PROCEEDINGS OF SCIENCE

The Higgs sector in the CP-violating MSSM at $\mathcal{O}(\alpha_t\alpha_s)$

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Radiative corrections in the MSSM involving non-vanishing CP-phases can induce mixing between the CP-even and the CP-odd states. In particular, the lightest Higgs boson is not purely CP-even but receives a CP-odd component. Results are presented for the leading two-loop contributions of $\mathcal{O}(\alpha_t\alpha_s)$ to the masses in the Higgs sector of the MSSM with non-vanishing CP-phases. They are derived in the Feynman-diagrammatic approach using on-shell renormalization and contain besides the leading logarithmic also subleading and non-logarithmic corrections.

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1. Introduction

In contrast to the Standard Model the MSSM contains two Higgs doublets ensuring that up- as well as down-type fermions obtain masses via the Higgs mechanism. Besides, the theory remains anomaly-free. In the Higgs sector at the tree-level, two phases can appear. The Higgs potential contains the soft breaking parameter m_{12}, which induces a mixing of components of both Higgs doublets and which is complex in general. This phase can be eliminated via a Peccei-Quinn transformation [1, 2]. The other phase describes a phase difference between the Higgs doublets and vanishes because of the minimum condition for the Higgs potential. Thus, at the Born level, there is no CP-violation in the Higgs sector and it contains two neutral CP-even, H, h, one neutral CP-odd, A, and two charged physical Higgs bosons, $H^{\pm}$. However taking into account radiative corrections can induce CP-violation via complex parameters entering the Higgs boson self-energies [3]. Especially, the lightest Higgs boson is no longer CP-even but may contain CP-odd components. The three neutral Higgs bosons are then labeled according to their masses, $M_{h_1} \leq M_{h_2} \leq M_{h_3}$.

The lightest Higgs boson in the MSSM is of special interest for two reasons. First, it has an upper theoretical mass bound, $M_{h_1} \lesssim 135$ GeV [4], and second, its mass depends on all MSSM parameters via the radiative corrections. Even already today, this allows to combine experimental data and theoretical predicitions and to give exclusion bounds for the MSSM parameter space. After the discovery of the lightest Higgs boson its mass will be an interesting precision observable. An accurate experimental determination and a precise theoretical prediction can put strong indirect limits on the MSSM parameter space.

While for the MSSM with no CP-phases, the mass of the lightest Higgs boson is calculated up to leading 3-loop order [4, 5, 6, 7], the status of the theoretical prediction of M_{h_1} is as follows: Using the effective potential approach the fermionic and sfermionic corrections are calculated up to two-loop leading-log contributions [8]. Also, investigations of the gaugino contributions [9] as well as effects of the imaginary parts of the self-energies [10] were perfomed at the one-loop level. Most recently, M_{h_1} was calculated including the full one-loop and the two-loop corrections of $\mathcal{O}(\alpha_t\alpha_s)$ [11, 12] with $\alpha_t = \lambda_t^2/(4\pi)$, λ_t being the top Yukawa coupling.

2. Determination of the Higgs masses

The values of the Higgs masses are governed by the two-point-function $\hat{\Gamma}$ which is given in terms of the momentum p and the mass matrix $\mathbf{M}$,

$$-i\hat{\Gamma}(p^2) = p^2\,\mathbb{1} - \mathbf{M}(p^2)\ . \tag{2.1}$$

The mass matrix $\mathbf{M}$ contains the Born masses, $M_{\phi_{\text{Born}}}$, and the renormalized self-energies, $\hat{\Sigma}_{\phi\chi}$, $\phi,\chi = H,h,A$,

$$\mathbf{M}(p^2) = \begin{pmatrix} M^2_{H_{\text{Born}}} - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{Hh}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{Hh}(p^2) & M^2_{h_{\text{Born}}} - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{HA}(p^2) & -\hat{\Sigma}_{hA}(p^2) & M^2_{A_{\text{Born}}} - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}\ . \tag{2.2}$$

For vanishing phases, the mixing between the CP-even and the CP-odd states vanish, $\hat{\Sigma}_{HA}(p^2) = \hat{\Sigma}_{hA}(p^2) = 0$.

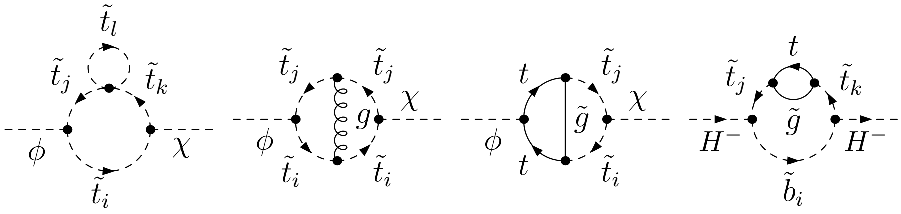

Figure 1: Sample of two-loop diagrams for the Higgs boson self-energies ($\phi,\chi = h,H,A;\ i,j,k,l = 1,2$).

The masses of the Higgs bosons are determined by calculating the zeros of the determinant of $\hat\Gamma$, $\det[p^2\mathbb{1} - \mathbf{M}(p^2)] = 0$. The numerical recipe used is to calculate first the eigenvalues $\lambda(p^2)$ of $\mathbf{M}(p^2)$ and to solve the equation $\lambda(p^2) - p^2 = 0$ iteratively.

3. Renormalized Higgs self-energies

The renormalized self-energies $\hat\Sigma_{\phi\chi}$ can be expressed as a sum of contributions of different loop order $\hat\Sigma^{(i)}_{\phi\chi}$ where (i) denotes the loop order,

$$\hat\Sigma_{\phi\chi} = \hat\Sigma^{(1)}_{\phi\chi} + \hat\Sigma^{(2)}_{\phi\chi} + \ldots \tag{3.1}$$

The calculation of the one-loop part of the self-energy was performed in [11]. Here we review the $\mathcal{O}(\alpha_t\alpha_s)$ contributions to the two-loop Higgs self-energy [12]. A sample of diagrams is shown in fig. 1. These contributions were evaluated using the approximation of vanishing electroweak gauge couplings and vanishing external momenta. In this approximation, the renormalized two-loop self-energies containing the unrenormalized Σ and the counterterm part $\delta m^{2(2)}_{\phi\chi}$ have the following form:

$$\hat\Sigma^{(2)}_{\phi\chi}(0) = \Sigma^{(2)}_{\phi\chi}(0) - \delta m^{2(2)}_{\phi\chi} \tag{3.2}$$

where $\delta m^{2(2)}_{\phi\chi}$ is a function of the tadpole counterterms $\delta T^{(2)}_{\phi}$ and the counterterm of the charged Higgs mass square $\delta m^{2(2)}_{H^\pm}$. The charged Higgs mass square is defined via an on-shell description yielding at the two-loop level within the used approximations:

$$\hat\Sigma^{(2)}_{H^+H^-}(0) = 0 \Rightarrow \delta m^{2(2)}_{H^\pm} = \Sigma^{(2)}_{H^+H^-}(0)\ . \tag{3.3}$$

The tadpoles are fixed by the requirement that the minimum of the Higgs potential is not shifted yielding the following counterterms at the two-loop level:

$$T^{(2)}_{\phi} + \delta T^{(2)}_{\phi} = 0 \Rightarrow \delta T^{(2)}_{\phi} = -T^{(2)}_{\phi}\ ,\ \phi = h,H,A\ . \tag{3.4}$$

The parameters of the (s)top and the (s)bottom sector have to be defined at the one-loop level. The top (s)quark masses are defined on-shell. The $\tilde b_1$-mass (= $\tilde b_L$-mass in the used limit of vanishing bottom quark mass) is determined by the $SU(2)$ relation so that $m_{\tilde b_1} = m_{\tilde b_1}(m_{\tilde t_1}, m_{\tilde t_2}, m_t, \theta_{\tilde t}, \varphi_{\tilde t})$. To fix the stop mixing angle and the corresponding phase the following condition is used:

$$\widetilde{\mathrm{Re}}\hat\Sigma_{\tilde t_{12}}(m^2_{\tilde t_1}) + \widetilde{\mathrm{Re}}\hat\Sigma_{\tilde t_{12}}(m^2_{\tilde t_2}) = 0\ \Rightarrow\ (\delta\theta_{\tilde t} + i\sin\theta_{\tilde t}\cos\theta_{\tilde t}\,\delta\varphi_{\tilde t})e^{i\varphi_{\tilde t}} = \frac{\widetilde{\mathrm{Re}}\Sigma_{\tilde t_{12}}(m^2_{\tilde t_1}) + \widetilde{\mathrm{Re}}\Sigma_{\tilde t_{12}}(m^2_{\tilde t_2})}{2(m^2_{\tilde t_1} + m^2_{\tilde t_2})} \tag{3.5}$$

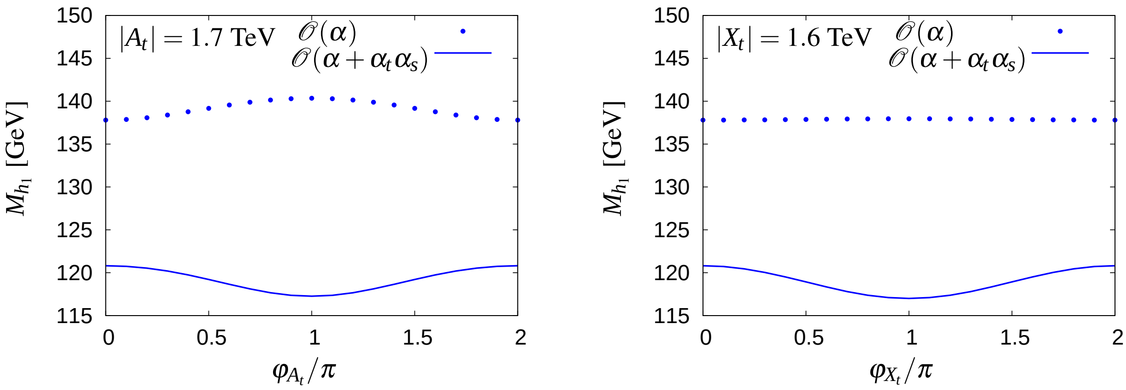

Figure 2: The lightest Higgs boson mass M_{h_1} as a function of φ_{A_t} and φ_{X_t} at the one- and two-loop level (see text). The other parameters are: $M_{H^\pm} = 500$ GeV, $\tan\beta = 10$, $M_{\rm SUSY} = M_3 = \mu = A_{f\neq t} = 1000$ GeV, $M_2 = 500$ GeV, $M_1 = 5/3\,\sin^2\theta_W/\cos^2\theta_W\,M_2$.

which is a generalization of the condition for real parameters used in [13]. The trilinear coupling A_t is then determined by a combination of the other parameters of the (s)top sector.

The two-loop Feynman diagrams were generated using the program `FeynArts` [14]. For the evaluation of traces and the tensor reduction the program `TwoCalc` [15] was used.

4. Numerical results

The two-loop corrections of $\mathscr{O}(\alpha_t\alpha_s)$ are implemented into the program `FeynHiggs2.6` [4, 5, 12]. For numerical examples see fig. 2 and fig. 3.

In fig. 2, a comparison of the M_{h_1}-dependence on the phase of the trilinear coupling, φ_{A_t}, (left plot) and on the phase of the stop mixing, φ_{X_t}, (right plot) is shown – the stop mixing is defined as $X_t = A_t - \mu^*\cot\beta$. The absolute value of A_t and X_t is chosen such that for vanishing phases the value of A_t is the same in both plots. The dotted curves depict the one-loop corrected Higgs mass while the Higgs mass M_{h_1} including contributions of $\mathscr{O}(\alpha_t\alpha_s)$ is shown as the drawn-through curves. The dependence of the one-loop corrected mass on φ_{X_t} is much smaller than on φ_{A_t}. For this specific example, the mass dependence including the two-loop corrections is similar in both plots while in general, the dependence on φ_{X_t} tends to be smaller than on φ_{A_t}, also in the two-loop case. This is mainly due to the fact that varying φ_{A_t} also changes the size of the stop masses while a change of φ_{X_t} keeps the stop masses constant.

In fig. 3, the M_{h_1}-dependence on φ_{X_t} is shown again, but for a smaller mass of the charged Higgs boson $M_{H^\pm}$. As in fig. 2, the dotted curves depict the one-loop corrected Higgs mass and the drawn-through curves the Higgs mass including two-loop contributions of $\mathscr{O}(\alpha_t\alpha_s)$. For small $M_{H^\pm}$, also the one-loop corrected Higgs mass shows a clear dependence on φ_{X_t}.

For the MSSM with real parameters, `FeynHiggs2.6` includes also the two-loop corrections of $\mathscr{O}(\alpha_t^2, \alpha_b\alpha_s, \alpha_t\alpha_b, \alpha_b^2)$ [7] in the evaluation of the Higgs boson self-energies. Only the leading-log part of these corrections are known for complex parameters [8]. The bands in fig. 3 yield an estimate of the size of these contributions including non-logarithmic terms. The boundaries of the

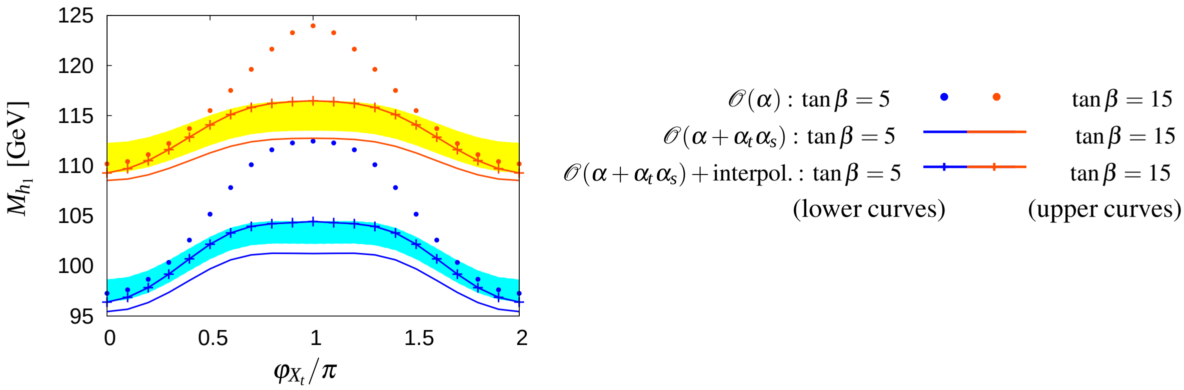

Figure 3: The M_{h_1}-dependence on φ_{X_t} at the one- and two-loop level (see text). The other parameters are: $M_{\rm SUSY} = M_3 = M_2 = 500$ GeV, $M_1 = 250$ GeV, $\mu = A_{f\neq t} = 1000$ GeV, $M_{H^\pm} = 150$ GeV, $|X_t| = 700$ GeV.

bands are given as

$$M_{h_1}^{\rm boundary}(\varphi_{X_t}) = M_{h_1}^{\rm corr.}(\varphi_{X_t}) + \Delta M_{h_1}(\varphi_{\rm boundary}) \qquad (4.1)$$

where $M_{h_1}^{\rm boundary}$ with $\varphi_{\rm boundary} = 0$ and $\varphi_{\rm boundary} = \pi$ respectively gives the lower and the upper boundary of the bands – in our numerical example is $\Delta M_{h_1}(0) \leqslant \Delta M_{h_1}(\pi)$. $M_{h_1}^{\rm corr.}$ denotes the mass of the lightest Higgs boson including the full one-loop and the two-loop $\mathcal{O}(\alpha_t\alpha_s)$ corrections. ΔM_{h_1} gives the size of the $\mathcal{O}(\alpha_t^2, \alpha_b\alpha_s, \alpha_t\alpha_b, \alpha_b^2)$ contributions. The crossed curve shows M_{h_1}, taking into account the one-loop and two-loop $\mathcal{O}(\alpha_t\alpha_s)$ corrections and interpolating ΔM_{h_1}, i.e. the contributions of $\mathcal{O}(\alpha_t^2, \alpha_b\alpha_s, \alpha_t\alpha_b, \alpha_b^2)$. These crossed curves lie between the lower and the upper boundary of the band which shows that the interpolation procedure is working well.

5. Conclusions

Quantum corrections can induce CP-violation in the MSSM Higgs sector. Their effects can be sizeable and they have to be taken into account for the prediction of Higgs masses. The two-loop $\mathcal{O}(\alpha_t\alpha_s)$ contributions with the complete phase dependence are included into `FeynHiggs2.6`.

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