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Disorder parameter for confinement and vacuum field strength correlators.

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Abstract.

The possibility is explored to relate confinement to properties of gauge invariant field strength correlators.

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1. Introduction

A disorder parameter for dual superconductivity of gauge theory vacuum has been developed [1] [2][3][4]. It is the *vev* , $\langle\mu\rangle$, of an operator carrying non zero magnetic charge $\frac{q}{2g}$. The Euclidean version is

$$\mu(\vec{x},t) = \exp[-\frac{q}{2g^2}\int d^3y \vec{E}(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})] \tag{1.1}$$

Here $\vec{b}_\perp(\vec{r}) = \frac{\vec{n}\wedge\vec{r}}{r(r-\vec{r}.\vec{n}}$ is the field of a monopole in the transverse gauge with

$\nabla\vec{b}_\perp = 0$

$\nabla\wedge\vec{b}_\perp = \frac{\vec{r}}{r^3} - 4\pi\theta(\vec{n}.\vec{r})\vec{n}\delta^2(\vec{r}_\perp)$.

The field $\vec{E}_\perp$ is the conjugate momentum to the transverse component of the potential $\vec{A}_\perp$ so that μ is nothing but a translation operator of $\vec{A}_\perp$, or

$$\mu(\vec{x},t)|\vec{A}_\perp(\vec{y},t)\rangle = |\vec{A}_\perp(\vec{y},t) + \frac{q}{2g}\vec{b}_\perp(\vec{x}-\vec{y})\rangle \tag{1.2}$$

It just creates a monopole.

One of the factors $\frac{1}{g}$ at the exponent of Eq(1.1) comes from the Dirac quantization condition for magnetic charge, the other one from the fact that the electric field in the lattice formulation has an additional multiplicative factor g . The operator μ can be written in the form

$$\mu = \exp[-\beta\Delta S] \tag{1.3}$$

with the usual notation $\beta = \frac{2N}{g^2}$ and $\Delta S = -\frac{q}{4N}\int d^3y\vec{E}(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})$. As a consequence $\langle\mu\rangle$ is the ratio of two partition functions and $\langle\mu\rangle = 1$ at $\beta = 0$.

$$\langle\mu\rangle = \frac{Z(S+\Delta S)}{Z(S)} \tag{1.4}$$

For compact $U(1)$ gauge theory a few theorems have been proved:

(1) $\mu(\vec{x},t)$ is a gauge invariant, Dirac like , magnetically charged operator , and obeys cluster property[4][5].

(2) $\langle\mu\rangle \neq 0$ for $\beta < \beta_c$ where there is confinement , $\langle\mu\rangle = 0$ for $\beta \geq \beta_c$ i.e. in the deconfined phase. β_c is the critical point.

In $U(1)$ gauge theory confinement is therefore produced by condensation of monopoles i.e. by dual superconductivity of the vacuum.

Instead of $\langle\mu\rangle$ it proves convenient to use the quantity

$$\rho = \frac{\partial\ln(\langle\mu\rangle)}{\partial\beta} \tag{1.5}$$

From Eq(1.4) it follows that $\rho = \langle S\rangle_S - \langle S+\Delta S\rangle_{S+\Delta S}$, the subscript indicating the action used in the statistical weight. Moreover because of the boundary condition $\mu = 1$ at $\beta = 0$,

$$\langle\mu\rangle = \exp[\int_0^\beta \rho(\beta')d\beta'] \tag{1.6}$$

For $SU(N)$ gauge theories, with and without quarks, $N-1$ operators μ^a can be defined ($a = 1,2,...N-1$), and the corresponding order parameters $\langle\mu^a\rangle$[2],[3]. The definition of μ^a has the same form as μ in Eq (1.1) with the field strength $\vec{E}_\perp$ replaced by $\vec{E}^a_\perp(\vec{y},t) \equiv Tr[\Phi^a \vec{E}(\vec{y},t)]_\perp$ where Φ^a selects the direction of the residual abelian gauge field in the abelian projected gauge. For a detailed discussion see Ref's[2],[3]. The choice of the abelian projection is irrelevant [6][7][8].

Fig(1) shows the numerical determination of $\langle\mu\rangle$ for compact $U(1)$ gauge theory, Fig(2) shows the corresponding quantity ρ , which presents a strong negative peak at the critical point β_c . The analysis goes as follows[2],[3]:

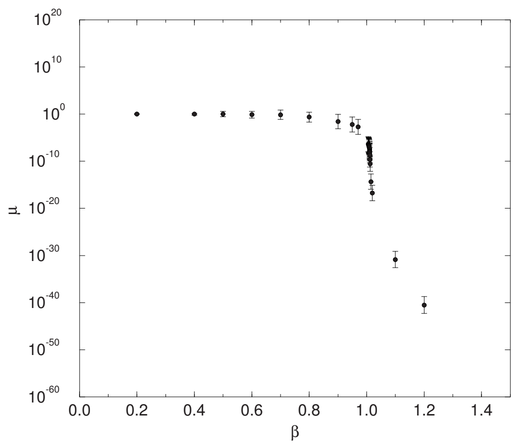

Figure 1: $\langle\mu\rangle$ for U(1) lattice gauge theory Ref[9].

1) For $\beta < \beta_c$ ρ^a tends to a finite limit in the thermodynamical limit $V = L_s^3 \to \infty$, and by use of Eq(1.6) $\langle\mu\rangle \neq 0$

2) For $\beta \geq \beta_c$ $\rho^a \approx -|c|L_s + c'$ with $c \neq 0$,or , by use of Eq(1.6) $lim_{L_s\to\infty}\langle\mu\rangle = 0$

3)At $\beta \approx \beta_c$ the correlation length goes large compared to the lattice spacing and the scaling

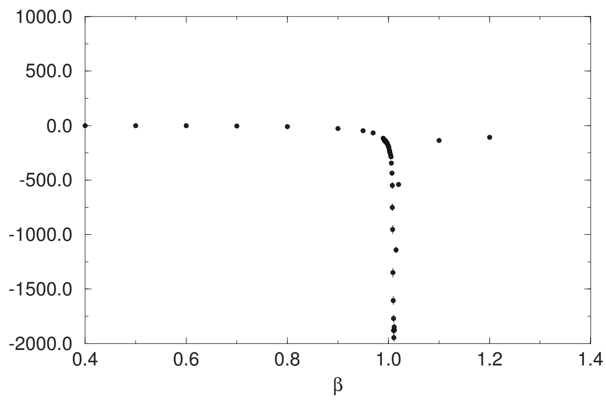

Figure 2: ρ versus β Ref.[1]

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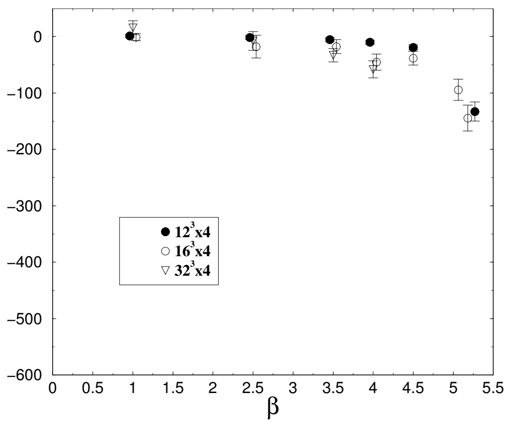

Figure 3: Strong coupling behavior of ρ at various lattice sizes and $am = 0.1335$ Ref.[11]

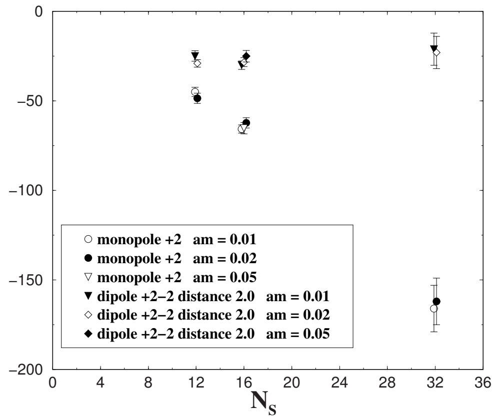

Figure 4: Volume dependence of ρ in the deconfined phase for different values of the magnetic charge.Ref [12]

law holds

$$\frac{\rho}{L_s^{\frac{1}{\nu}}} = f(\tau L_s^{\frac{1}{\nu}}) \tag{1.7}$$

where $\tau \equiv 1 - \frac{T}{T_c}$. ν is the critical index of the correlation length , and is typical of the universality class of the transition. For weak first order $\frac{1}{\nu} = 3$ [Quenched $SU(3)$[3] , $N_f = 2$ QCD [10]], for 3d-Ising $\frac{1}{\nu} = 1.6$ [SU(2) [2]], for $3d - O(4)$ $\frac{1}{\nu} = 1.336$.

The properties 1), 2), 3) as observed in different systems are shown in Figs (3), (4), (5)

The question we address [13] in this paper is whether $\langle \mu \rangle$ can be computed in the frame of the Stochastic Vacuum Model of QCD[14][15] [16] . The model consists in expressing physical quantities in terms of gauge invariant correlators of field strengths , making a cluster expansion of them and keeping only the two point cluster. The model provides a good description of many aspects of *QCD* and it would be interesting to know if the distinction between confined and deconfined phase could be read in the behavior of the correlators.

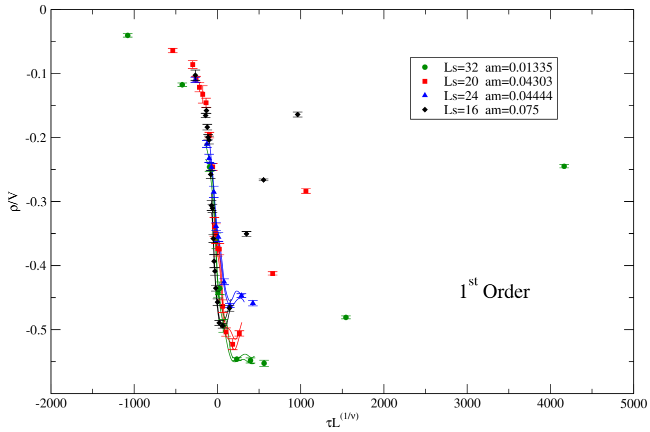

Figure 5: Scaling of ρ assuming first order for the deconfining transition. $N_f = 2$ *QCD*. Ref. [10]

2. Cluster expansion of $\langle \mu^a \rangle$

The series expansion of the exponential in Eq(1.1) reads [13]

$$\langle \exp[-\frac{q}{2g^2}\int d^3y \vec{E}^a(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})] \rangle = \tag{2.1}$$

$$\Sigma_n \frac{1}{n!}(-\frac{q}{2g})^n \int d^3x_1 \int d^3x_2 ... \int d^3x_n \vec{b}^{i_1}_\perp(\vec{x}-\vec{y}_1).......\vec{b}^{i_n}_\perp(\vec{x}-\vec{y}_n)\langle E^a_{i_1}(\vec{y}_1,t).....E^a_{i_n}(\vec{y}_n,t)\rangle$$

According to the stochastic vacuum model one performs a cluster expansion of the correlators: the one point cluster is zero by symmetry, and clusters of order higher than 2 are neglected. Keeping the correct combinatorics into account [13] the net result is

$$\langle \mu \rangle = \exp[-\frac{q^2}{8g^2}\int d^3y_1 d^3y_2 b^{i_1}_\perp(\vec{y}_1-\vec{x}) b^{i_2}_\perp(\vec{y}_2-\vec{x})\langle E^a_{i_1}(\vec{y}_1,t)E^a_{i_2}(\vec{y}_2,t)\rangle] \tag{2.2}$$

Higher clusters are $O(q^4)$ at the exponent. Here $\vec{E}^a = Tr[\Phi^a \vec{E}]$. The gauge invariant correlator at the exponent of Eq(2.2)

$$\langle E^a_{i_1}(\vec{y}_1,t)E^a_{i_2}(\vec{y}_2,t)\rangle = \Phi^a_{i_1 i_2}(\vec{y}_1-\vec{y}_2) \tag{2.3}$$

in principle depends on the path C used to parallel transport from $\vec{y}_1$ to $\vec{y}_2$ but this dependence is irrelevant to the study of the ultraviolet and infrared behavior. Since $\beta \equiv \frac{2N}{g^2}$, by use of Eq's (1.5), (2.2) we get

$$\rho^a = \frac{\partial}{\partial \beta}[-\frac{q^2}{16N}\beta \int d^3y_1 \int d^3y_2 b^{i_1}_\perp(\vec{y}_1) b^{i_2}_\perp(\vec{y}_2)\Phi^a_{i_1 i_2}(\vec{y}_1-\vec{y}_2)] \tag{2.4}$$

In Fourier transform

$$\vec{b}_\perp(\vec{k}) = \frac{\vec{n}\wedge\vec{k}}{k(k-\vec{k}.\vec{n}+i\varepsilon)}, \vec{H}(\vec{k}) = -i[\frac{\vec{k}}{k^2} - \frac{\vec{n}}{(\vec{n}\vec{k}-i\varepsilon)}] \tag{2.5}$$

$$\Phi^a_{ij}(\vec{k}) = (k^2\delta_{ij} - K_i k_j) f(k^2) \tag{2.6}$$

independent on a [16].

$$(k^2\delta_{ij} - k_i k_j) b^i_\perp(\vec{k}) b^j_\perp(-\vec{k}) = |\vec{H}(\vec{k})|^2 = \frac{1}{k_z^2} - \frac{1}{k^2} \tag{2.7}$$

The equation follows for ρ

$$\rho = -\frac{q^2}{16N}\frac{\partial}{\partial\beta}[\beta\int\frac{d^3k}{(2\pi)^3} f(k^2)(\frac{1}{k_z^2} - \frac{1}{k^2})] \tag{2.8}$$

In the deconfined phase the perturbative expression can be used $\frac{f(k^2)}{(2\pi)^3} = \frac{1}{2k}$ and

$$\rho^a = \frac{\pi q^2}{8N}[-\sqrt{2}L_s + 2\ln(L_s) + constant] \tag{2.9}$$

with L_s the spatial size of the lattice , and

$$L_s = \frac{(L_s a)}{a} \equiv \frac{IR - cutoff}{UV - cutoff} \tag{2.10}$$

In the thermodynamical limit $L_s \to \infty$,$\rho \to -\infty$ as in Fig(4), and by use of Eq(1.5), $\mu \to 0$.

In the confined phase one expects the same UV behavior, which is dictated by OPE ,but aL_s will be replaced by some IR cutoff Λ so that ,at fixed lattice spacing ρ^a is volume independent as in Fig(3) .

This will never be the case if Eq(2.8) holds, no matter how IR well behaved is the correlator: the term $\propto \frac{1}{k_z^2}$ due to the Dirac string will always diverge.

This means that the stochastic approach is inadequate in the confined phase. Indeed in that phase the vacuum is a Bogolubov-Valatin superposition of states with different magnetic charge and the operator μ will connect sectors differing by q units of magnetic charge : The Dirac string will then end on an antimonopole and the integral will be IR cut-off by a massive propagator and be volume independent.

This can be checked in $U(1)$ theory in the dual formulation of Polyakov [17].The potential of the dual field χ is there proportional to $cos(\chi)$ which in the weak coupling is equivalent to a mass term, and gives a gaussian distribution like the stochastic vacuum model in QCD. In the strong coupling regime the tunneling between the minima of $cos(\chi)$ provides a Bogolubov-Valatin vacuum.

Like the Polyakov line the order parameter $\langle\mu\rangle$ is singular in the continuum limit $a \to 0$, but its IR behavior at any finite UV cutoff detects confinement or deconfinement.

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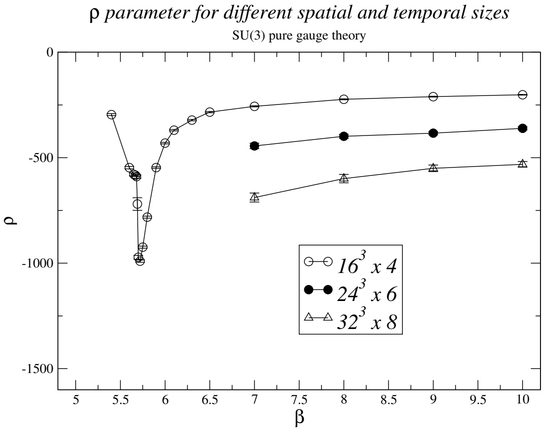

Figure 6: Check of Eq(2.9)

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