tag{3.2}$$

with $n_{ref} = \frac{\nu_{ref}}{\nu_0}$. The two parameters K and α that completely describe the lognormal process will be determined using the excess-rms correlation and structure functions. The reference frequency is conventionnaly fixed to 10^{-4}Hz.

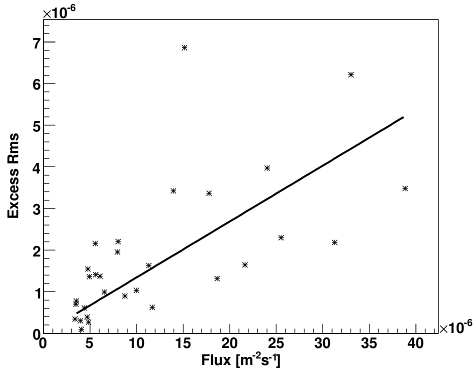

Figure 2: *Excess rms versus flux, each point represents a measurement over 20 minutes. The solid line is a least-squares fit to the data provided to guide the eye (also in Fig.3).*

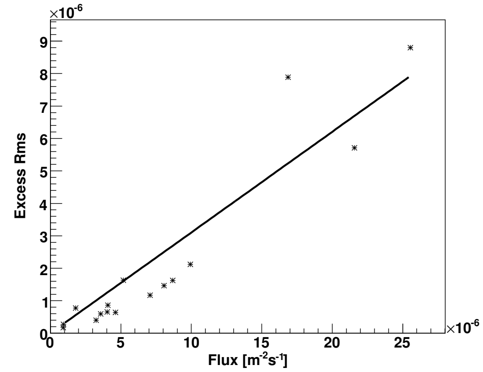

Figure 3: *Excess rms versus flux, each point represents a measurement over 80 minutes.*

3.1 Determination of the parameters using the rms-flux correlation

For a given set (K,α), a comparison between the excess rms-flux correlation plots derived from 500 simulated light curves (see Fig.5) and the experimental one is characterized by a likelihood function which is further maximized with respect to K and α. The probability of obtaining the experimental fluxes and fractional variability distribution given a chosen (K,α) set is calculated. The number of intervals with positive excess variance is also taken into account. Fig.4 shows the experimental histograms of fluxes and fractional variability for the intervals of 1200 seconds.

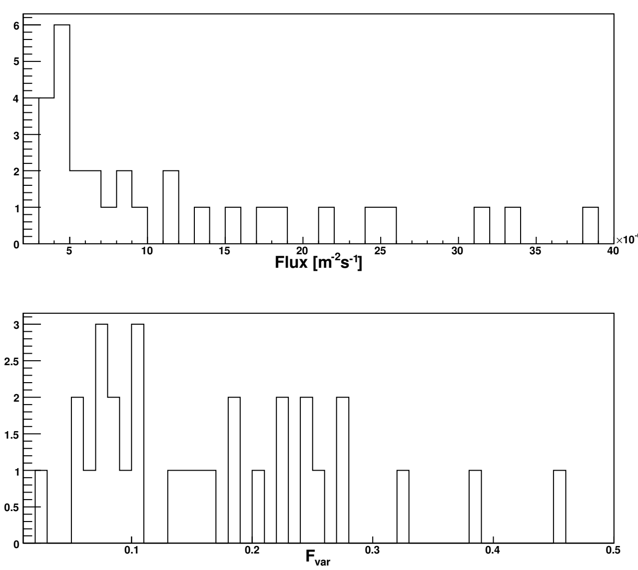

Figure 4: *Histograms of fluxes and fractional variability for the intervals of 1200 seconds derived from a light curve with a one minute sampling.*

Confidence regions at 95% are established and shown for the two duration intervals in Fig.6: red contours are obtained with light curve segments of 20 minute duration and blue contours are obtained with light curve segments of 80 minute duration.

The contours are in good agreement within each other and well constrain the lognormal process.

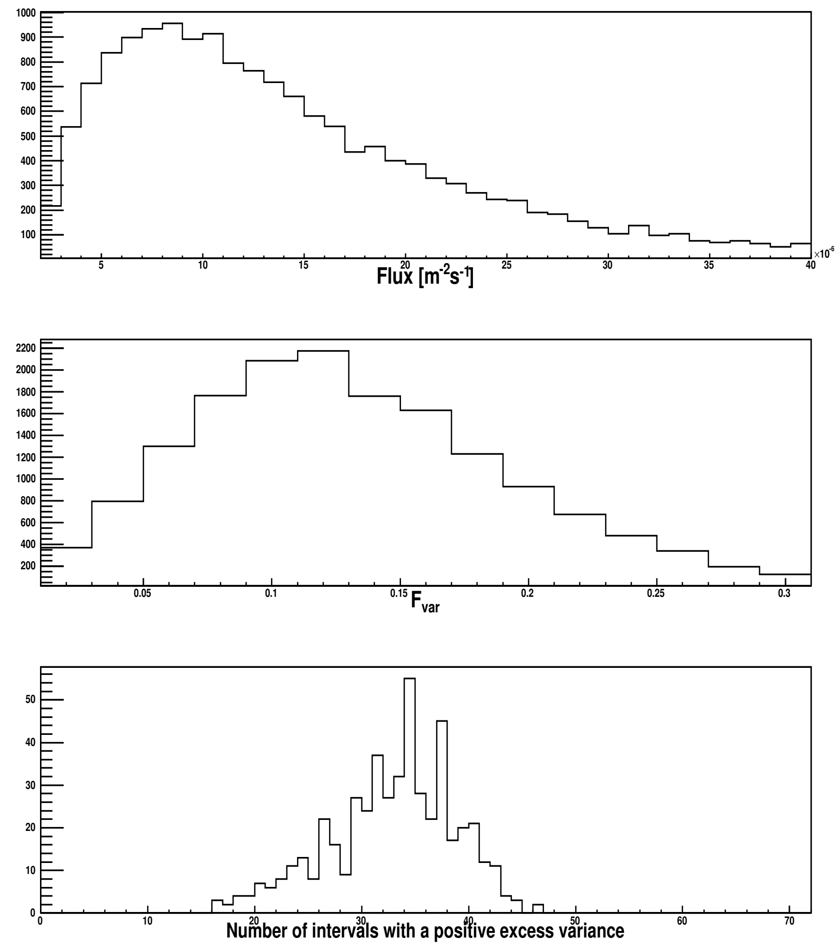

Figure 5: *Histograms of fluxes and fractional variability obtained from 500 simulations of 1 minute sampled light curves derived from a lognormal process with a spectral variability index $\alpha = 2$ and a normalization factor $K = 500\ Hz^{-1}$. The third line shows the distribution of the number of intervals with a positive excess variance. It is equal to $\frac{33\pm6}{59}$ which is compatible with the experimental number ($\frac{30}{59}$).*

3.2 Structure function

The Kolmogorov structure functions are a useful tool in the study of red noise and have been extensively used in telecommunication engineering as well as in astrophysics ([5],[6],[7]). Given a signal $X(t_k)$ measured at regular time intervals (k = 1 to N) the first order structure function is defined as follow:

$$S(\tau) = \overline{X(t_k)^2} - 2\overline{X(t_k)X(t_k+\tau)} + \overline{X(t_k+\tau)^2} \tag{3.3}$$

where the bar on the symbols denote an average over the N measurements. Considering a random Gaussian process X(t) defined by equation 3.1, it can be shown that structure functions averaged over an ensemble of light curves are expected to vary as $\tau^{\alpha-1}$. Note that here, X(t) is the logarithm of the flux. However, this only holds for the ensemble of light curves, whereas the structure function for a given value of τ is submitted to large fluctuations; furthermore windowing and sampling effects distort the single structure function. Therefore, we shall compare the structure functions

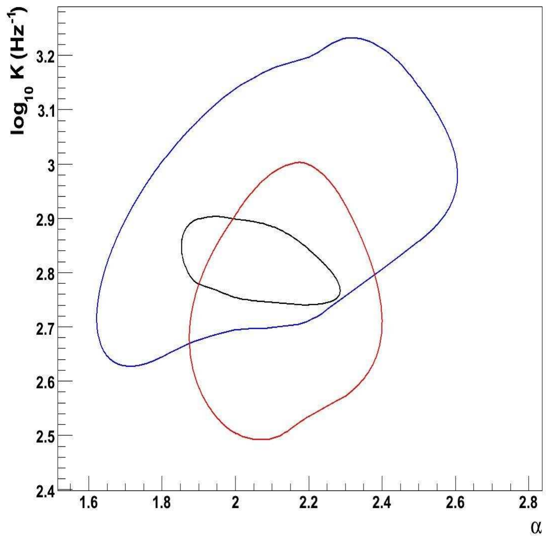

Figure 6: *Contours are given for 95% confidence domains in the (K,α) plane. Red contours are obtained by the method explained in section 3.1, using light curve segments of 20 minute duration sampled every minute. Blue contours are similarly obtained with light curve segments of 80 minute duration, sampled every 4 minutes. Black contours are obtained from the study of structure functions explained in section 3.2.*

derived from the experimental light curve to those obtained from a large number of simulated light curves corresponding to given values of α and K. Simulated time series with the same windowing function, the same sampling intervals and realistic measurement errors have been extracted from much longer ones. For a given value of τ, the distributions of $\log_{10} SF(\tau)$ from the simulated ensemble were found to almost follow Gaussian distributions, with expectation values $\lambda(\tau)$ and standard deviations $\Delta\lambda(\tau)$. Figure 7 shows the experimental structure function calculated over the logarithm of the flux for the 4 minute sampled light curve. The dotted lines represent 68% confidence intervals corresponding to the process defined by $\alpha = 2$ and $\log_{10}(K/Hz^{-1}) = 2.8$. These limits take account of the fluctuations from series to series generated from the same process and the errors affecting each simulated light curve, namely flux measurement errors and statistical errors due to the limited number of measurements on the light curve.

The experimental structure functions can thus be compared to those of the simulated ensemble by means of χ^2 variables defined as follow:

$$\chi^2(\alpha, K) = \sum_k \left\{ \frac{\log_{10}[S_{\exp}(\tau_k)] - \lambda(\tau_k)}{\Delta\lambda(\tau_k)} \right\}^2 \tag{3.4}$$

This χ^2 is minimized with respect to K and α and is used to define confidence domains in the (K, α) plane (see Figure 6, black contours).

4. Conclusion

We show that a lognormal process can explain the observed variability of PKS 2155-304 be-

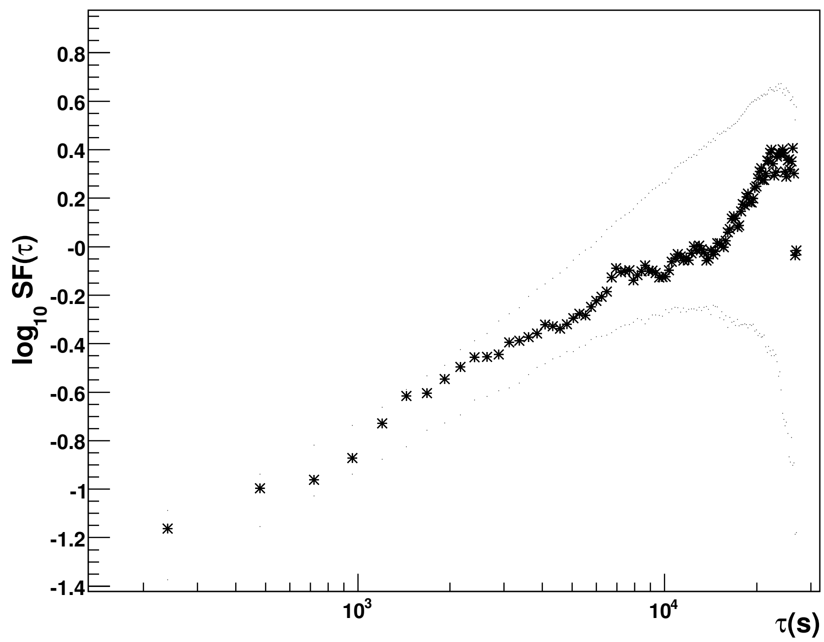

Figure 7: *Experimental structure function with the corresponding limits obtained from a process characterized by* $\alpha = 2$ *and* $\log_{10}(K/Hz^{-1}) = 2.8$

tween MJD 53944 and 53947, favouring multiplicative scenarios. We have presented two different studies that characterize the PDS which give similar values for the variability index ($\alpha = 2.06 \pm 0.21$) and for the power spectral density ($\log_{10}(K/Hz^{-1}) = 2.82 \pm 0.08$) at 10^{-4}Hz. These results provide another quantitative similarity with variability observed in XRBs and AGN, and might indicate that similar processes are at play in the inner regions.

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