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Light quark masses and pseudoscalar decay constants from $N_f = 2$ twisted mass QCD

Vittorio Lubicz
Dip. di Fisica, Università di Roma Tre and INFN, Sez. di Roma Tre,
Via della Vasca Navale 84, I-00146 Roma, Italy
E-mail: `lubicz@fis.uniroma3.it`

Silvano Simula
INFN, Sez. di Roma Tre,
Via della Vasca Navale 84, I-00146 Roma, Italy
E-mail: `silvano.simula@roma3.infn.it`

Cecilia Tarantino[∗†]
Dip. di Fisica, Università di Roma Tre and INFN, Sez. di Roma Tre,
Via della Vasca Navale 84, I-00146 Roma, Italy
E-mail: `tarantino@fis.uniroma3.it`

for the European Twisted Mass Collaboration (ETMC)

We present the results of the lattice QCD calculation of the average up-down and strange quark masses and of the light meson pseudoscalar decay constants, recently performed with $N_f = 2$ dynamical fermions by the ETM Collaboration. The simulation is carried out at a single value of the lattice spacing with the twisted mass fermionic action at maximal twist, which guarantees automatic $\mathcal{O}(a)$-improvement of the physical quantities. Quark masses are renormalized by implementing the non perturbative RI-MOM renormalization procedure. Our results for the light quark masses are $m_{ud}^{\overline{\rm MS}}(2\ \text{GeV}) = 3.85 \pm 0.12 \pm 0.40$ MeV, $m_s^{\overline{\rm MS}}(2\ \text{GeV}) = 105 \pm 3 \pm 9$ MeV and $m_s/m_{ud} = 27.3 \pm 0.3 \pm 1.2$. We also obtain $f_K = 161.7 \pm 1.2 \pm 3.1$ MeV and the ratio $f_K/f_\pi = 1.227 \pm 0.009 \pm 0.024$. From this ratio, by using the experimental determination of $\Gamma(K \to \mu \bar{\nu}_\mu(\gamma))/\Gamma(\pi \to \mu \bar{\nu}_\mu(\gamma))$ and the average value of $|V_{ud}|$ from nuclear beta decays, we obtain $|V_{us}| = 0.2192(5)(45)$, in agreement with the determination from K_{l3} decays and the unitarity constraint.

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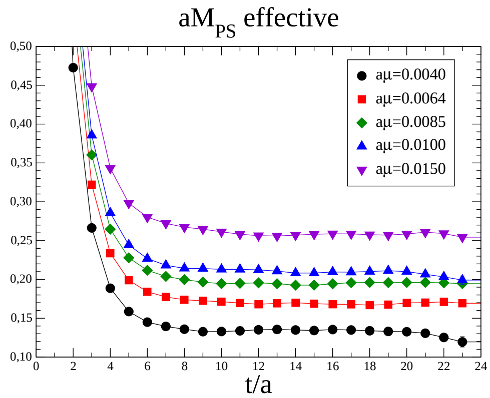

Figure 1: *Effective masses of pseudoscalar mesons with $\mu_S = \mu_1 = \mu_2$, as a function of the time.*

1. Introduction

We present our recent determination [1] of the light quark masses (strange quark mass m_s and average up-down quark mass m_{ud}), of the kaon pseudoscalar decay constant f_K, and of the ratio f_K/f_π. In order to investigate the properties of the K meson, we have simulated the theory with $N_f = 2$ degenerate dynamical and two valence quarks, considering a partially quenched setup with the valence quark masses μ_1 and μ_2 different between each other and from the sea quark mass μ_S.

The calculation is based on a set of (ETMC) gauge field configurations generated with the tree-level improved Symanzik gauge action at $\beta = 3.9$, corresponding to $a = 0.087(1)$ fm ($a^{-1} \simeq 2.3$ GeV) [2], and the twisted mass fermionic action at maximal twist. We have simulated 5 values of the bare sea quark mass, $a\mu_S = \{0.0040, 0.0064, 0.0085, 0.0100, 0.0150\}$ and 8 values, $a\mu_{1,2} = \{0.0040, 0.0064, 0.0085, 0.0100, 0.0150, 0.0220, 0.0270, 0.0320\}$, for the valence quark mass. The first five masses, equal to the sea masses, lie in the range $1/6\, m_s \lesssim \mu_{1,2} \lesssim 2/3\, m_s$, being m_s the physical strange quark mass, while the heaviest three are around the strange quark mass.

At each value of the sea quark mass we have computed the two-point correlation functions of charged pseudoscalar mesons, on a set of 240 independent gauge field configurations, separated by 20 HMC trajectories one from the other. To improve the statistical accuracy, we have evaluated the meson correlators using a stochastic method with a $Z(2)$-noise to include all spatial sources [3, 4]. Statistical errors on meson masses and decay constants are evaluated using the jackknife procedure, while those on the fit results, based on data obtained at different sea quark masses, are evaluated using a bootstrap procedure. Further details on the numerical simulation can be found in refs. [1, 5].

The use of twisted mass fermions presents several advantages [6]: i) the pseudoscalar meson masses and decay constants are automatically improved at $\mathcal{O}(a)$; ii) at maximal twist, the physical quark mass is directly related to the twisted mass parameter of the action, and it is subject only to multiplicative renormalization; iii) the determination of the pseudoscalar decay constant does not require the introduction of any renormalization constant, and it is based on the relation

$$f_{PS} = (\mu_1 + \mu_2) \frac{|\langle 0|P^1(0)|P\rangle|}{M_{PS}^2} \,. \tag{1.1}$$

The meson mass M_{PS} and the matrix element $|\langle 0|P^1(0)|P\rangle|$ have been extracted from a fit of the two-point pseudoscalar correlation function in the time interval $t/a \in [10, 21]$. In order to illustrate the quality of the data, we show in fig. 1 the effective masses of pseudoscalar mesons, as a function of the time, in the degenerate cases $\mu_S = \mu_1 = \mu_2$.

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2. Quark mass dependence of pseudoscalar meson masses and decay constants

The determination of the physical properties of K mesons requires to study the corresponding observables over a large range of masses, from the physical strange quark down to the light up-down quark. In ref. [1], we have studied the quark mass dependence of pseudoscalar meson masses and decay constants by considering two different functional forms: i) the dependence predicted by continuum partially quenched chiral perturbation theory (PQChPT), ii) a polynomial dependence.

PQChPT fits: Within PQChPT we have considered the full next-to-leading order (NLO) expressions with the addition of the local NNLO contributions, i.e. terms quadratic in the quark masses, which turn out to be needed for a good description of the data up to the region of the strange quark. The PQChPT predictions [7] can be written as

$$\begin{aligned} M^2_{PS}(\mu_S,\mu_1,\mu_2) \;&=\; B_0\,(\mu_1+\mu_2)\cdot\left[1+\frac{\xi_1\,(\xi_S-\xi_1)\ln 2\xi_1}{(\xi_2-\xi_1)}-\frac{\xi_2\,(\xi_S-\xi_2)\ln 2\xi_2}{(\xi_2-\xi_1)}+\right.\\ &\left.+a_V\,\xi_{12}+a_S\,\xi_S+a_{VV}\,\xi_{12}^2+a_{SS}\,\xi_S^2+a_{VS}\,\xi_{12}\,\xi_S+a_{VD}\,\xi_{D12}^2\right]\,, \\ f_{PS}(\mu_S,\mu_1,\mu_2) \;&=\; f\cdot\left[1-\xi_{1S}\ln 2\xi_{1S}-\xi_{2S}\ln 2\xi_{2S}+\frac{\xi_1\,\xi_2-\xi_S\,\xi_{12}}{2\,(\xi_2-\xi_1)}\ln\left(\frac{\xi_1}{\xi_2}\right)+\right.\\ &\left.+(b_V+1/2)\,\xi_{12}+(b_S-1/2)\,\xi_S+b_{VV}\,\xi_{12}^2+b_{SS}\,\xi_S^2+b_{VS}\,\xi_{12}\,\xi_S+b_{VD}\,\xi_{D12}^2\right]\,, \end{aligned} \tag{2.1}$$

where $\xi_i = 2B_0\mu_i/(4\pi f)^2$, $\xi_{ij} = B_0(\mu_i+\mu_j)/(4\pi f)^2$ and $\xi_{Dij} = B_0(\mu_i-\mu_j)/(4\pi f)^2$. The parameters B_0 and f are the LO low energy constants (LECs) [1], whereas a_V, a_S, b_V and b_S are related to the NLO LECs by $a_V = 4\alpha_8 - 2\alpha_5$, $a_S = 8\alpha_6 - 4\alpha_4$, $b_V = \alpha_5$, $b_S = 2\alpha_4$. The quadratic mass terms in eq. (2.1) represent the local NNLO contributions. The chiral logarithms, also known at two loops in the partially quenched theory [8], involve a larger number of NLO LECs whose values cannot be fixed from phenomenology in the $N_f = 2$ theory. Introducing their contribution would increase significantly the number of free parameters, thus limiting the predictive power of the calculation.

Aiming at a percent precision, the impact of finite size corrections cannot be neglected in our study, where the lattice spatial extension is $L = 24a \simeq 2.2$ fm and $M_{PS}L \geq 3.2$. Since we have not performed yet a systematic study on different lattice volumes, we have estimated the finite size effects by including in the fits the corrections predicted by one-loop PQChPT [9] (for their explicit expressions see ref. [1]).

Polynomial fits: The inclusion of the local NNLO contributions in the PQChPT predictions of eq. (2.1) is required by the observation that the pure NLO predictions are not accurate enough to describe the quark mass dependence of pseudoscalar meson masses and decay constants up to the strange quark region. Not having considered the full NNLO chiral predictions, we have evaluated the associated systematic uncertainty, considering as an alternative description a simple polynomial dependence on the quark masses, for both pseudoscalar meson masses and decay constants:

$$\begin{aligned} M^2_{PS}(\mu_S,\mu_1,\mu_2) &= B_0\,(\mu_1+\mu_2)\cdot\left[1+a_V\xi_{12}+a_S\xi_S+a_{VV}\xi_{12}^2+a_{SS}\xi_S^2+a_{VS}\xi_{12}\xi_S+a_{VD}\xi_{D12}^2\right]\,, \\ f_{PS}(\mu_S,\mu_1,\mu_2) &= f\cdot\left[1+(b_V+1/2)\xi_{12}+(b_S-1/2)\xi_S+b_{VV}\xi_{12}^2+b_{SS}\xi_S^2+b_{VS}\xi_{12}\xi_S+b_{VD}\xi_{D12}^2\right]\,. \end{aligned} \tag{2.2}$$

The differences between the results obtained by performing either chiral or polynomial fits have been included in the final estimates of the systematic errors.

[1]The pseudoscalar decay constant f is normalised such that $f_\pi = 130.7$ MeV at the physical pion mass.

3. Chiral extrapolations

The input data of our analysis [1] are the lattice results for the pseudoscalar meson masses and decay constants obtained at each value of the sea quark mass, with both degenerate and non degenerate valence quarks. We have excluded from the fits the heaviest mesons having both the valence quark masses in the strange mass region, i.e. with $a\mu_{1,2} = \{0.0220, 0.0270, 0.0320\}$, considering therefore 150 combinations of quark masses. The number of free parameters in the combined fit of M_{PS}^2 and f_{PS} is 14, but a first analysis shows that some of them (from 1 to 5 depending on the fit) are compatible with zero within one standard deviation, and are kept fixed to zero.

In order to extrapolate the pseudoscalar meson masses and decay constants to the points corresponding to the physical pion and kaon, we have considered three different fits:

- **Polynomial fit:** a polynomial dependence on the quark masses is assumed for pseudoscalar meson masses and decay constants, according to eq. (2.2).
- **PQChPT fit:** pseudoscalar meson masses and decay constants are fitted according to the PQChPT predictions of eq. (2.1) including the finite volume corrections derived in ref. [9].
- **Constrained PQChPT fit:** this fit, denoted as C-PQChPT in the following, deserves a more detailed explanation. The main uncertainty in using eqs. (2.1) and (2.2) to describe the quark mass dependence of M_{PS}^2 and f_{PS} is related to the extrapolation toward the physical up-down quark mass. On the other hand, we have shown in ref. [2] that pure NLO ChPT, with the inclusion of finite volume corrections, is sufficiently accurate in describing the lattice pseudoscalar meson masses and decay constants when the analysis is restricted to our lightest four quark masses in the unitary setup (i.e. $\mu_1 = \mu_2 = \mu_S$). In order to take advantage of this information, when performing the C-PQChPT fit we first determine the LO parameters B_0 and f and the NLO combinations $a_V + a_S$ and $b_V + b_S$ from a fit based on pure NLO ChPT performed on the lightest four unitary points. By using these constraints, the other parameters entering the chiral expansions of M_{PS}^2 and f_{PS} are then obtained from a fit to eq. (2.1) over the non unitary points. For consistency with the previous unitary fit, we exclude also in this case from the analysis the data at the highest value of sea quark mass, $a\mu_S = 0.0150$.

We find that, though the quality of the fit is better in the polynomial case, all three analyses provide a good description of the lattice data, in the whole region of masses explored in the simulation, once the terms quadratic in the quark masses are taken into account.

A potential problem in the partially quenched theory is the divergence of the chiral logarithms in the limit in which the light valence quark mass goes to zero at fixed sea quark mass (see eq. (2.1)). This divergence does not affect the extrapolation of the lattice results to the physical point, since sea and the light valence quark masses are degenerate in this case. However, in order to verify that this unphysical behaviour of the partially quenched chiral logarithms does not modify the result of the extrapolation, we have repeated the analysis restricting both the polynomial and the chiral fits to the 30 quark mass combinations (26 in the case of the C-PQChPT fit) that, satisfying the constraint $\mu_2 \geq \mu_1 = \mu_S$, are not affected by dangerous chiral logarithms. The comparison between the results obtained by considering the two different sets of quark masses is reassuring, as it shows that the effects of potentially divergent chiral logarithms are well under control in our analysis.

The mass dependence of the pseudoscalar meson masses and decay constants is illustrated in fig. 2, where lattice data are compared with the results of the polynomial, PQChPT and C-

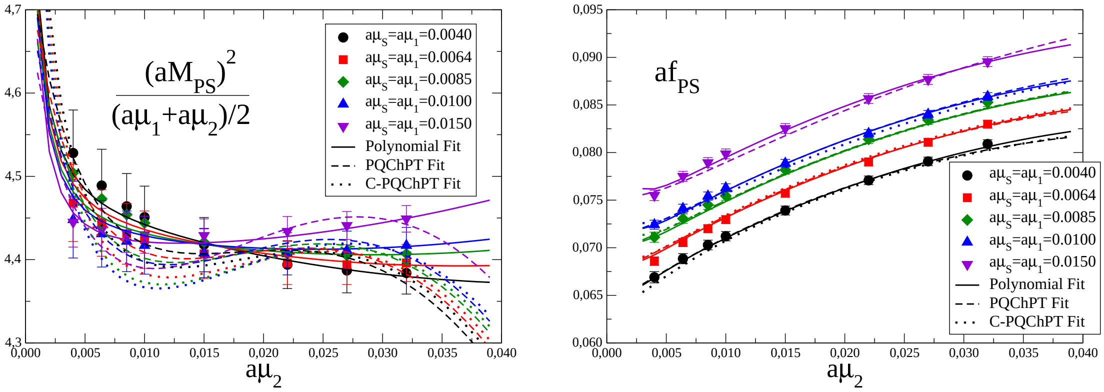

Figure 2: *Lattice results for $a^2M_{PS}^2/\frac{1}{2}(a\mu_1 + a\mu_2)$ (left) and af_{PS} (right) as a function of the valence quark mass $a\mu_2$, with $a\mu_1 = a\mu_S$. The solid, dashed and dotted curves represent the results of the three fits.*

PQChPT fits. We have shown in the plots the cases in which one of the valence quark mass (μ_1) is equal to the sea quark mass, and the results are presented as a function of the second valence quark mass (μ_2). The points corresponding to the physical pion and kaon are thus obtained by extrapolating/interpolating the results shown in fig. 2 to the limits $\mu_1 \to m_{ud}$ and $\mu_2 \to m_s$.

To investigate the impact of finite volume corrections we have compared, for the pseudoscalar meson masses and decay constants, the PQChPT fits obtained with or without including these corrections. The differences turn out to be small [1]; however, to better quantify the systematic error due to finite size effects, we plan to extend our analysis on lattices with different spatial sizes.

By having determined the fit parameters, we have then extrapolated eqs. (2.1) and (2.2) to the physical pion and kaon, as follows. We have first used the experimental values of the ratios M_π/f_π and M_K/M_π to determine the average up-down and the strange quark mass respectively. Once these masses have been determined, we have used again eqs. (2.1) and (2.2) to compute the values of the pion and kaon decay constants as well as their ratio f_K/f_π.

4. Physical results

In order to convert into physical units the results obtained for the strange quark mass and the kaon decay constants we have fixed the scale within each analysis (polynomial, PQChPT and C-PQChPT fits) by using f_π as physical input. In the case of the polynomial and PQChPT fits we conservatively introduce for the dimensionful quantities a 6% and 3% of systematic error to take into account the different scale estimate derived in the analysis over the lightest four unitary points [1, 2].

The determination of the physical strange and up-down quark masses also requires implementing a renormalization procedure. The relation between the bare twisted mass at maximal twist, μ_q, and the renormalized quark mass, m_q, is given by $m_q(\mu_R) = Z_m(g^2, a\mu_R)\,\mu_q(a)$, where μ_R is the renormalization scale, conventionally fixed to 2 GeV for the light quarks. Z_m is the inverse of the flavour non-singlet pseudoscalar density renormalization constant, $Z_m = Z_P^{-1}$. We have used the non-perturbative RI-MOM determination of Z_P, which gives $Z_P^{\rm RI-MOM}(1/a) = 0.39(1)(2)$ at $\beta = 3.9$ [10], and converted the result to the $\overline{\rm MS}$ scheme at the scale $\mu_R = 2$ GeV by using renormalization group improved continuum perturbation theory at the N^{3}LO [11].

Fit	$m_{ud}^{\overline{\rm MS}}$ (MeV)	$m_s^{\overline{\rm MS}}$ (MeV)	m_s/m_{ud}	f_K (MeV)	f_K/f_π
Polynomial	4.07(9)(33)	109(2)(9)	26.7(2)(0)	158.7(11)(89)	1.214(8)(0)
PQChPT	3.82(15)(25)	107(3)(7)	27.9(2)(0)	160.2(15)(54)	1.225(11)(0)
C-PQChPT	3.74(13)(21)	102(3)(6)	27.4(3)(0)	161.8(10)(0)	1.238(7)(0)

Table 1: *Results for the light quark masses and pseudoscalar decay constants, in physical units, from the polynomial, PQChPT and C-PQChPT fits, analysing only the combinations of quark masses satisfying $\mu_2 \geq \mu_1 = \mu_S$. The quoted errors are statistical (first) and systematic (second), the latter coming from the uncertainties in the determination of the lattice scale and of the quark mass renormalization constant.*

In table 1 we collect the results for the light quark masses and pseudoscalar decay constants, in physical units, and for the ratios m_s/m_{ud} and f_K/f_π, as obtained from the polynomial, PQChPT and C-PQChPT fits. To be conservative, we consider the results obtained from the analysis of the quark mass combinations satisfying the constraint $\mu_2 \geq \mu_1 = \mu_S$ which, though being affected by larger statistical errors, are safe from the effects of the potentially divergent chiral logarithms. In table 1 we quote as a systematic error within each fit the uncertainty associated with the determination of the lattice spacing and of the quark mass renormalization constant.

In order to derive our final estimates for the quark masses and decay constants, we perform a weighted average of the results of the three analyses presented in table 1 and conservatively include the whole spread among them in the systematic uncertainty. In this way, we obtain as our final estimates of the light quark masses the results

$$m_{ud}^{\overline{\rm MS}}(2\ {\rm GeV}) = 3.85 \pm 0.12 \pm 0.40\ {\rm MeV} \quad , \quad m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 105 \pm 3 \pm 9\ {\rm MeV}\,, \tag{4.1}$$

and the ratio

$$m_s/m_{ud} = 27.3 \pm 0.3 \pm 1.2\,, \tag{4.2}$$

where the first error is statistical and the second systematic. For the kaon decay constant and the ratio f_K/f_π we obtain the accurate determinations

$$f_K = 161.7 \pm 1.2 \pm 3.1\ {\rm MeV} \quad , \quad f_K/f_\pi = 1.227 \pm 0.009 \pm 0.024\,. \tag{4.3}$$

An interesting comparison of our results for the strange quark mass and the ratio f_K/f_π with other lattice QCD determinations is illustrated in fig. 3 (see ref. [1] for the full list of references).

An important finding of our analysis [1] is that the use of non-perturbative renormalization turns out to play a crucial role in the determination of the quark masses. The estimate $Z_P^{\rm RI-MOM}(1/a) = 0.39(1)(2)$ obtained with the RI-MOM method is in fact significantly smaller than the prediction $Z_P^{\rm BPT}(1/a) \simeq 0.57(5)$ given by one-loop boosted perturbation theory (in the same RI-MOM renormalization scheme) [10]. Had we used the perturbative estimate of Z_P we would have obtained $m_{ud}^{\overline{\rm MS}}(2\ {\rm GeV}) = 2.63 \pm 0.08 \pm 0.36$ MeV and $m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 72 \pm 2 \pm 9$ MeV. As shown in fig. 3 (left), our prediction for the strange quark mass in eq. (4.1) is in good agreement with other determinations based on a non-perturbative evaluation of the mass renormalization constant. The non-perturbative renormalization method, therefore, is found to have an important impact that can be even larger than the quenching effect and that should be kept in mind, particularly when combining the lattice results to produce the quark mass final averages.

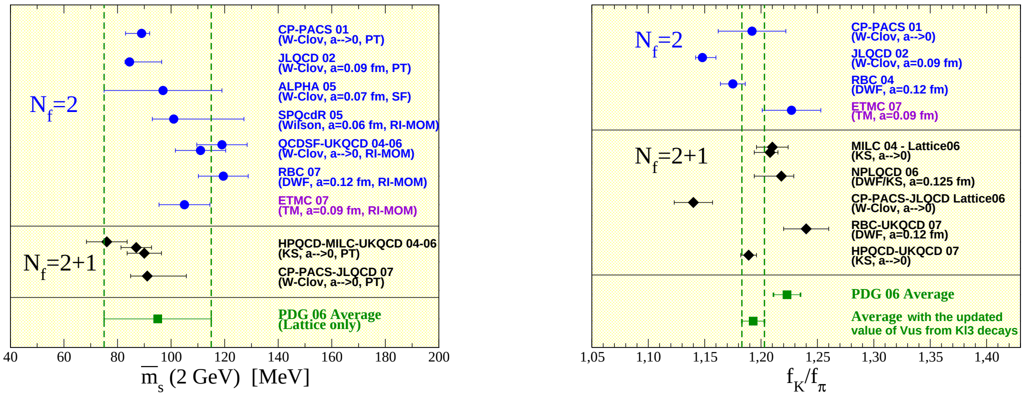

Figure 3: *Lattice QCD determinations of the strange quark mass (left) and of the ratio f_K/f_π (right) obtained from simulations with $N_f = 2$ and $N_f = 2+1$ dynamical fermions. The results are also compared with the PDG 06 averages [12] and, for f_K/f_π, with the average from the $K_{\ell 3}$ determination of V_{us} [13].*

Our result for the ratio f_K/f_π can be combined with the experimental measurement of $\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))/\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))$ [12] to get a determination of the ratio $|V_{us}|/|V_{ud}|$ [14]. We obtain $|V_{us}|/|V_{ud}| = 0.2251(5)(47)$, where the first error is the experimental one and the second is the theory error coming from the uncertainty on f_K/f_π. It yields, combined with the determination $|V_{ud}| = 0.97377(27)$ [15] from nuclear beta decays, the estimate $|V_{us}| = 0.2192(5)(45)$, in agreement with the value extracted from $K_{\ell 3}$ decays, $|V_{us}| = 0.2255(19)$ [13], and leads to the constraint due to the unitarity of the CKM matrix $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = (-3.7 \pm 2.0) \cdot 10^{-3}$.

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