

Radiation from relativistic jets

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Nonthermal radiation observed from astrophysical systems containing relativistic jets and shocks, e.g., gamma-ray bursts (GRBs), active galactic nuclei (AGNs), and Galactic microquasar systems usually have power-law emission spectra. Recent PIC simulations of relativistic electron-ion (electron-positron) jets injected into a stationary medium show that particle acceleration occurs within the downstream jet. In the presence of relativistic jets, instabilities such as the Buneman instability, other two-streaming instability, and the Weibel (filamentation) instability create collisionless shocks, which are responsible for particle (electron, positron, and ion) acceleration. The simulation results show that the Weibel instability is responsible for generating and amplifying highly nonuniform, small-scale magnetic fields. These magnetic fields contribute to the electron's transverse deflection behind the jet head. The "jitter" radiation from deflected electrons in small-scale magnetic field. This jitter radiation, a case of diffusive synchrotron radiation, may be important to understand the complex time evolution and/or spectral structure in gamma-ray bursts, relativistic jets, and supernova remnants.

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1. Introduction

Shocks are believed to be responsible for prompt emission from gamma-ray bursts (GRBs) and their afterglows, for variable emission from blazars, and for particle acceleration processes in jets from active galactic nuclei (AGN) and supernova remnants (SNRs). The predominant contribution to the observed emission spectra is often assumed to be synchrotron- and inverse Compton radiation from these accelerated particles for gamma-ray bursts [1-7] and for AGN jets [8-13]. It is assumed that turbulent magnetic fields in the shock region lead to Fermi acceleration, producing higher energy particles [14, 15]. To make progress in understanding emission from these object classes, it is essential to place modeling efforts on a firm physical basis. This requires studies of the microphysics of the shock process in a self-consistent manner [16, 17].

2. Method of calculation

Three-dimensional relativistic particle-in-cell (RPIC) simulations have been used to study the microphysical processes in relativistic shocks. Such PIC simulations show that rapid acceleration takes place in situ in the downstream jet [18-33]. Three independent simulation studies confirm that relativistic counter-streaming jets do excite the Weibel instability [34], which generates current filaments and associated magnetic fields [35], and accelerates electrons [18-22].

In order to determine the luminosity and spectral energy density (SED) of synchrotron radiation, it is general practise to simply assume that a certain fraction $\varepsilon_{\rm B}$ of the post-shock thermal energy density is carried by the magnetic field, that a fraction $\varepsilon_{\rm e}$ is carried by electrons, and that the energy distribution of the electrons is a power-law, $d \log n_{\rm e}/d \log \varepsilon = p$ (above some minimum energy $E_{\rm m}$ which is determined by $\varepsilon_{\rm e}, \varepsilon_{\rm B}$ and p). In this approach, $\varepsilon_{\rm B}$, $\varepsilon_{\rm e}$, and p are treated as free parameters, to be determined by observations. However, more constrainting data now require additional free parameters such as the introduction of broken power-law to reproduce the spectral energy distributions of TeV blazars for instance [11]. Due to the lack of a first principle theory of collisionless shocks, a purely phenomenological approach to modeling radiation is applied, but one must recognize that emission is then calculated without a full understanding of the processes responsible for particle acceleration and magnetic field generation [17]. It is important to clarify that the constraints imposed on these parameters by the observations are independent of any assumptions regarding the nature of the shocks and the processes responsible for particle acceleration or magnetic field generation. Any model proposed for the actual shock micro-physics must be consistent with these phenomenological constraints.

Since magnetic fields are generated by the current structures produced in the filamentation (Weibel) instability, it is possible that "jitter" radiation [36-44] is an important emission process in GRB and AGN jets. It should be noted that synchrotron- and 'jitter'-radiation are fundamentally the same physical processes (emission of accelerated charges in a magnetic field), but the relative importance of the two regimes depends on the comparison of the deflection angle and the emission angle of the charges [36]. Emission via synchrotron- or "jitter"-radiation from relativistic shocks is determined by the magnetic field strength and structure and the electron energy distribution behind the shock, which can be computed self-consistently with RPIC simulations. The full RPIC

simulations may actually help to determine whether the emission is more synchrotron-like or jitterlike.

The characteristic differences between Synchrotron- and jitter radiation are relevant for a more fundamental understanding of the complex time evolution and/or spectral properties of GRBs (prompt and afterglows) [45]. For example, jitter radiation has been proposed as a solution of the puzzle that below their peak frequency GRB spectra are sometimes steeper than the "line of death" spectral index associated with synchrotron emission [35-38], i.e., the observed SED scales as $F_V \propto v^{2/3}$, whereas synchrotron SEDs should follow $F_V \propto v^{1/3}$, or even more shallow (i.e. $F_V \propto v^{\alpha}$ where $\alpha \leq 1/3$, e.g., [37]). Thus, it is crucial to calculate the emerging radiation by tracing electrons (positrons) in self-consistently evolved electromagnetic fields. This highly complex analytical and computational task requires sophisticated tools, such as multi-dimensional, relativistic, PIC methods.

2.1 New Computing Method of Calculating Synchrotron and Jitter Emission from Electron Trajectories in Self-consistently Generated Magnetic Fields

Consider a particle at position $\mathbf{r}_0(t)$ at time t (Fig. 1). At the same time, we observe the associated electric field from position \mathbf{r} . Because of the finite propagation velocity of light, we actually observe the particle at an earlier position $\mathbf{r}_0(t')$ along its trajectory, labeled with the retarded time $t' = t - \delta t' = t - \mathbf{R}(t')/c$. Here $\mathbf{R}(t') = |\mathbf{r} - \mathbf{r}_0(t')|$ is the distance from the charge (at the retarded time t') to the observer's position.

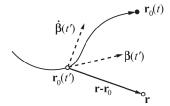


Figure 1: Definition of the retardation effect. From an observers point, r, one sees the particle at position $\mathbf{r}_0(t')$ where it was at retarded time t' (from Figure 2.2 in [46]).

The retarded electric field from a charged particle moving with instantaneous velocity β under acceleration $\dot{\beta}$ is expressed as [48],

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0} \left[\frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2 (1 - \mathbf{n} \cdot \boldsymbol{\beta})^3 \mathbf{R}^2} \right]_{\text{ret}} + \frac{q}{4\pi\varepsilon_0 c} \left[\frac{\mathbf{n} \times \{(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \mathbf{n} \cdot \boldsymbol{\beta})^3 \mathbf{R}} \right]_{\text{ret}}$$
(2.1)

Here, $\mathbf{n} \equiv \mathbf{R}(t')/|\mathbf{R}(t')|$ is a unit vector that points from the particle's retarded position towards the observer. The first term on the right hand side, containing the velocity field, is the Coulomb field from a charge moving without influence from external forces. The second term is a correction term that arises when the charge is subject to acceleration. Since the velocity-dependent field falls off in distance as R^{-2} , while the acceleration-dependent field scales as R^{-1} , the latter becomes dominant when observing the charge at large distances ($R \gg 1$).

The choice of unit vector **n** along the direction of propagation of the jet (hereafter taken to be the Z-axis) corresponds to head-on emission. For any other choice of **n** (e.g., $\theta \leq 1/\gamma$), off-axis emission is seen by the observer. The observer's viewing angle is set by the choice of **n** $(n_x^2 + n_y^2 + n_z^2 = 1)$. After some calculation and simplifying assumptions (for detailed derivation see

[46]) the total energy W radiated per unit solid angle per unit frequency can be expressed as

$$\frac{d^2 W}{d\Omega d\omega} = \frac{\mu_0 c q^2}{16\pi^3} \left| \int_{-\infty}^{\infty} \frac{\mathbf{n} \times \left[(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right]}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^2} e^{i\omega(t' - \mathbf{n} \cdot \mathbf{r}_0(t')/c)} dt' \right|^2$$
(2.2)

This equation contains the retarded electric field from a charged particle moving with instantaneous velocity β under acceleration $\dot{\beta}$, and only the acceleration field is kept since the velocity field decreases rapidly as $1/R^2$. The distribution over frequencies of the emitted radiation depends on the particle energy, radius of curvature, and acceleration. These quantities are readily obtained from the trajectory of each charged particle.

Since the jet plasma has a large velocity Z-component in the simulation frame, the radiation from the particles (electrons and positrons) is heavily beamed along the Z-axis as jitter radiation [36, 37, 38, 44].

3. Radiation from relativistic electrons: a simple case to test computing method

Here we have calculated the radiation from two electrons with Lorentz factor ($\gamma = 15.8, 40.8$) [29, 30]. The electrons gyrate in the x - z plane with the uniform magnetic field (B_y) and the results are shown in Figures 2 & 3.

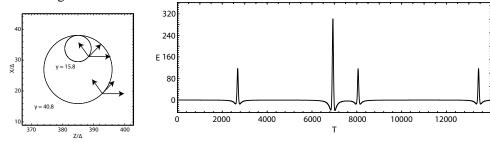


Figure 2: The paths of two charged particles moving in a fixed homogenous magnetic field (left panel) ($\gamma = 15.8, 40.8$). The particles produce a time dependent electric field. An observer situated at great distance along the n-vector sees the retarded electric field from the gyrating particles (right panel). As a result of relativistic beaming, the field is seen as pulses peaking when the particles move directly towards the observer.

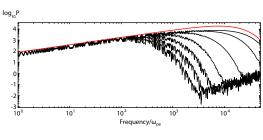


Figure 3: The observed power spectrum from two charged particles, gyrating in a magnetic field at different viewing angles. The viewing angles are 0° , 1° , 2° , 3° , 4° , 5° , and 6° ($n_y \neq 0$). With larger angles the frequencies above the Nyquist frequency should be strongly damped, however they increase due to aliasing [46]. The units on both axes are arbitrary. The theoretical synchrotron spectrum for a viewing angle equal to 0° is plotted for comparison as a red curve for the electron with $\gamma = 40.8$ (multiplied by 2 for clarity).

The spectra observed far from the electron at angles with respect to the *z* direction are shown in Fig. 3. The higher frequencies (> f_c) are strongly damped with increasing angles as $e^{(-f/f_c)}$, see [48]. Since the critical frequency $f_c = \frac{3}{2}\gamma^3 \left(\frac{c}{\rho}\right) = 2309$, where $\rho = 11.03$ for the electron with $\gamma = 40.8$ is larger than that for $\gamma = 15.8$, the radiation from the electron $\gamma = 40.8$ is dominant [30]. The electron with $\gamma = 15.8$ gyrates about three times in this period, the ripples in the spectrum shows the electron cyclotron frequency. However, in order to resolve it much longer time is required [46]. We have very good agreement between the spectrum obtained from the simulation and the theoretical synchrotron spectrum expectation (red curve) from eq. 3 (eq. 7.10 [46]).

Synchrotron radiation with the full angular dependency for the parallel polarization component is given by [48],

$$\frac{d^2 W_{||}}{d\omega d\Omega} = \frac{\mu_0 c q^2 \omega^2}{12\pi} \left(\frac{r_{\rm L} \theta_\beta^2 \beta^2}{c} \right)^2 \frac{|K_{\frac{2}{3}}(\chi/\sqrt{\cos\theta\beta^3})|^2}{(\cos\theta\beta^3)^2},\tag{3.1}$$

where θ is the angle between **n** and the orbital plane $\theta_{\beta}^2 \equiv 2(1 - \beta \cos \theta)$, $\chi = \omega r_{\rm L} \theta_{\beta}^3/3c$ and $r_{\rm L}$ the gyro-radius $\gamma m v/(qB)$. For $\beta \to 1$ and $\theta \to 0$ this expression converges toward the solution one normally finds in text books [48, 47].

It should be noted that the method based on the integration of the retarded electric fields calculated by tracing many electrons described in this section can provide a proper spectrum in turbulent electromagnetic fields. On the other hand, if the formula for the frequency spectrum of radiation emitted by a relativistic charged particle in instantaneous circular motion is used [47, 48], the complex particle accelerations and trajectories are not properly accounted for and the jitter radiation spectrum is not properly obtained (for details see [46, 49]). The results described above validate the technique used in our code as described previous section [29, 30, 46, 49].

4. Discussion

We have started to calculate emission directly from our simulations using the same method described in the previous section. In order to calculate the (jitter-like) synchrotron radiation from the particles in the electromagnetic fields generated by the filamentation instability, the retarded electric field from a single particle is Fourier-transformed to give the individual particle spectrum as described in the previous section. The individual particle spectra are added together to produce a total spectrum over a particular simulation time span [46, 49]. It should be noted that for this calculation very large simulations over a long time (t_s) are required using a small time step (Δt) in order to increase the upper frequency limit to the spectrum (Nyquist frequency $\omega_N = 1/2\Delta t$). Frequency resolution is limited by the time span ($\Delta \omega = 1/t_s$) [46, 49]. For a case with the time step $\Delta t = 0.01/\omega_{pe}$ and the time span $t_s = 50/\omega_{pe}$, a calculated spectrum will have the highest frequency, $50\omega_{pe}$ and the frequency resolution (the lowest frequency), $0.02\omega_{pe}$. ω_{pe} is calculated with an appropriate plasma density. Simulations over a long time allow us to obtain multiple spectra at sequential time spans so the spectral evolution can be calculated. Synthetic spectra obtained in the way we have described above should be compared with GRB prompt and afterglow observations.

In the case of AGN jets, diffusive synchrotron radiation has already been invoked by several works [39, 41, 43] to reproduce spectra of 3C273, M87 and Cen A knots from radio to X-rays. For TeV blazars, taking into account the relative importance of the energy densities contained in the small-scale and large-scale magnetic fields may be an elegant alternative to the choice of a broken power-law for the energy distribution of radiating electrons.

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