PROCEEDINGS OF SCIENCE

Theoretical progress on the V_{us} determination from τ decays

Elvira Gámiz,[a] **Matthias Jamin,**[bc] **Antonio Pich**[*,d] **Joaquim Prades**[ef] **and Felix Schwab**[c]

[a]*Department of Physics, University of Illinois, Urbana IL 61801, USA*
[b]*ICREA*
[c]*IFAE, Departament de Física Teòrica, Universitat Autonòma de Barcelona, E-08193 Bellaterra, Barcelona, Spain*
[d]*Departament de Física Teòrica, IFIC, Universitat de València–CSIC, Apt. de Correus 22085, E-46071 València, Spain*
[e]*Theory Unit, Physics Department, CERN, CH-1211 Genève 23, Switzerland*
[f]*CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada, Campus de Fuente Nueva, E-18002 Granada, Spain*
E-mail: megamiz@uiuc.edu, jamin@ifae.es, antonio.pich@ific.uv.es, prades@ugr.es, schwab@ifae.es

A very precise determination of V_{us} can be obtained from the semi-inclusive hadronic decay width of the τ lepton into final states with strangeness. The ratio of the Cabibbo-suppressed and Cabibbo-allowed τ decay widths directly measures $(V_{us}/V_{ud})^2$, up to very small SU(3)-breaking corrections which can be theoretically estimated with the needed accuracy. Together with previous LEP and CLEO data, the recent measurements by Babar and Belle of some Cabibbo-suppressed τ decays imply $V_{us} = 0.2165 \pm 0.0026_{\text{exp}} \pm 0.0005_{\text{th}}$, which is already competitive with the standard extraction from K_{l3} decays. The uncertainty is largely dominated by experimental errors and should be easily reduced with the high statistics of the B factories, providing the most accurate determination of this parameter. A 1% experimental precision on the Cabibbo-suppressed τ decay width would translate into a 0.6% uncertainty on V_{us}.

Kaon International Conference
May 21-25, 2007
Laboratori Nazionali di Frascati dell'INFN

[*]Speaker.

PoS(KAON)008

1. The hadronic τ decay width

The hadronic decays of the τ lepton provide a very clean laboratory to perform precise tests of the Standard Model [1]. The inclusive character of the total τ hadronic width renders possible an accurate calculation of the ratio [2–6]

$$R_\tau \equiv \frac{\Gamma[\tau^- \to \nu_\tau \,\text{hadrons}\,(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}\,, \tag{1.1}$$

using analyticity constraints and the Operator Product Expansion. One can separately compute the contributions associated with specific quark currents: $R_{\tau,V}$ and $R_{\tau,A}$ correspond to the Cabibbo-allowed decays through the vector and axial-vector currents, while $R_{\tau,S}$ contains the remaining Cabibbo-suppressed contributions.

The theoretical prediction for $R_{\tau,V+A}$ can be expressed as [4]

$$R_{\tau,V+A} = N_C\,|V_{ud}|^2\, S_{\text{EW}}\,\{1 + \delta_{\text{P}} + \delta_{\text{NP}}\}\,, \tag{1.2}$$

where $N_C = 3$ is the number of quark colours and $S_{\text{EW}} = 1.0201 \pm 0.0003$ contains the electroweak radiative corrections [7–9]. The dominant correction ($\sim 20\%$) is the perturbative QCD contribution δ_{P}, which is fully known to $O(\alpha_s^3)$ [4] and includes a resummation of the most important higher-order effects [5, 10].

Non-perturbative contributions are suppressed by six powers of the τ mass [4] and, therefore, are very small. Their numerical size has been determined from the invariant-mass distribution of the final hadrons in τ decay, through the study of weighted integrals [11],

$$R_\tau^{kl} \equiv \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}\,, \tag{1.3}$$

which can be calculated theoretically in the same way as R_τ. The predicted suppression [4] of the non-perturbative corrections has been confirmed by ALEPH [12], CLEO [13] and OPAL [14]. The most recent analysis [12] gives

$$\delta_{\text{NP}} = -0.0043 \pm 0.0019\,. \tag{1.4}$$

The QCD prediction for $R_{\tau,V+A}$ is then completely dominated by the perturbative contribution; non-perturbative effects being smaller than the perturbative uncertainties from uncalculated higher-order corrections. The result turns out to be very sensitive to the value of $\alpha_s(m_\tau)$, allowing for an accurate determination of the fundamental QCD coupling [3, 4]. The experimental measurement $R_{\tau,V+A} = 3.471 \pm 0.011$ implies [15]

$$\alpha_s(m_\tau) = 0.345 \pm 0.004_{\text{exp}} \pm 0.009_{\text{th}}\,. \tag{1.5}$$

The strong coupling measured at the τ mass scale is significantly larger than the values obtained at higher energies. From the hadronic decays of the Z, one gets $\alpha_s(M_Z) = 0.1186 \pm 0.0027$ [16]. Evolving up to the scale M_Z [17], the strong coupling constant in (1.5) decreases to [15]

$$\alpha_s(M_Z) = 0.1215 \pm 0.0012\,, \tag{1.6}$$

in excellent agreement with the direct measurements at the Z peak and with a similar accuracy. The comparison of these two determinations of α_s in two extreme energy regimes, m_τ and M_Z, provides a beautiful test of the predicted running of the QCD coupling; i.e., a very significant experimental verification of *asymptotic freedom*.

2. Cabibbo-suppressed τ decay width

The separate measurement of the $|\Delta S|=0$ and $|\Delta S|=1$ τ decay widths allows us to pin down the SU(3)-breaking effect induced by the strange quark mass [18 – 27], through the differences

$$\delta R_\tau^{kl} \equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} \,. \tag{2.1}$$

Since QCD is flavour blind, these quantities vanish in the SU(3) limit. The only non-zero contributions are proportional to powers of the quark mass-squared difference $m_s^2(m_\tau) - m_d^2(m_\tau)$ or to vacuum expectation values of SU(3)-breaking operators such as $\langle\delta O_4\rangle \equiv \langle 0|m_s\bar{s}s - m_d\bar{d}d|0\rangle \approx (-1.4\pm 0.4)\cdot 10^{-3}\ \mathrm{GeV}^4$ [19, 26]. The dimensions of these operators are compensated by corresponding powers of m_τ^2, which implies a strong suppression of δR_τ^{kl} [19]:

$$\delta R_\tau^{kl} \approx 24\, S_{\mathrm{EW}} \left\{ \frac{m_s^2(m_\tau)}{m_\tau^2}\left(1-\varepsilon_d^2\right)\Delta_{kl}(\alpha_s) - 2\pi^2\,\frac{\delta O_4}{m_\tau^4}\, Q_{kl}(\alpha_s)\right\} , \tag{2.2}$$

where $\varepsilon_d \equiv m_d/m_s = 0.053\pm 0.002$ [28]. The perturbative QCD corrections $\Delta_{kl}(\alpha_s)$ and $Q_{kl}(\alpha_s)$ are known to $O(\alpha_s^3)$ and $O(\alpha_s^2)$, respectively [19, 27].

The moments δR_τ^{k0} $(k=0,1,2,3,4)$ have been measured by ALEPH [29] and OPAL [30]. In spite of the large experimental uncertainties, the corresponding QCD analysis has allowed to perform a rather competitive determination of the strange quark mass [20, 26]. However, the extracted value depends sensitively on the modulus of the Cabibbo–Kobayashi–Maskawa matrix element $|V_{us}|$, because the small differences δR_τ^{kl} result from a strong cancellation between two nearly-equal quantities. It appears then natural to turn things around and, with an input for m_s obtained from other sources, to actually determine $|V_{us}|$ [26]. The most sensitive moment is the unweighted difference of decay widths $\delta R_\tau \equiv \delta R_\tau^{00}$:

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,\mathrm{th}}} \,. \tag{2.3}$$

The SU(3)-breaking quantity $\delta R_\tau \sim 0.25$ is one order of magnitude smaller than the ratio $R_{\tau,V+A}/|V_{ud}|^2 = 3.661\pm 0.012$, where we have taken for V_{ud} the PDG advocated value $|V_{ud}| = 0.97377\pm 0.00027$ [31]. Therefore, to a first approximation V_{us} can be directly obtained from experimental measurements, without any theoretical input. With $R_{\tau,S} = 0.1686\pm 0.0047$ [15], one gets in the SU(3) limit:

$$\delta R_\tau = 0 \qquad \longrightarrow \qquad |V_{us}|^{\mathrm{SU(3)}} = 0.215\pm 0.003\,. \tag{2.4}$$

This rather remarkable determination is only slightly shifted by the small SU(3)-breaking corrections. For instance, taking $\delta R_\tau \approx 0.25$ increases the result to $|V_{us}| \approx 0.222$. Thus, an estimate of $\delta R_{\tau,\mathrm{th}}$ with an accuracy of around 10% translates into a final theoretical uncertainty for $|V_{us}|$ of only 0.4% (± 0.0008). The final precision on the τ determination of $|V_{us}|$ is then a purely experimental issue, in contrast to the standard extraction from K_{l3} which is already limited by theoretical errors.

PoS(KAON)008

3. Theoretical evaluation of δR_τ

The theoretical analysis of R_τ [2–4] involves the two-point correlation functions

$$\Pi^{\mu\nu}_{ij,\mathscr{J}}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|T[\mathscr{J}^\mu_{ij}(x)\mathscr{J}^{\nu\dagger}_{ij}(0)]|0\rangle = \left[q^\mu q^\nu - q^2 g^{\mu\nu}\right]\Pi^T_{ij,\mathscr{J}}(q^2) + q^\mu q^\nu\, \Pi^L_{ij,\mathscr{J}}(q^2) \tag{3.1}$$

of vector, $\mathscr{J}^\mu_{ij} = V^\mu_{ij} \equiv \bar{q}_j\gamma^\mu q_i$, and axial-vector, $\mathscr{J}^\mu_{ij} = A^\mu_{ij} \equiv \bar{q}_j\gamma^\mu\gamma_5 q_i$, quark currents ($i,j = u,d,s$). The invariant-mass distribution of the final hadrons in the τ decay is proportional to the imaginary parts of the correlators $\Pi^{T/L}_{ij,\mathscr{J}}(q^2)$, where the superscript in the transverse and longitudinal components denotes the corresponding angular momentum $J=1$ (T) and $J=0$ (L). Employing the analytic properties of these correlators one can express R_τ as a contour integral running counter-clockwise around the circle $|s| = m_\tau^2$ in the complex s-plane:

$$\begin{aligned} R_\tau &= 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^T(s) + \mathrm{Im}\,\Pi^L(s)\right] \\ &= -i\pi \oint_{|s|=m_\tau^2} \frac{ds}{s}\left[1-\frac{s}{m_\tau^2}\right]^3 \left\{3\left[1+\frac{s}{m_\tau^2}\right] D^{L+T}(s) + 4D^L(s)\right\}. \end{aligned} \tag{3.2}$$

We have used integration by parts to rewrite R_τ in terms of the logarithmic derivatives

$$D^{L+T}(s) \equiv -s\frac{d}{ds}\Pi^{L+T}(s), \qquad D^L(s) \equiv \frac{s}{m_\tau^2}\frac{d}{ds}\left[s\,\Pi^L(s)\right], \tag{3.3}$$

where the relevant combination of two-point correlation functions is given by

$$\Pi^J(s) \equiv |V_{ud}|^2\left\{\Pi^J_{ud,V}(s)+\Pi^J_{ud,A}(s)\right\} + |V_{us}|^2\left\{\Pi^J_{us,V}(s)+\Pi^J_{us,A}(s)\right\}. \tag{3.4}$$

The two terms proportional to $|V_{ud}|^2$ contribute to $R_{\tau,V}$ and $R_{\tau,A}$, respectively, while $R_{\tau,S}$ contains the remaining contributions proportional to $|V_{us}|^2$.

At large enough Euclidean $Q^2 \equiv -s$, both $\Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ can be computed within QCD, using well-established operator product expansion techniques. The result is organised in a series of local gauge-invariant operators of increasing dimension, times the appropriate inverse powers of Q^2. Performing the complex integration (3.2), one can then express R_τ as an expansion in inverse powers of m_τ^2 [4]. The perturbative correction δ_P in eq. (1.2) corresponds to the dimension-zero contributions. The dominant SU(3)-breaking contributions shown in eq. (2.2) are associated with operators of dimension two (m^2) and four (δO_4).

A very detailed theoretical analysis of δR_τ was performed in refs. [19]. The perturbative QCD corrections to the relevant dimension-two and four operators turn out to take the values $\Delta_{00}(\alpha_s) = 2.0 \pm 0.5$ and $Q_{00}(\alpha_s) = 1.08 \pm 0.03$. In order to predict δR_τ, one needs an input value for the strange quark mass; we will adopt the range

$$m_s(m_\tau) = (100 \pm 10)\,\mathrm{MeV} \qquad [m_s(2\,\mathrm{GeV}) = (96 \pm 10)\,\mathrm{MeV}], \tag{3.5}$$

which includes the most recent determinations of m_s from QCD sum rules and lattice QCD [32]. This gives $\delta R_\tau = 0.227 \pm 0.054$, which implies

$$|V_{us}| = 0.2216 \pm 0.0031_{\mathrm{exp}} \pm 0.0017_{\mathrm{th}} = 0.2216 \pm 0.0036. \tag{3.6}$$

The largest theoretical uncertainty on δR_τ originates in the longitudinal contribution ($J=L$) to the dimension-two correction $\Delta_{00}(\alpha_s)$. The corresponding perturbative series, which is known to $O(\alpha_s^3)$, shows a very pathological behaviour with clear signs of being non-convergent; this induces a large theoretical error in $\Delta_{00}(\alpha_s)$. Fortunately, the longitudinal contribution to R_τ can be estimated phenomenologically with a much higher accuracy, because it is dominated by far by the well-known pion and kaon poles,

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi^L_{ud,A}(s) \,=\, 2f_\pi^2\,\delta(s-m_\pi^2)\,, \qquad\qquad \frac{1}{\pi}\,\mathrm{Im}\,\Pi^L_{us,A}(s) \,=\, 2f_K^2\,\delta(s-m_K^2)\,, \tag{3.7}$$

which are determined by the $\pi^-\to l^-\bar\nu_l$ and $K^-\to l^-\bar\nu_l$ decay widths. Although much smaller, the leading contribution to the scalar spectral function can be also obtained from s-wave $K\pi$ scattering data [32, 33]. Taking into account additional tiny corrections from higher-mass pseudoscalar resonances [34], one obtains the following phenomenological determination of the longitudinal contribution to δR_τ [26, 32]:

$$\delta R_\tau|^L \,=\, 0.1544\pm 0.0037\,. \tag{3.8}$$

The pion and kaon contributions amount to 79% of $\delta R_\tau|^L$. For comparison, taking the strange quark mass in the range (3.5), the direct QCD calculation of this quantity gives $\delta R_\tau|^L = 0.166\pm 0.051$; in agreement with (3.8), but with a much larger error.

The perturbative QCD series $\Delta^{L+T}_{00}(\alpha_s)$ is much better behaved, although it starts to show an asymptotic character at $O(\alpha_s^3)$. Following the prescription advocated in refs. [19], one finds $\frac{3}{4}\Delta^{L+T}_{00}(\alpha_s) = 0.75\pm 0.12$ and $Q^{L+T}_{00}(\alpha_s) = 0.08\pm 0.03$, which imply $\delta R_\tau|^{L+T} = 0.062\pm 0.015$. Adding the longitudinal contribution (3.8), gives $\delta R_\tau = 0.216\pm 0.016$, which agrees with the pure QCD determination and is 3.4 times more precise. This allows us to obtain an improved V_{us} determination:

$$|V_{us}| \,=\, 0.2212\pm 0.0031_{\,\mathrm{exp}}\pm 0.0005_{\,\mathrm{th}} \,=\, 0.2212\pm 0.0031\,. \tag{3.9}$$

4. Improved evaluation of $\Gamma(\tau^-\to\nu_\tau \mathbf{K}^-/\pi^-)$

The phenomenological determination of $\delta R_\tau|^L$ contains a hidden dependence on V_{us} through the input value of the kaon decay constant f_K. Although the numerical impact of this dependence is negligible, it is worth while to take it explicitly into account. At the same time, we can determine the $\tau^-\to\nu_\tau K^-/\pi^-$ decay widths with better accuracy than the present direct experimental measurements, through the ratios ($P=K,\pi$)

$$R_{\tau/P} \,\equiv\, \frac{\Gamma(\tau^-\to\nu_\tau P^-)}{\Gamma\left(P^-\to\bar\nu_\mu\mu^-\right)} \,=\, \frac{m_\tau^3}{2m_P m_\mu^2}\,\frac{\left(1-m_P^2/m_\tau^2\right)^2}{\left(1-m_\mu^2/m_P^2\right)^2}\,\left(1+\delta R_{\tau/P}\right)\,, \tag{4.1}$$

where $\delta R_{\tau/\pi} = (0.16\pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90\pm 0.22)\%$ are the estimated electroweak radiative corrections [35, 36]. Using the measured $K^-/\pi^-\to\bar\nu_\mu\mu^-$ decay widths and the τ lifetime [31], one gets then:

$$\mathrm{Br}(\tau^-\to\nu_\tau K^-) = (0.715\pm 0.004)\%\,, \qquad\qquad \mathrm{Br}(\tau^-\to\nu_\tau\pi^-) = (10.90\pm 0.04)\%\,, \tag{4.2}$$

in good agreement with the less accurate PDG averages $\mathrm{Br}(\tau^-\to\nu_\tau K^-) = (0.691\pm 0.023)\%$ and $\mathrm{Br}(\tau^-\to\nu_\tau\pi^-) = (10.90\pm 0.07)\%$.

PoS(KAON)008

Following ref. [15], we will use the improved estimate of the electronic branching fraction $\mathrm{Br}(\tau^- \to \nu_\tau \bar{\nu}_e e^-)_{\mathrm{univ}} = (17.818 \pm 0.032)\%$, which is obtained by averaging the direct measurements of the electronic an muonic branching fractions and the τ lifetime, assuming universality. The resulting kaon and pion contributions to R_τ take then the values:

$$R_\tau|^{\tau^-\to\nu_\tau K^-} = (0.04014 \pm 0.00021)\,, \qquad R_\tau|^{\tau^-\to\nu_\tau \pi^-} = (0.6123 \pm 0.0025)\,. \tag{4.3}$$

In our theoretical analysis of δR_τ, we have considered the separate contributions from the $J = L$ and $J = L+T$ pieces, defined through the two terms on the rhs of eqs. (3.2). The corresponding splitting of the kaon and pion contributions is given by:

$$\begin{aligned} R_\tau|_L^{\tau^-\to\nu_\tau K^-} &= R_\tau|^{\tau^-\to\nu_\tau K^-} - R_\tau|_{L+T}^{\tau^-\to\nu_\tau K^-} = -2\,\frac{m_K^2}{m_\tau^2}\, R_\tau|^{\tau^-\to\nu_\tau K^-} = -(0.006196 \pm 0.000033)\,, \\ R_\tau|_L^{\tau^-\to\nu_\tau \pi^-} &= R_\tau|^{\tau^-\to\nu_\tau \pi^-} - R_\tau|_{L+T}^{\tau^-\to\nu_\tau \pi^-} = -2\,\frac{m_\pi^2}{m_\tau^2}\, R_\tau|^{\tau^-\to\nu_\tau \pi^-} = -(0.007554 \pm 0.000031)\,. \end{aligned} \tag{4.4}$$

Subtracting the longitudinal contributions from eq. (2.3), we can give an improved formula to determine V_{us} with the best possible accuracy:

$$|V_{us}|^2 = \frac{\tilde{R}_{\tau,S}}{\frac{\tilde{R}_{\tau,V+A}}{|V_{ud}|^2} - \delta\tilde{R}_{\tau,\mathrm{th}}} \equiv \frac{R_{\tau,S} - R_\tau|_L^{\tau^-\to\nu_\tau K^-}}{\frac{R_{\tau,V+A} - R_\tau|_L^{\tau^-\to\nu_\tau\pi^-}}{|V_{ud}|^2} - \delta\tilde{R}_{\tau,\mathrm{th}}}\,, \tag{4.5}$$

where

$$\delta\tilde{R}_{\tau,\mathrm{th}} \equiv \delta\tilde{R}_\tau|^L + \delta R_{\tau,\mathrm{th}}|^{L+T} = (0.033 \pm 0.003) + (0.062 \pm 0.015) = 0.095 \pm 0.015\,. \tag{4.6}$$

The subtracted longitudinal correction $\delta\tilde{R}_\tau|^L$ is now much smaller because it does not contain any pion or kaon contribution.

Using the same input values for $R_{\tau,S}$ and $R_{\tau,V+A}$, one recovers the V_{us} determination obtained before in eq. (3.9), with a slightly improved error of ± 0.0030.

5. Experimental update of $\mathbf{R_{\tau,S}}$

Within the Standard Model, where charged-current universality holds, the electron branching fraction $B_e \equiv \mathrm{Br}(\tau^- \to \nu_\tau \bar{\nu}_e e^-)_{\mathrm{univ}} = (17.818 \pm 0.032)\%$ determines the hadronic one, i.e., $R_\tau = \frac{1}{B_e} - 1.972564 = 3.640 \pm 0.010$. Since $R_{\tau,V+A} = R_\tau - R_{\tau,S}$, the only additional experimental input which is needed is $R_{\tau,S}$. Up to now, we have been using the value $R_{\tau,S} = 0.1686 \pm 0.0047$, from a recent compilation of LEP and CLEO data [15].

Babar and Belle have recently reported their first measurements of Cabibbo-suppressed τ decays: $\mathrm{Br}(\tau^- \to \nu_\tau \phi K^-) = (4.05 \pm 0.25 \pm 0.26)\cdot 10^{-5}$ [37], $\mathrm{Br}(\tau^- \to \nu_\tau K_S \pi^-) = (0.404 \pm 0.002 \pm 0.013)\%$ [38], $\mathrm{Br}(\tau^- \to \nu_\tau K^- \pi^0) = (0.416 \pm 0.003 \pm 0.018)\%$ [39], $\mathrm{Br}(\tau^- \to \nu_\tau K^- \pi^- \pi^+) = (0.273 \pm 0.002 \pm 0.009)\%$ [40] and $\mathrm{Br}(\tau^- \to \nu_\tau \phi K^-) = (3.39 \pm 0.20 \pm 0.28)\cdot 10^{-5}$ [40]. The last mode includes $\mathrm{Br}(\tau^- \to \nu_\tau K^- K^- K^+) = (1.58 \pm 0.13 \pm 0.12)\cdot 10^{-5}$ [40], which is found to be consistent with going entirely through $\tau^- \to \nu_\tau \phi K^-$. The changes induced in $R_{\tau,S}$ have been nicely summarized at this workshop by Swagato Banerjee [41]. Taking for $\mathrm{Br}(\tau^- \to \nu_\tau K^-)$ the value (4.2) and including the small $\tau^- \to \nu_\tau \phi K^-$ decay mode, one finds a total probability for the τ to decay into strange final states of $(2.882 \pm 0.071)\%$, which implies the updated values

$$R_{\tau,S} = 0.1617 \pm 0.0040\,, \qquad\qquad R_{\tau,V+A} = 3.478 \pm 0.011\,. \tag{5.1}$$

Although consistent within the quoted uncertainties, the new Babar and Belle measurements are all smaller than the previous world averages, which translates into a smaller value of $R_{\tau,S}$ and V_{us}. Using eq. (4.5), one finds now

$$|V_{us}| \,=\, 0.2165 \pm 0.0026_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}} \,=\, 0.2165 \pm 0.0026\,. \tag{5.2}$$

Sizeable changes on the experimental determination of $R_{\tau,S}$ are to be expected from the full analysis of the huge Babar and Belle data samples. In particular, the high-multiplicity decay modes are not well known at present and their effect has been just roughly estimated or simply ignored. Thus, the result (5.2) could easily fluctuate in the near future. However, it is important to realize that the final error of the V_{us} determination from τ decay is completely dominated by the experimental uncertainties. If $R_{\tau,S}$ is measured with a 1% precision, the resulting V_{us} uncertainty will get reduced to around 0.6%, i.e. ± 0.0013, making τ decay the best source of information about V_{us}.

An accurate experimental measurement of the invariant-mass distribution of the final hadrons in Cabibbo-suppressed τ decays could make possible a simultaneous determination of V_{us} and the strange quark mass. However, the extraction of m_s suffers from theoretical uncertainties related to the convergence of the perturbative series $\Delta_{00}^{L+T}(\alpha_s)$. A better understanding of these corrections is needed [42].

Acknowledgments

This work has been supported by the European Commission MRTN FLAVIA*net* [MRTN-CT-2006-035482], the MEC (Spain) and FEDER (EC) [FPA2005-02211 (M.J.), FPA2004-00996 (A.P.) and FPA2006-05294 (J.P.)], the Deutsche Forschungsgemeinschaft (F.S.), the Junta de Andalucía [P05-FQM-101 (J.P.), P05-FQM-437 (E.G. and J.P.) and Sabbatical Grant PR2006-0369 (J.P.)] and the Generalitat Valenciana [GVACOMP2007-156 (A.P.)].

References

[1] A. Pich, *Tau Physics 2006: Summary & Outlook*, in proceedings of the *Ninth International Workshop on Tau Lepton Physics* (TAU06, Pisa, Italy, 19–22 September 2006), *Nucl. Phys. B (Proc. Suppl.)* **169** (2007) 393 [arXiv:hep-ph/0702074].

[2] E. Braaten, *Phys. Rev. Lett.* **60** (1988) 1606; *Phys. Rev.* **D39** (1989) 1458.

[3] S. Narison and A. Pich, *Phys. Lett.* **B211** (1988) 183.

[4] E. Braaten, S. Narison and A. Pich, *Nucl. Phys.* **B373** (1992) 581.

[5] F. Le Diberder and A. Pich, *Phys. Lett.* **B286** (1992) 147.

[6] A. Pich, *Nucl. Phys. B (Proc. Suppl.)* **39B,C** (1995) 326 [arXiv:hep-ph/9412273].

[7] W.J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **61** (1988) 1815.

[8] E. Braaten and C.S. Li, *Phys. Rev.* **D42** (1990) 3888.

[9] J. Erler, *Rev. Mex. Phys.* **50** (2004) 200 [arXiv:hep-ph/0211345].

[10] A.A. Pivovarov, *Z. Phys.* **C53** (1992) 461.

[11] F. Le Diberder and A. Pich, *Phys. Lett.* **B289** (1992) 165.

[12] ALEPH Collaboration, *Phys. Rep.* **421** (2005) 191 [arXiv:hep-ex/0506072]; *Eur. Phys. J.* **C4** (1998) 409; *Phys. Lett.* **B307** (1993) 209.

[13] CLEO Collaboration, *Phys. Lett.* **B356** (1995) 580.

[14] OPAL Collaboration, *Eur. Phys. J.* **C7** (1999) 571 [arXiv:hep-ex/9808019].

[15] M. Davier, A. Höcker and Z. Zhang, *Rev. Mod. Phys.* **78** (2006) 1043 [arXiv:hep-ph/0507078].

[16] The LEP Collaborations ALEPH, DELPHI, L3, OPAL and the LEP Electroweak Working Group, arXiv:hep-ph/0612034; http://www.cern.ch/LEPEWWG.

[17] G. Rodrigo, A. Pich and A. Santamaria, *Phys. Lett.* **B424** (1998) 367 [arXiv:hep-ph/9707474].

[18] M. Davier, *Nucl. Phys. B (Proc. Suppl.)* **55C** (1997) 395.

[19] A. Pich and J. Prades, *JHEP* **9910** (1999) 004 [arXiv:hep-ph/9909244]; **9806** (1998) 013 [arXiv:hep-ph/9804462].

[20] S. Chen et al., *Eur. Phys. J.* **C22** (2001) 31 [arXiv:hep-ph/0105253]. M. Davier et al., *Nucl. Phys. B (Proc. Suppl.)* **98** (2001) 319.

[21] K.G. Chetyrkin, J.H. Kühn and A.A. Pivovarov, *Nucl. Phys.* **B533** (1998) 473 [arXiv: hep-ph/9805335].

[22] J.G. Körner, F. Krajewski and A.A. Pivovarov, *Eur. Phys. J.* **C20** (2001) 259 [arXiv:hep-ph/0003165].

[23] K. Maltman and C.E. Wolfe, *Phys. Lett.* **B639** (2006) 283 [arXiv:hep-ph/0603215].

[24] J. Kambor and K. Maltman, *Phys. Rev.* **D62** (2000) 093023 [arXiv:hep-ph/0005156]; **D64** (2001) 093014 [arXiv:hep-ph/0107187].

[25] K. Maltman, *Phys. Rev.* **D58** (1998) 093015 [arXiv:hep-ph/9804298].

[26] E. Gámiz et al., *Phys. Rev. Lett.* **94** (2005) 011803 [arXiv:hep-ph/0408044]; *JHEP* **0301** (2003) 060 [arXiv:hep-ph/0212230].

[27] P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, *Phys. Rev. Lett.* **95** (2005) 012003 [arXiv:hep-ph/0412350].

[28] H. Leutwyler, *Phys. Lett.* **B378** (1996) 313 [arXiv:hep-ph/9602366].

[29] ALEPH Collaboration, *Eur. Phys. J.* **C11** (1999) 599 [arXiv:hep-ex/9903015]; **C10** (1999) 1 [arXiv:hep-ex/9903014].

[30] OPAL Collaboration, *Eur. Phys. J.* **C35** (2004) 437 [arXiv:hep-ex/0406007].

[31] W.-M. Yao et al., *The Review of Particle Physics*, *Journal of Physics* **G33** (2006) 1.

[32] M. Jamin, J.A. Oller and A. Pich, *Phys. Rev.* **D74** (2006) 074009 [arXiv:hep-ph/0605095].

[33] M. Jamin, J.A. Oller and A. Pich, *Eur. Phys. J.* **C24** (2002) 237 [arXiv:hep-ph/0110194]; *Nucl. Phys.* **B622** (2002) 279 [arXiv:hep-ph/0110193]; **B587** (2000) 331 [arXiv:hep-ph/0006045].

[34] K. Maltman and J. Kambor, *Phys. Rev.* **D65** (2002) 074013 [arXiv:hep-ph/0108227].

[35] W.J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **71** (1993) 3629.

[36] R. Decker and M. Finkemeier, *Nucl. Phys.* **B438** (1995) 17 [arXiv:hep-ph/9403385]; *Phys. Lett.* **B334** (1994) 199.

[37] Belle Collaboration, *Phys. Lett.* **B643** (2006) 5 [arXiV:hep-ex/0609018].

[38] Belle Collaboration, arXiV:0706.2231 [hep-ex].

[39] Babar Collaboration, arXiV:0707.2922 [hep-ex].

[40] Babar Collaboration, arXiV:0707.2981 [hep-ex].

[41] S. Banerjee, arXiV:0707.3058v3 [hep-ex].

[42] E. Gámiz et al., work in progress.