PROCEEDINGS OF SCIENCE

# Four-fermion production near the W-pair production threshold

**Pietro Falgari**[*]
*Institut für Theoretische Physik E, RWTH-Aachen*
*E-mail:* falgari@physik.rwth-aachen.de

I report on recent results for the total production cross section of the process $e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}X$ near the $W$-pair production threshold up to next-to-leading order in $\Gamma_W/M_W \sim \alpha \sim v^2$ obtained in the framework of unstable-particle effective field theory. Remaining theoretical uncertainties and their impact on the experimental determination of the $W$ mass are discussed.

*8th International Symposium on Radiative Corrections*
*October 1-5, 2007*
*Florence, Italy*

[*]Speaker.

 http://pos.sissa.it/

PoS(RAD COR 2007)003

## 1. Introduction

An accurate measurement of the $W$ mass is of primary interest for precision tests of the Standard Model and for search of New-Physics effects through virtual-particle exchange. The total error on $M_W$ could be lowered to 6 MeV by measuring the four-fermion production cross section near the $W$-pair production threshold [1] at a future International Linear Collider (ILC), provided that the theoretical uncertainties are well below 1%. This is a difficult task, requiring gauge-invariant inclusion of finite-width effects and calculation of QCD and electroweak radiative corrections to the full $2 \to 4$ process. Previous NLO calculations in the double-pole approximation [2] were supposed to break down near threshold for kinematical reasons. The recent computation of the complete NLO corrections to $e^-e^+ \to 4f$ in the complex-mass scheme [3] is valid both near threshold and in the continuum, but is technically difficult, requiring the computation of one-loop six-point functions.

Here I present NLO results for the total cross section of the process

$$e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}X \tag{1.1}$$

near the $W$-pair production threshold [4] computed with effective field theory (EFT) techniques [5, 6, 7]. Section 2 reviews briefly the formalism, while the calculation of the Born cross section and of radiative corrections is outlined in Sections 3 and 4. Section 5 presents numerical results together with an estimate of the remaining theoretical uncertainties and a comparison with [3].

## 2. Unstable-particle effective field theory

The EFT approach [7] exploits the hierarchy of scales $M\Gamma \ll M^2$ which characterizes processes involving unstable particles, $M$ and $\Gamma$ being the mass and width of the intermediate resonance. The degrees of freedom of the full theory are classified according to their scaling into short-distance ($k^2 \sim M^2$) and long-distance ($k^2 \lesssim M\Gamma$) modes. The fluctuations at the small scale (resonant particles, soft and Coulomb photons,...) represent the field content of the effective Lagrangian $\mathscr{L}_{\text{eff}}$. "Hard" fluctuations with $k^2 \sim M^2$ are not part of the effective theory and are integrated out. Their effect is included in $\mathscr{L}_{\text{eff}}$ through short-distance matching coefficients, computed in standard *fixed-order* perturbation theory. The systematic inclusion of finite-width effects is relevant for modes with virtuality $k^2 \lesssim M\Gamma$ and is obtained through complex short-distance coefficients in $\mathscr{L}_{\text{eff}}$ [7].

The specific process (1.1) is primarily mediated by production of a pair of resonant $W$s. The total cross section is extracted from appropriate cuts of the forward-scattering amplitude [4], which after integrating out the hard modes with $k^2 \sim M_W^2$ reads [7]

$$i\mathscr{A} = \sum_{k,l} \int d^4x \,\langle e^-e^+|\mathrm{T}[i\mathscr{O}_p^{(k)\dagger}(0)\, i\mathscr{O}_p^{(l)}(x)]|e^-e^+\rangle + \sum_k \langle e^-e^+|i\mathscr{O}_{4e}^{(k)}(0)|e^-e^+\rangle. \tag{2.1}$$

The operators $\mathscr{O}_p^{(l)}$ ($\mathscr{O}_p^{(k)\dagger}$) in the first term on the right-hand side of (2.1) produce (destroy) a pair of non-relativistic resonant $W$ bosons. The second term accounts for the remaining non-resonant contributions. The computation of $\mathscr{A}$ is split into the determination of the matching coefficients of the operators $\mathscr{O}_p^{(l)}$, $\mathscr{O}_{4e}^{(k)}$ and the calculation of the matrix elements in (2.1). Both quantities are computed as power series in the couplings $\alpha$, $\alpha_s$, the ratio $\Gamma_W/M_W$ and the non-relativistic velocity of the intermediate resonant $W$ pair $v^2 \equiv (\sqrt{s}-2M_W)/(2M_W)$, collectively referred to as $\delta \sim \alpha_s^2 \sim \alpha \sim \Gamma_W/M_W \sim v^2$.



PoS(RAD COR 2007)003

The effective Lagrangian describing the non-relativistic $W$ bosons up to NLO in $\delta$ is [6]

$$\mathscr{L}_{\rm NRQED} = \sum_{a=\mp}\left[\Omega_a^{\dagger i}\left(iD^0 + \frac{\vec{D}^2}{2M_W} - \frac{\Delta}{2}\right)\Omega_a^i + \Omega_a^{\dagger i}\,\frac{(\vec{D}^2 - M_W\Delta)^2}{8M_W^3}\,\Omega_a^i\right]. \tag{2.2}$$

$\Delta$ is the matching coefficient $\Delta \equiv (\bar{s} - M_W^2)/M_W$, where $\bar{s}$ is the complex pole of the $W$ propagator. The field $\Omega_\pm^i = \sqrt{2M_W}W_\pm^i$ describes the three physical polarizations of non-relativistic $W$s, and the covariant derivative $D_\mu\Omega_\pm = (\partial_\mu \mp ieA_\mu)\Omega_\pm$ contains the interaction of the resonant fields $\Omega_\pm$ with *soft* and *potential* photons (see Section 4). To complete $\mathscr{L}_{\rm eff}$ one has to add to (2.2) the effective production vertices $\mathscr{O}_p^{(l)}$ and the four-fermion operators $\mathscr{O}_{4e}^{(k)}$ with the corresponding matching coefficients computed to the desired order in $\delta$. These are presented in Sections 3 and 4.

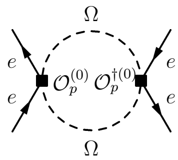
## 3. EFT approximation to the Born cross section

The lowest-order production operator of two non-relativistic resonant $W$s is [6]

$$\mathscr{O}_p^{(0)} = \frac{\pi\alpha_{ew}}{M_W^2}\left(\bar{e}_{c_2,L}(\gamma^i n^j + \gamma^j n^i)e_{c_1,L}\right)\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right). \tag{3.1}$$

Its matching coefficient is extracted from the *on-shell* process $e^-e^+ \to W^-W^+$, where "on-shell" means $k^2 = \bar{s}$. The four-fermion operators $\mathscr{O}_{4e}^{(k)}$ do not contribute to $\mathscr{A}$ at this order, and the forward-scattering amplitude is simply

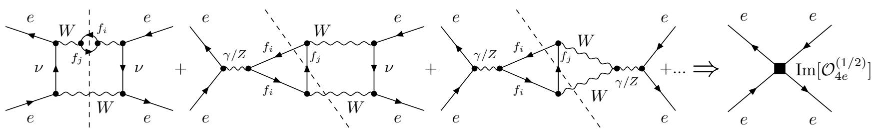
$$i\mathscr{A}^{(0)} = \int d^4x \langle e^-e^+|T[i\mathscr{O}_p^{(0)\dagger}(0)i\mathscr{O}_p^{(0)}(x)]|e^-e^+\rangle =$$




$$= -\frac{i\pi\alpha^2}{s_w^4}\sqrt{-\frac{E + i\Gamma_W^{(0)}}{M_W}}, \tag{3.2}$$

with $E = \sqrt{s} - 2M_W$ and $s_w = \sin\theta_W$. The total cross section for (1.1) is extracted from appropriate cuts of (3.2). At lowest order this is correctly done by multiplying the imaginary part of $\mathscr{A}^{(0)}$ with the LO branching ratios of the decays $W^- \to \mu^-\bar{\nu}_\mu$, $W^+ \to u\bar{d}$, so that $\sigma^{(0)} = \frac{1}{27s}\,{\rm Im}\,\mathscr{A}^{(0)}$.

Beyond the leading term $\sigma^{(0)}$ there are contributions which can be identified with terms of the expansion in $\delta$ of a full-theory Born result computed with a fixed-width prescription. The first class of corrections arises from *four-electron operators* in (2.1). The imaginary part of their matching coefficients are extracted from suitable cuts of *hard* two-loop SM diagrams [4]:




(3.3)

Compared to the LO cross section $\sigma^{(0)} \sim \alpha^2\sqrt{\delta}$ the new term is suppressed by $\alpha/\sqrt{\delta} \sim \sqrt{\delta}$ and is denoted as "$\sqrt{\rm N}$LO". True NLO contributions to $\mathscr{A}^{(0)}$ arise from *higher-dimension production operators* and *propagator corrections*. The former come from the matching of the effective theory on the on-shell process $e^-e^+ \to W^-W^+$ at order $v$ ($\mathscr{O}_p^{(1/2)}$) and $v^2$ ($\mathscr{O}_p^{(1)}$) [6]. The latter correspond to the term $(\vec{\partial}^2 - M_W\Delta)^2/(8M_W^3)$ in (2.2). A comparison of the EFT Born approximations with the full result computed with Whizard [8] shows a good convergence of the series [4]. However partial inclusion of N$^{3/2}$LO corrections is necessary to obtain an agreement of $\sim 0.1\%$ at 170 GeV and $\sim 10\%$ at 155 GeV [4].

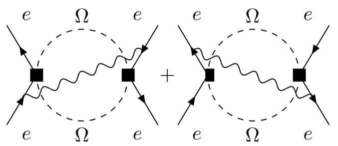
## 4. Radiative corrections

A complete NLO prediction must include radiative corrections to the Born result. These are electroweak and QCD corrections to the matching coefficient of $\mathscr{O}_p^{(0)}$ and loop contributions to the EFT matrix elements. At NLO the flavor-specific final state is selected by multiplying the total cross section with NLO branching ratios. The $O(\alpha)$ correction to the matching coefficient of (3.1) is obtained from the one-loop amplitude of $e^-e^+ \to W^-W^+$. Many of the 180 one-loop diagrams do not contribute due to threshold kinematics and the result reads [4]:

$$C_p^{(1)} = \frac{\alpha}{2\pi}\left[\left(-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}\right)\left(-\frac{4M_W^2}{\mu^2}\right)^{-\varepsilon} + c_p^{(1,\text{fin})}\right] \tag{4.1}$$

The one-loop corrections to the matrix elements arise from exchange of *potential* ($(q_0, |\vec{q}|) \sim M_W(\delta, \sqrt{\delta})$) and *soft* ($(q_0, |\vec{q}|) \sim M_W(\delta, \delta)$) photons. Loops containing $n$ potential photons are enhanced by inverse powers of $v$, $\Delta\mathscr{A} \sim \mathscr{A}^{(0)}\alpha^n v^{-n} \sim \mathscr{A}^{(0)}\alpha^{n/2}$, so that the first and second Coulomb corrections must be included in a NLO calculation. Near threshold they amount respectively to $\sim 5\%$ and $\sim 0.2\%$ of $\sigma^{(0)}$ [4].

Two-loop diagrams with soft photons connecting different hard subprocesses of (3.1) give the so-called *non-factorizable* corrections. As a consequence of the residual gauge-invariance of $\mathscr{L}_{\text{eff}}$, and in agreement with previous results [9], only the initial-initial state interferences survive:




$$= \frac{4\pi^2\alpha^2}{s_w^4 M_W^2}\frac{\alpha}{\pi}\int \frac{d^d r}{(2\pi)^d}\frac{1}{\eta_-\eta_+}\left[\left(\frac{1}{\varepsilon^2}+\frac{5}{12}\pi^2\right)\left(-\frac{2\eta_-}{\mu}\right)^{-2\varepsilon}\right], \tag{4.2}$$

with $\eta_- = r_0 - \frac{|\vec{r}|^2}{2M_W} + i\frac{\Gamma_W^{(0)}}{2}$ and $\eta_+ = E - r_0 - \frac{|\vec{r}|^2}{2M_W} + i\frac{\Gamma_W^{(0)}}{2}$.

## 5. Results and remaining theoretical uncertainties

Because of the approximation $m_e = 0$, the sum of the corrections calculated in Section 4 is not infrared safe, containing uncanceled $\varepsilon$-poles. The result should be convoluted with $\overline{\text{MS}}$ electron distribution functions after minimal subtraction of the pole. Since the distributions available in the literature are computed in a different scheme, which assumes $m_e$ as infrared regulator, it is more convenient to convert our result from $\overline{\text{MS}}$ to this scheme. This is done by adding contributions from the *hard-collinear* ($k^2 \sim m_e^2$) and *soft-collinear* ($k^2 \sim m_e^2 \frac{\Gamma_W}{M_W}$) regions. These cancel the $\varepsilon$-poles, but introduce large logs of $2M_W/m_e$ [4]. The large logs are resummed by convoluting the NLO cross section with the structure functions $\Gamma_{ee}^{\text{LL}}$ used in [2] after subtracting the double counting terms [4]. Since only leading logs are resummed in $\Gamma_{ee}^{\text{LL}}$, one can equivalently choose to convolute only the Born cross section with the structure functions, as done for example in [3], the difference being formally NLL. Fig. 1 shows the percentual correction to the Born result due to initial-state radiation alone (solid black), full NLO corrections with ISR improvement of the Born cross-section only (dot-dashed red), and complete NLO corrections with full ISR improvement (dashed blue). The contribution of genuine electroweak and QCD corrections amounts to $\sim 8\%$ at threshold. It must also be noted that the difference between the two implementations of ISR is numerically important, reaching $\sim 2\%$ at threshold. A comparison of the EFT approximation with [3] reveals a discrepancy which is never larger than $\sim 0.6\%$ in the range $161\,\mathrm{GeV} < \sqrt{s} < 170\,\mathrm{GeV}$. More precisely we have for the full calculation $\sigma_{4\mathrm{f}}(161\,\mathrm{GeV}) = 118.12(8)\,\mathrm{fb}$, $\sigma_{4\mathrm{f}}(170\,\mathrm{GeV}) = 401.8(2)\,\mathrm{fb}$ [3], while in the EFT one obtains $\sigma_{\mathrm{EFT}}(161\,\mathrm{GeV}) = 117.38(4)\,\mathrm{fb}$, $\sigma_{\mathrm{EFT}}(170\,\mathrm{GeV}) = 399.9(2)\,\mathrm{fb}$ [4].

The dominant remaining theoretical uncertainty comes from an incomplete NLL treatment of ISR. This translates into an uncertainty on the $W$ mass of $\sim 31\,\mathrm{MeV}$ [4]. Further uncertainties come from $\mathrm{N}^{3/2}\mathrm{LO}$ corrections in the EFT. The missing $O(\alpha)$ corrections to the four-electron operator (3.3), which are included in [3], contributes an estimated uncertainty of $\sim 8\,\mathrm{MeV}$ [4], while interference of potential and soft photon exchange accounts for additional $\sim 5\,\mathrm{MeV}$ [4]. This means that with a NLL treatment of initial-state radiation, which seems realistically achievable in the near future, and further inputs from [3] the total theoretical error on $M_W$ could be reduced to the level required for phenomenological applications at linear colliders.

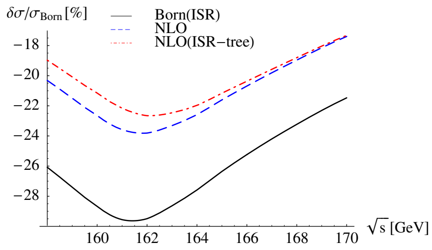



**Figure 1:** Size of the relative NLO corrections for different implementations of ISR

## Acknowledgments

I thank M. Beneke, C. Schwinn, A. Signer and G. Zanderighi for the collaboration on [4] and for comments on the manuscript.

## References

[1] G. Wilson, in *2nd ECFA/DESY Study*, pp. 1498–1505, Desy LC note LC-PHSM-2001-009.

[2] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, *Nucl. Phys.* **B587**, 67 (2000), [hep-ph/0006307]; *Phys. Lett.* **B475**, 127 (2000), [hep-ph/9912261]; W. Beenakker, F. A. Berends and A. P. Chapovsky, *Nucl. Phys.* **B548**, 3 (1999), [hep-ph/9811481].

[3] A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, *Nucl. Phys.* **B724**, 247 (2005), [hep-ph/0505042]; *Phys. Lett.* **B612**, 223 (2005), [hep-ph/0502063].

[4] M. Beneke, P. Falgari, C. Schwinn, A. Signer, G. Zanderighi, *Nucl. Phys.* **B792**, 89 (2008) [arXiv:0707.0773 [hep-ph]]; C. Schwinn, [arXiv:0708.0730 [hep-ph]].

[5] A. P. Chapovsky, V. A. Khoze, A. Signer and W. J. Stirling, *Nucl. Phys.* **B621**, 257 (2002), [hep-ph/0108190].

[6] M. Beneke, N. Kauer, A. Signer and G. Zanderighi, *Nucl. Phys. Proc. Suppl.* **152**, 162 (2006), [hep-ph/0411008].

[7] M. Beneke, A. P. Chapovsky, A. Signer and G. Zanderighi, *Nucl. Phys.* **B686**, 205 (2004), [hep-ph/0401002].

[8] W. Kilian, in *2nd ECFA/DESY Study*, pp. 1924–1980, DESY LC-Note LC-TOOL-2001-039.

[9] V. S. Fadin, V. A. Khoze and A. D. Martin, *Phys. Rev.* **D49**, 2247 (1994); K. Melnikov and O. I. Yakovlev, *Phys. Lett.* **B324**, 217 (1994), [hep-ph/9302311].