

## Pion cloud and sea quark flavor asymmetry in the impact parameter representation

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We study large-distance contributions to the nucleon parton densities in the transverse coordinate (impact parameter) representation based on generalized parton distributions (GPDs). Chiral dynamics generates a distinct component of the partonic structure, located at momentum fractions  $x \lesssim M_\pi/M_N$  and transverse distances  $b \sim 1/M_\pi$ . We analyze the phenomenological “pion cloud” model of the flavor asymmetry  $\bar{d}(x) - \bar{u}(x)$  and quantify what fraction of the calculated asymmetry results from the universal large-distance region. We show that in this model the region  $b > b_{\text{core}} \approx 0.55$  fm (the transverse “quark core” radius estimated from the nucleon’s axial charge radius) produces only  $\sim 30\%$  of the asymmetry measured in the FNAL E866 experiment at  $x \sim 0.1$ . Our findings indicate that the asymmetry originates mostly from the “quark core” at small transverse distances.

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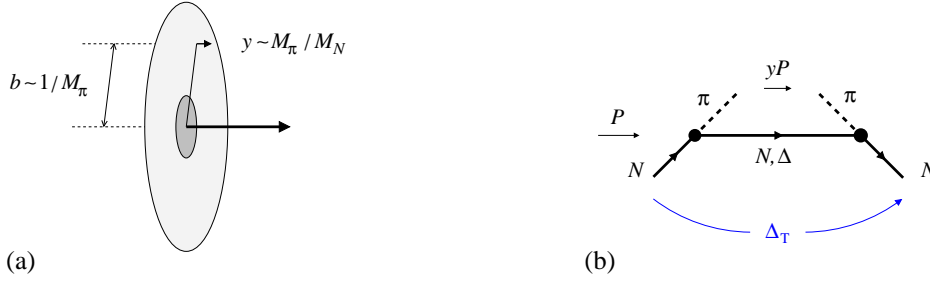
## 1. Introduction

Experiments in deep–inelastic lepton–nucleon scattering [1] and Drell–Yan pair production in nucleon–nucleon collisions [2, 3] have unambiguously shown that the light antiquark distributions in the proton are not flavor–symmetric,  $\bar{d}(x) - \bar{u}(x) > 0$ . The asymmetry is clearly of non-perturbative origin and exhibits only weak scale dependence, and therefore represents an interesting quasi–observable which directly probes the QCD quark structure of the nucleon at low resolution scales. The existence of a large flavor asymmetry had been predicted [4] on the basis of the contribution of the nucleon’s pion cloud to deep–inelastic processes [5]. The ‘bare’ nucleon couples to configurations containing a virtual pion, and transitions  $p \rightarrow n\pi^+$  are more likely than  $p \rightarrow \Delta^{++}\pi^-$ , resulting in an excess of  $\pi^+$  over  $\pi^-$  in the ‘dressed’ proton. This basic idea was implemented in a variety of dynamical models, which incorporate finite–size effects by hadronic form factors associated with the  $\pi NN$  and  $\pi N\Delta$  vertices; see Refs. [6] for a review of the extensive literature. It was noted long ago [7] that in order to reproduce the fast decrease of the observed asymmetry with  $x$  one needs  $\pi NN$  form factors much softer than those commonly used in meson exchange parametrizations of the  $NN$  interaction; however, even with such soft form factors pion virtualities (four–momenta squared) generally extend up to values  $\gg M_\pi^2$  [8]. This raises the question to what extent such models actually describe large–distance effects associated with soft pion exchange (momenta  $\sim M_\pi$ ), and what part of their predictions is simply a parametrization of small–distance dynamics which should more naturally be associated with non-hadronic degrees of freedom. More generally, one faces the question how to formulate the concept of the ‘pion cloud’ in the nucleon’s partonic structure in a manner consistent with chiral dynamics in QCD.

A framework which allows one to address these questions in a systematic fashion is the transverse coordinate (impact parameter) representation, in which the distribution of partons is studied as a function of the longitudinal momentum fraction,  $x$ , and the transverse distance,  $b$ , of the parton from the transverse center of momentum of the nucleon [9]. In this representation, chiral dynamics can be associated with a distinct component of the partonic structure, located at  $x \lesssim M_\pi/M_N$  and  $b \sim 1/M_\pi$  [10]. In the gluon (more generally, in any flavor–singlet) distribution this large–distance component is sizable and contributes to the increase of the nucleon’s average transverse size,  $\langle b^2 \rangle$ , with decreasing  $x$  [10]. Here we study the large–distance component in the antiquark flavor asymmetry  $\bar{d}(x) - \bar{u}(x)$  (*i.e.*, the non-singlet sector). Using general arguments, we first identify the region where parton distributions are governed by chiral dynamics. We then quantify what fraction of the asymmetry obtained in the phenomenological pion cloud model actually arises from the universal large–distance region. Finally, we show that this contribution accounts only for a small part of the observed asymmetry, indicating that most of it is due to non–chiral dynamics at small distances. (A more detailed account of our studies will be given in a forthcoming article.)

## 2. Chiral dynamics and partonic structure

The region where parton densities are governed by chiral dynamics can be established from general considerations. The central object is the pion longitudinal momentum and transverse coordinate distribution in a fast–moving nucleon,  $f_\pi(y, b)$  ( $y$  is the pion momentum fraction); the precise meaning of this concept and its region of applicability will be explained in the following.



**Figure 1:** (a) Parametric region where the pion distribution in the nucleon is governed by chiral dynamics (in longitudinal momentum fraction  $y$ , and transverse distance  $b$ ). (b) The pion GPD in the nucleon. The distributions  $f_{\pi B}(y, b)$  ( $B = N, \Delta$ ) are obtained as the 2-dimensional Fourier transform  $\Delta_T \rightarrow \mathbf{b}$  ( $b \equiv |\mathbf{b}|$ ).

Chiral dynamics generally governs large-distance contributions to nucleon observables, resulting from exchange of “soft” pions in the nucleon rest frame; in the time-ordered formulation of relativistic dynamics these are pions with energies  $E_\pi \sim M_\pi$  and momenta  $|\mathbf{k}_\pi| \sim M_\pi$ . Chiral symmetry provides that such pions couple weakly to the nucleon and to each other, so that their effects can be computed perturbatively. (The distance scale  $1/M_\pi$  is assumed to be much larger than all other hadronic length scales in question.) Boosting these weakly interacting  $\pi N$  configurations to a frame in which the nucleon is moving with large velocity, we find that they correspond to longitudinal pion momentum fractions of the order<sup>1</sup>

$$y \sim M_\pi/M_N. \quad (2.1)$$

At the same time, the soft pions’ transverse momenta, which are not affected by the boost, correspond to a motion over transverse distances of the order

$$b \sim 1/M_\pi. \quad (2.2)$$

Together, Eqs. (2.1) and (2.2) determine the parametric region where the pion momentum and coordinate distribution is governed by model-independent chiral dynamics, and the soft pion can be regarded as a parton in the nucleon’s wave function in the usual sense (see Fig. 1a).

The condition Eq. (2.1) implies that the pion momentum fraction in the nucleon is parametrically small,  $y \ll 1$ . As a consequence, one can generally neglect the recoil of the spectator system and identify the distance  $b$  with the separation of the pion from the transverse center of momentum of the spectator system,  $r = b/(1 - y)$ . This makes for an important simplification in the spatial interpretation of chiral contributions to the parton densities.

In its region of applicability defined by Eqs. (2.1) and (2.2), the  $b$ -dependent pion “parton” distribution can be defined unambiguously as the transverse Fourier transform of the pion GPD in the nucleon (see Fig. 1b). The latter can be evaluated using either time-ordered or covariant perturbation theory, and implies summation over all relevant baryonic intermediate states. An important point is that, because the pion wavelength is large compared to the typical nucleon/baryon radius, only the lowest-mass excitations can effectively contribute to the GPD in the region of Eqs. (2.1)

<sup>1</sup>In covariant perturbation theory soft pions have virtualities  $-k_\pi^2 \sim M_\pi^2$ , and Eq. (2.1) results from the condition that the minimum pion virtuality required by the longitudinal momentum fraction,  $y$ , be of the order  $M_\pi^2$ .

and (2.2). We therefore retain only the  $N$  and  $\Delta$  intermediate states in the sum. The inclusion of the  $\Delta$ , whose mass splitting with the nucleon introduces a non-chiral scale which is numerically comparable to the pion mass, represents a slight departure from strict chiral dynamics but is justified by the numerical importance of this contribution (*cf.* the discussion of the  $N_c \rightarrow \infty$  limit in QCD below). Calculation of the large- $b$  asymptotics of the pion distribution from the graph of Fig. 1b shows exactly the behavior described by Eqs. (2.1) and (2.2) [10]. For the  $N$  intermediate state

$$f_{\pi N}(y, b) \propto e^{-\kappa_{\pi N}(y)b}/b, \quad \text{with } \kappa_{\pi N}(y) \sim M_\pi \text{ for } y \sim M_\pi/M_N, \quad (2.3)$$

*i.e.*, for parametrically small  $y$  the transverse distribution has a ‘‘Yukawa tail’’ with a  $y$ -dependent range of the order  $\sim 1/M_\pi$  (the limiting value of  $\kappa_{\pi N}$  for  $y \rightarrow 0$  is  $2M_\pi$ ). For values  $y \sim 1$  one finds an exponential decay with a range of the order  $\sim 1/M_N$ , which is not a chiral contribution, in agreement with the restriction Eq. (2.1).

The chiral contribution to the nucleon’s parton densities is then obtained as the convolution of the pion momentum distribution in the nucleon with the relevant parton distribution in the pion. For the antiquark flavor asymmetry it takes the form (we suppress the QCD scale dependence)

$$[\bar{d} - \bar{u}](x, b)_{\text{chiral}} = \int_x^1 \frac{dy}{y} \left[ \frac{2}{3} f_{\pi N}(y, b) - \frac{1}{3} f_{\pi \Delta}(y, b) \right] v_\pi(z), \quad z \equiv x/y. \quad (2.4)$$

Here  $f_{\pi N}$  and  $f_{\pi \Delta}$  are the isoscalar pion distributions with  $N$  and  $\Delta$  intermediate states in the conventions of Refs. [8, 10]; the isovector nature of the asymmetry is encoded in the numerical prefactors. Furthermore,  $v_\pi(z)$  denotes the ‘‘valence’’ quark/antiquark distribution in the pion,

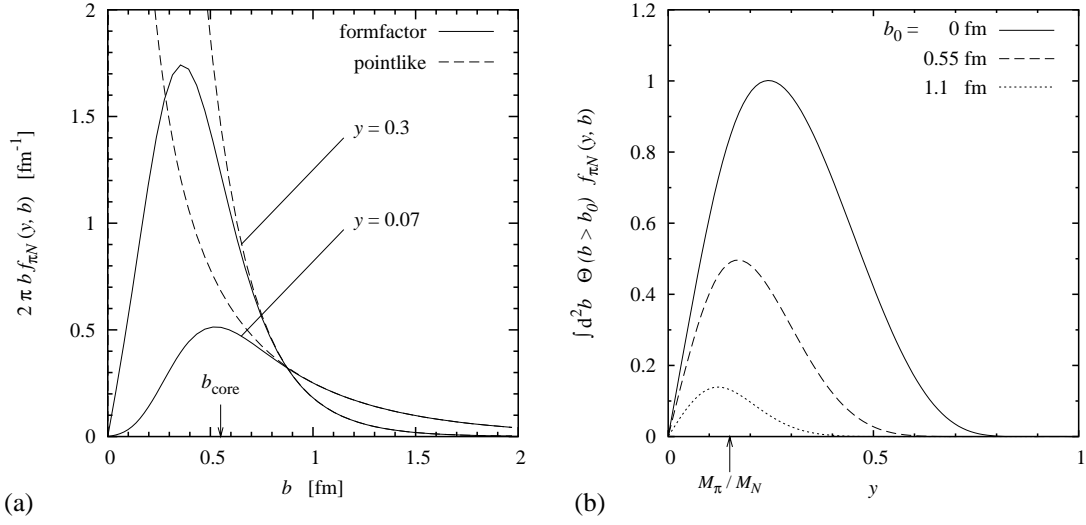
$$v_\pi(z) = [u_{\pi^+} - \bar{u}_{\pi^+}](z) = [\bar{d}_{\pi^+} - d_{\pi^+}](z), \quad \int_0^1 dz v_\pi(z) = 1. \quad (2.5)$$

Equation (2.4) applies to quark momentum fractions of the order  $x \lesssim M_\pi/M_N$ , and transverse distances  $b \sim 1/M_\pi$ . In arriving at Eq. (2.4) we have assumed that the ‘‘decay’’ of the pion into quarks and antiquarks happens locally on the transverse distance scale of the chiral  $b$ -distribution,  $b \sim 1/M_\pi$  (see Fig. 1a). This is justified if  $x$  is not too small, as in this case the antiquark momentum fraction  $z$  in the pion does not reach small values ( $x < z < 1$  in the convolution integral) and one can neglect chiral effects which cause the transverse size of the pion itself to grow at small  $z$ ; such effects were recently studied in Refs. [11] within a novel resummation approach.

### 3. Impact parameter analysis of the pion cloud model

We now turn to the phenomenological pion cloud model of the asymmetry and investigate what part of its predictions actually correspond to the large-distance region governed by universal chiral dynamics. There are several variants of this model, employing different types of form factors to regularize the transverse momentum integral in the pion distribution (*cf.* Fig. 1b). We consider here the scheme in which the spectator baryon is on mass-shell and the pion virtualities are restricted by form factors; the relation to other schemes (invariant mass cutoff in the time-ordered approach) will be discussed elsewhere.

The pion momentum distributions  $f_{\pi N, \pi \Delta}(y, b)$  in this model are calculated by evaluating the pion GPD from the loop integral Fig. 1b with form factors, and performing the transformation to



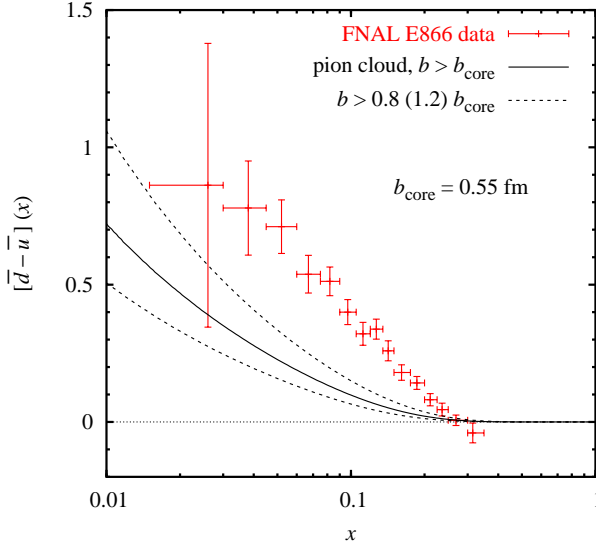
**Figure 2:** (a) The transverse spatial distribution of pions in the nucleon,  $f_{\pi N}(y, b)$ , as a function of  $b$ , for values  $y = 0.07$  and  $0.3$ . Shown is the radial distribution  $2\pi b f_{\pi N}$ , whose integral (area) gives the pion momentum distribution. *Solid lines:* Pion cloud model with virtuality cutoff (exponential form factor,  $\Lambda_{\pi N} = 1.0\text{GeV}$ ) [8]. *Dashed line:* Distribution for pointlike particles, regulated by subtraction at  $\Delta_T^2 = 0$  (the integral over  $b$  does not exist in this case). (b) Momentum distribution of pions in the nucleon. *Solid line:* Full distribution  $f_{\pi N}(y)$  obtained in pion cloud model (exponential form factor,  $\Lambda_{\pi N} = 1.0\text{GeV}$ ). *Dashed line:* Contribution from  $b > b_{\text{core}} = 0.55$  fm. *Dotted line:* Contribution from  $b > 2b_{\text{core}} = 1.1$  fm.

the impact parameter representation. Figure 2a shows  $f_{\pi N}(y, b)$  as a function of  $b$  for  $y = 0.07$  and  $0.3$  (which is  $1/2$  and  $2$  times  $M_\pi/M_N$ , respectively). Also shown are the distributions obtained with pointlike particles (no form factors), in which the loop integral was regularized by subtraction at  $\Delta_T^2 = 0$ ; this subtraction of a  $\Delta_T^2$ -independent term in the GPD corresponds to a modification of the impact parameter distribution by a delta function term  $\propto \delta^{(2)}(\mathbf{b})$ , which is ‘invisible’ at finite  $b$  [10]. One sees that for  $b \gtrsim 0.5$  fm the results of the two calculations coincide, showing that in this region the pion distribution is not sensitive to finite size effects. It is interesting that the  $b$  value where the universal behavior sets in is numerically close to the transverse size of the ‘quark core’ inferred from the nucleon axial form factor,  $b_{\text{core}} = [\frac{2}{3}\langle r^2 \rangle_{\text{axial}}]^{1/2} \approx 0.55$  fm [10]. This indicates that a two-component description of the partonic structure in transverse space, as advocated in Ref. [10], is natural from the numerical point of view. Finally, we note that for large  $b$  both distributions in Fig. 2a exhibit the universal asymptotic behavior derived earlier [10].

We can now quantify which transverse distances contribute to the pion momentum distribution in the pion cloud model with form factors. Figure 2b shows the momentum distribution of pions obtained by integrating  $f_{\pi N}(y, b)$  from a lower cutoff,  $b_0$ , to infinity,

$$\int d^2b \Theta(b > b_0) f_{\pi B}(y, b) \quad (B = N, \Delta). \quad (3.1)$$

Restricting the  $b$  integration to values  $b > b_{\text{core}} = 0.55$  fm strongly suppresses large pion momentum fractions and shifts the distribution toward smaller  $y$ , in agreement with the general considerations described in Sec. 2. Overall, we see that less than half of the pions in the phenomenological pion cloud model arise from the ‘safe’ large-distance region.



**Figure 3:** *Solid line:* Large-distance contribution to the antiquark flavor asymmetry  $[\bar{d} - \bar{u}](x)$ , calculated in the pion cloud model with  $b > b_{\text{core}} = 0.55$  fm, cf. Eq. (3.1). *Dotted lines:* Same with  $b > 0.8 b_{\text{core}}$  (upper line) and  $1.2 b_{\text{core}}$  (lower line). The model parameters in this calculation were chosen as  $\Lambda_{\pi N} = 1.0$  GeV,  $\Lambda_{\pi\Delta} = 0.8$  GeV (exponential cut-off) [8], and the valence quark density in the pion was taken from Ref. [12]. *Data:* FNAL E866 Drell-Yan experiment; see second article of Ref. [3].

#### 4. Spatial decomposition of the flavor asymmetry

The above results allow us to make a first attempt at a spatial decomposition of the antiquark flavor asymmetry in the proton. To this end, we calculate the convolution integral Eq. (2.4) with the  $b$ -integrated pion distribution Eq. (3.1), where the lower limit,  $b_0$ , is taken sufficiently large to exclude the model-dependent small-distance region (see Fig. 2a). Figure 3 shows the result obtained with  $b_0$  taken as the phenomenological “core” radius,  $b_{\text{core}} = 0.55$  fm (solid line), as well as the band covered when  $b_0$  is changed from this value by  $\pm 20\%$  (dotted lines). Also shown in the figure are the final data from the FNAL E866 experiment [3]. One sees that the large-distance contribution to the asymmetry obtained from the pion cloud model is practically zero for  $x > 0.3$ , as expected from the general considerations of Sec. 2.<sup>2</sup> At  $x \sim 0.1$  the large-distance contribution accounts for only  $\sim 30\%$  of the measured asymmetry, indicating that most of it results from the “quark core” at small transverse distances. This conclusion is robust and does not depend on the form factors or other regulators employed in the pion cloud model (see Fig. 2a).

At small  $x$  ( $\sim 0.01$ ) the large-distance contribution in the pion cloud model comes closer to the data; however, the details depend on the precise value of the lower limit in  $b$  (see Figure 3) and partly on the model parameters; also, the quality of the present data is rather poor. It is thus difficult to infer the magnitude of the required “core” contribution by comparing the present estimate of the large-distance contribution to the data in this region of  $x$ .

#### 5. Summary and discussion

The transverse coordinate representation based on GPDs is a most useful framework for studying the role of chiral dynamics in the nucleon’s partonic structure. Parametrically, the chiral contributions are located at momentum fractions  $x \lesssim M_\pi/M_N$  and transverse distances  $b \sim 1/M_\pi$ . We

<sup>2</sup>Without the restriction to  $b > b_{\text{core}}$  the pion cloud model with the given parameters would produce an asymmetry which is non-zero and positive also for  $x > 0.3$ ; however, this could partly be avoided by tuning the  $\pi NN$  and  $\pi N\Delta$  form factors in the model [8].

have shown that the results of the phenomenological pion cloud model become independent of the  $\pi NN$  form factors at transverse distances  $b \gtrsim 0.5$  fm and represent contributions governed by universal large-distance dynamics. The lower limit in  $b$  approximately coincides with the “quark core” radius  $b_{\text{core}} = 0.55$  fm, inferred previously from other phenomenological considerations, suggesting a natural “two-component” description of the partonic structure in transverse coordinate space [10]. Our findings provide a starting point for more detailed modeling of the nucleon’s partonic structure along these lines. A complete analysis should consider also the large-distance contributions to the flavor-singlet sea quark distribution  $\bar{u}(x) + \bar{d}(x)$  and  $SU(3)$  flavor symmetry breaking in the sea [8].

The pion cloud contribution to the flavor asymmetry  $\bar{d}(x) - \bar{u}(x)$  involves strong cancellations between  $\pi N$  and  $\pi \Delta$  intermediate states. A systematic way to deal with this problem is provided by the  $1/N_c$  expansion of QCD. In particular, the degeneracy of  $N$  and  $\Delta$  in the  $N_c \rightarrow \infty$  limit ensures the proper  $1/N_c$  scaling of the pion cloud contribution to the flavor asymmetry [10], showing that the latter is a legitimate part of the nucleon’s partonic structure in large- $N_c$  QCD. The connection between the pion cloud contribution to  $\bar{d}(x) - \bar{u}(x)$  and the dynamical picture of the nucleon as a chiral soliton in the large- $N_c$  limit remains an interesting problem for further study [10].

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