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NNLO QCD analysis of the virtual photon structure functions

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The next-to-next-to-leading order (NNLO) QCD analysis is performed for the virtual photon structure functions which can be measured in the double-tag events in two-photon processes in e^+e^- collisions. We investigate the perturbative QCD evaluation of $F_2^\gamma(x,Q^2,P^2)$ to NNLO and $F_L^\gamma(x,Q^2,P^2)$ to NLO with and without taking into account the target mass effects, which are relevant for the large x region. We also carry out the phenomenological analysis for the experimentally accessible effective structure function $F_{\rm eff}^\gamma = F_2^\gamma + (3/2)F_L^\gamma$.

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1. Introduction

The perturbative QCD is now in a stage of improving its predictions more precise, in order to perform the accurate estimation of the strong interaction effects at LHC processes. What I would like to discuss in this talk is the next-to-next-to-leading order QCD analysis of the virtual photon structure functions. We consider the photon structure functions F_2^γ and F_L^γ which can be measured in the two-photon process in e^+e^- collision at high energies. In the future new kinematical regime is expected to be available in the linear collider, ILC. Here we investigate the double-tag events where both of the outgoing e^+ and e^- are detected, then one can study the virtual photon structure functions, which describe the deep inelastic scattering off the virtual photon. In particular we consider the kinematical region in which one of the photon ('probe'photon) is far off-shell while the other one ('target'photon) is less off-shell but much bigger than the QCD scale parameter Λ:

$$\Lambda^2 \ll P^2 \ll Q^2 \ , \tag{1.1}$$

where Q^2 (P^2) is the mass squared of the probe (target) photon. In this situation the structure functions are perturbatively calculable not merely Q^2 dependence but also the shape and magnitude.

In the framework based on the operator product expansion (OPE) supplemented by the renormalization (RG) group method, Witten [1] obtained the leading order (LO) QCD contributions to F_2^γ and F_L^γ and, shortly after, the next-to-leading order (NLO) QCD corrections to F_2^γ were calculated by Bardeen and Buras [2]. The structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$ for the case of a virtual photon target ($P^2 \neq 0$)were studied in the LO and in the NLO by pQCD [3]. In ref.[4], we have studied the unpolarized virtual photon structure functions, $F_2^\gamma(x,Q^2,P^2)$ up to the NNLO and $F_L^\gamma(x,Q^2,P^2)$ up to the NLO, in pQCD for the kinematical region (1.1). In this talk we present the perturbative QCD evaluation of the above structure functions with and without taking into account the target mass effects. We also discuss the experimentally accessible effective structure function $F_{\rm eff}^\gamma = F_2^\gamma + (3/2)F_L^\gamma$.

2. Virtual photon structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$

We analyze the virtual photon structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$ using the theoretical framework based on the OPE and the RG method. The absorptive part of the forward virtual photon scattering amplitude for $\gamma(q)+\gamma(p)\to\gamma(q)+\gamma(p)$ is related to the structure tensor:

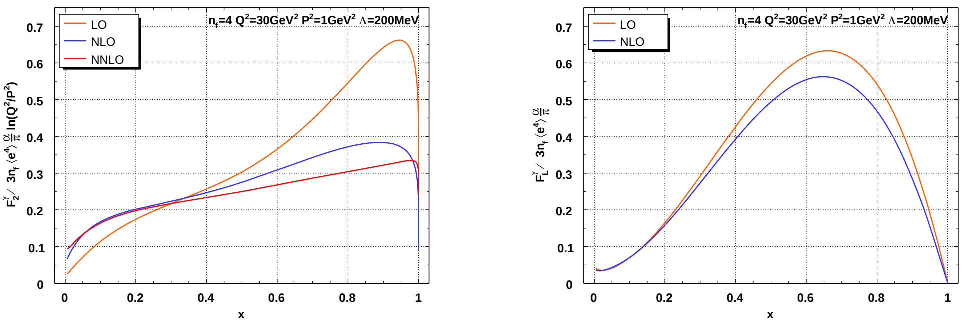
$$W_{\mu\nu}^\gamma(p,q) = \frac{1}{2}\int d^4x e^{iqx}\langle\gamma(p)|J_\mu(x)J_\nu(0)|\gamma(p)\rangle_{\rm spin\ av.}\ , \tag{2.1}$$

which is expressed in terms of two independent structure functions

$$W_{\mu\nu}^\gamma = e_{\mu\nu}\left\{\frac{1}{x}F_L^\gamma + \frac{p^2q^2}{(p\cdot q)^2}\frac{1}{x}F_2^\gamma\right\} + d_{\mu\nu}\frac{1}{x}F_2^\gamma\ , \tag{2.2}$$

where we have kept the target mass squared $p^2 = -P^2$ and the two tensor structures are given by

$$e_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad d_{\mu\nu} \equiv -g_{\mu\nu} + \frac{p_\mu q_\nu + p_\nu q_\mu}{p\cdot q} - \frac{p_\mu p_\nu q^2}{(p\cdot q)^2} \tag{2.3}$$



(a) (b)

Figure 1: (a)$F_2^\gamma(x,Q^2,P^2)$ to NNLO and (b) $F_L^\gamma(x,Q^2,P^2)$ to NLO for $Q^2=30\mathrm{GeV}^2$, $P^2=1\mathrm{GeV}^2$ with $\Lambda=0.2\mathrm{GeV}$.

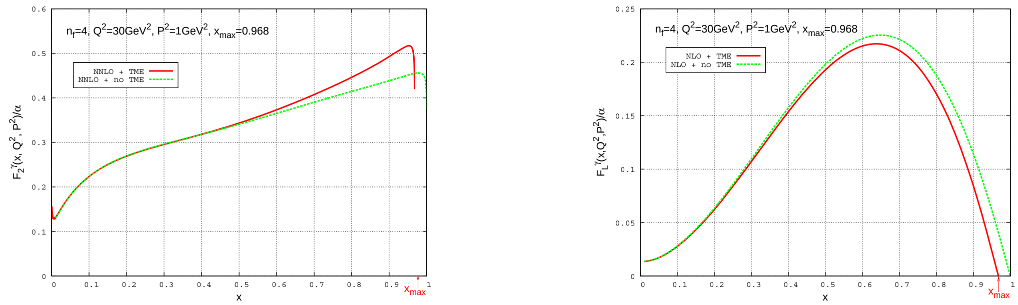
and x is the Bjorken variable defined by $x=Q^2/2p\cdot q$ with $q^2=-Q^2$.

The n-th moment of $F_2^\gamma(x,Q^2,P^2)$ up to NNLO without the target mass effects is given as

$$\begin{aligned}
M_{2,n}^\gamma(Q^2,P^2) &\equiv \int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2,P^2)\\
&= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{\frac{4\pi}{\alpha_s(Q^2)}\sum_i \mathscr{L}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\sum_i \mathscr{A}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right]\right.\\
&\quad +\sum_i \mathscr{B}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\mathscr{C}^n+\frac{\alpha_s(Q^2)}{4\pi}\left(\sum_i \mathscr{D}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n-1}\right]\right.\\
&\quad \left.\left.+\sum_i \mathscr{E}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right]+\sum_i \mathscr{F}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]+\mathscr{G}^n\right)+\mathscr{O}(\alpha_s^2)\right\}, \quad (2.4)
\end{aligned}$$

where the index i runs over $+,-,NS$ and $d_i^n=\lambda_i^n/2\beta_0$ and λ_i^n denotes the eigenvalues of 1-loop anomalous dimension matrices. The terms with $\mathscr{L}_i^n$ are the LO (α/α_s) contributions [1, 3]. The NLO (α) corrections are the terms with $\mathscr{A}_i^n$, $\mathscr{B}_i^n$ and $\mathscr{C}^n$ [2, 3]. The coefficients $\mathscr{D}_i^n$, $\mathscr{E}_i^n$, $\mathscr{F}_i^n$ and $\mathscr{G}^n$ give the NNLO $(\alpha\alpha_s)$ corrections and they are new. The explicit expressions of $\mathscr{D}_i^n$, $\mathscr{E}_i^n$, $\mathscr{F}_i^n$ and $\mathscr{G}^n$ are given in Eqs.(2.34)-(2.37) of Ref.[4]. For the 3-loop anomalous dimensions, we could use the recently calculated results of the three-loop anomalous dimensions for the quark and gluon operators and of the three-loop photon-quark and photon-gluon splitting functions [5]. For the longitudinal strucuture function $F_L^\gamma(x,Q^2,P^2)$ we can similarly derive the n-th moment up to NLO. The NLO terms are $(\alpha\alpha_s)$ corrections which are new results, given in Eqs.(6.3)-(6.8) of Ref.[4].

The LO, NLO and NNLO QCD results, as well as the box contribution, for the case of $F_2^\gamma(x,Q^2,P^2)$ $\left(F_L^\gamma(x,Q^2,P^2)\right)$ at $Q^2=30$ GeV^2 and $P^2=1$ GeV^2 with $n_f=4$, are shown in Fig.1(a) (Fig.1.(b)). For F_2^γ we observe that there exist notable NNLO QCD corrections at larger x. The corrections are negative and the NNLO curve comes below the NLO one for $0.3\lesssim x<1$. We see from Fig.1(b) that the NLO QCD corrections for F_L^γ are negative and the NLO curve comes below the LO one in the region $0.2\lesssim x<1$.



(a) (b)

Figure 2: (a) $F_2^\gamma(x,Q^2,P^2)$ with TME to NNLO and (b) $F_L^\gamma(x,Q^2,P^2)$ with TME to NLO

3. Target mass effects

If the target is a real photon ($P^2 = 0$), there is no need to consider target mass corrections. But when the target becomes off-shell, and for relatively low Q^2, we need to take into account target mass effects (TME). TME is important also by another reason. For the virtual photon target, the maximal value of the Bjorken variable x is not 1 but $x_{\max} = 1/(1+\frac{P^2}{Q^2})$, due to the constraint $(p+q)^2 \geq 0$, which is in contrast to the nucleon case where $x_{\max} = 1$. The structure functions should vanish at $x = x_{\max}$. However the numerical analysis in Fig.1 shows that the predicted graphs do not vanish but remains finite at $x = x_{\max}$. This flaw is due to the fact that TME have not been taken into account in the analysis. TME can be treated by considering the Nachtmann moments [6] to extract the definite spin contribution by expanding the amplitude in Gegenbauer polynomials. The Nachtmann moments for the definite spin-n contributions, $M_{2,n}^\gamma$ and $M_{L,n}^\gamma$ are given by [7]

$$\mu_{2,n}^\gamma(Q^2,P^2) \equiv \int_0^{x_{\max}} dx \frac{1}{x^3}\xi^{n+1}\left[\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\right] F_2^\gamma(x,Q^2,P^2) = M_{2,n}^\gamma \qquad (3.1)$$

$$\mu_{L,n}^\gamma(Q^2,P^2) \equiv \int_0^{x_{\max}} dx \frac{1}{x^3}\xi^{n+1}\Big[F_L^\gamma(x,Q^2,P^2) + \frac{4P^2x^2}{Q^2}\frac{(n+3)-(n+1)\xi^2P^2/Q^2}{(n+2)(n+3)}F_2^\gamma(x,Q^2,P^2)\Big] = M_{L,n}^\gamma. \qquad (3.2)$$

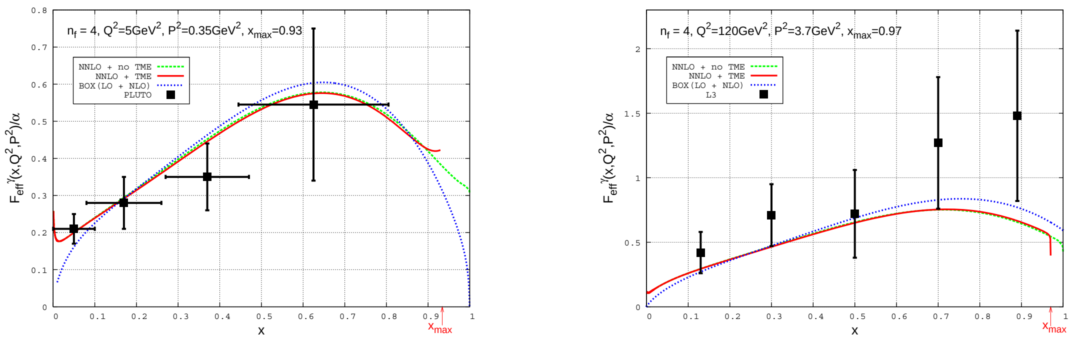
where ξ is the so-called ξ-scaling variable: $\xi = 2x/(1+r)$, with $r = \sqrt{1-4P^2x^2/Q^2}$. Inverting the Nachtmann moments to get the structure function $F_{2(L)}^\gamma(x,Q^2,P^2)$ as a function of x, we have

$$F_2^\gamma(x,Q^2,P^2) = \frac{x^2}{r^3}F(\xi) - 6\kappa\frac{x^3}{r^4}H(\xi) + 12\kappa^2\frac{x^4}{r^5}G(\xi) \qquad (3.3)$$

$$F_L^\gamma(x,Q^2,P^2) = \frac{x^2}{r}F_L(\xi) - 4\kappa\frac{x^3}{r^2}H(\xi) + 8\kappa^2\frac{x^4}{r^3}G(\xi) \qquad (3.4)$$

where $\kappa = P^2/Q^2$. The four functions $F(\xi)$, $H(\xi)$, $G(\xi)$, and $F_L(\xi)$ are given by the inverse Mellin transforms of the moments $M_{2,n}^\gamma$ and $M_{L,n}^\gamma$ divided by certain powers of n as given in Ref.[7].

We have plotted the F_2^γ (F_L^γ) with and without TME in Fig.2(a) (Fig2.(b)) for $Q^2 = 30\text{GeV}^2$, $P^2 = 1\text{GeV}^2$. We observe that TME become sizable at larger x region. While TME enhances F_2^γ at larger x, it reduces F_L^γ. In fact, F_2^γ becomes maximum at x very close to the maximal value of x, $x_{\max}$ (1) for the case with (without) TME. In the case of F_L^γ the maximum is attained at middle x.



(a) (b)

Figure 3: (a) $F_{\rm eff}^{\gamma}(x,Q^2,P^2)$ and data from PLUTO [8] and (b) $F_{\rm eff}^{\gamma}(x,Q^2,P^2)$ and data from L3 [9].

4. Concluding remarks

Finally let us compare our theoretical prediction for the virtual photon structure functions with the existing experimental data. In Fig.3(a) and Fig.3(b), we have plotted the experimental data from PLUTO Collaboration [8] and also those from L3 Collaboration [9] on the so-called "effective photon structure function" defined as $F_{\rm eff}^{\gamma} = F_2^{\gamma} + \frac{3}{2}F_L^{\gamma}$, together with the theoretical predictions, NNLO QCD with and without TME and also Box diagram calculations. Although the experimental error bars are rather large, the data are considered to be roughly consistent with the theoretical expectations, except for the larger x region in the case of L3 data. In the present analysis, we have treated the active flavors as massless quarks, and ignored the mass effects of the heavy flavors, which should remain as a future subject. We should also investigate the power corrections due to the higher-twist effects as well as possible resummation of large logs as x approaches $x_{\rm max}$.

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