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Neutral mesons and disconnected diagrams in Twisted Mass QCD

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We evaluate properties of neutral mesons in $N_f = 2$ dynamical simulations of TMQCD at maximal twist. The pion is explored - establishing the size of the isospin splitting (an order a^2 effect). We investigate the η' (the $N_f = 2$ flavour singlet pseudoscalar meson) and neutral ρ and scalar mesons. We show that disconnected diagrams can be evaluated very efficiently in TMQCD using variance reduction methods.

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1. Introduction

Here we discuss TMQCD at maximal twist with $N_f = 2$ degenerate sea quarks using configurations from ETMC [1−3]. In particular we focus on neutral mesons, which have some unusual properties in TMQCD. Thus we evaluate the disconnected contributions which are needed for a study of neutral mesons [4]. We present a new method which, for twisted mass, allows many disconnected contributions to be evaluated very efficiently.

We then present results for pions (exploring the charge splitting between π^0 and π^+). We also present results for flavour singlet pseudoscalar mesons (the η_2 meson), for vector mesons (charge splitting and decays), and for flavour-singlet scalar mesons.

2. Disconnected diagrams - variance reduction

TMQCD has a degenerate pair of $u,\ d$ quarks with fermion matrix: $M_{u,d} = M_W \pm i\mu\gamma_5$ where M_W is the Wilson-Dirac matrix. Hence $1/M_u - 1/M_d = -2i\mu(1/M_d)\gamma_5(1/M_u)$.

Consider the disconnected contribution $\sum X(1/M_u - 1/M_d)$ where X is some γ-matrix and/or colour-matrix and the sum is over space. The conventional method involves solving $\phi_r = (1/M_u)\xi_r$ with stochastic volume sources ξ_r, then $\sum X/M_u = \sum\langle\xi^* X\phi\rangle_r$ where the average is over noise samples (labelled r). However, the case mentioned above can be evaluated efficiently using the 'one-end-trick' [5, 6]. Then the required disconnected loop is given by

$$\sum X(1/M_u - 1/M_d) = -2i\mu\sum\langle\phi^* X\gamma_5\phi\rangle_r$$

This has signal/noise $V/\sqrt{V^2} = 1$ which is much more favourable than the conventional method with signal/noise $1/\sqrt{V}$ (here $V = L^3T$).

For example, at $\beta = 3.9$, $\mu = 0.004$, $L = 24$, (where $M(\pi) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm) taking $X = i$ which is appropriate for the η_2 correlator, we evaluate the momentum-zero loop at a time-slice. We find a standard deviation of $\sigma = 18$ from inherent gauge-time variation whereas the stochastic noise is $\sigma = 87$ from 24 samples of volume source (conventional method) but only $\sigma = 7.5$ from above method (with 12 samples). So 12 inversions give the disconnected correlator from all t to all t' with no significant increase in errors from the stochastic evaluation.

For cases where this method cannot be used, eg. π^0 with $\bar\psi\gamma_5\tau_3\psi \to \bar\chi I\chi$ at maximal twist, we use hopping parameter variance reduction [7] instead.

3. Pion order(a^2) effects

For ETMC data with $M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm and $La = 2.1$ fm, we show the ratios of correlators in fig. 1. The disconnected pieces are seen to be relatively large - and reduce the charge splitting as found previously [4]. The π^0 is lighter than the π^+, unlike a previous preliminary study of dynamical fermions [8]. From a 4×4 fit to these correlations, we obtain $\Delta ma = 0.027(7)$. Since we expect $r_0^2(m(\pi^0)^2 - m(\pi^+)^2) = c(a/r_0)^2$, we compare this expression with results from several lattice data sets [2] in fig. 2.

We see that, as expected, the flavour splitting decreases as a^2. The sign and behaviour are consistent with Chiral PT and the nature of the phase transition [1] where $m(\pi^0) = 0$. We can use

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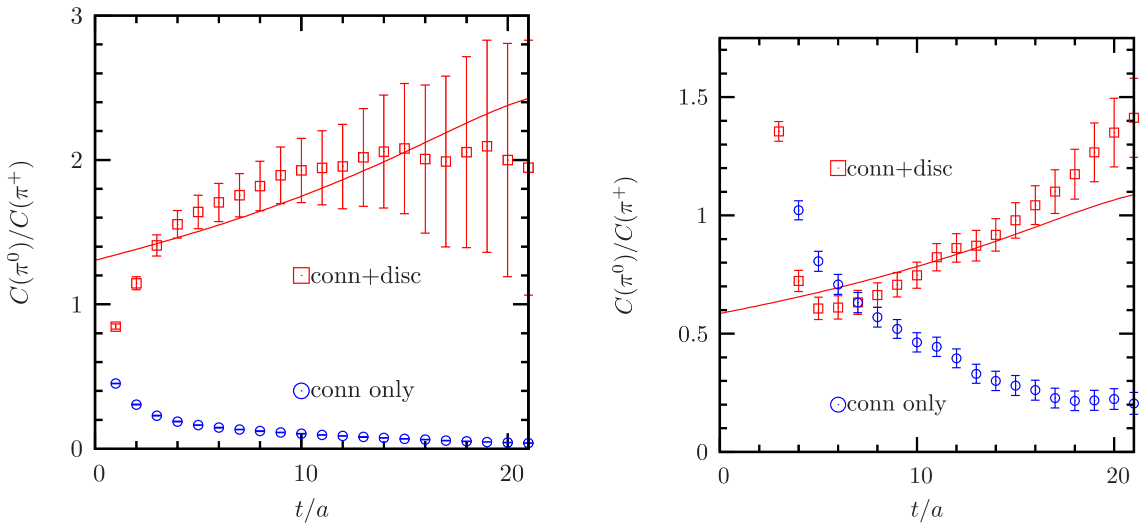

Figure 1: Ratio of correlators between neutral and charged pions. The left plot is for local operators $\bar{\chi}I\chi$ for π^0 and $\bar{\chi}\gamma_5\chi$ for π^+ and the right plot for local operators $\bar{\chi}\gamma_4\gamma_5\tau_3\chi$ for π^0 and $\bar{\chi}\gamma_4\chi$ for π^+. The curves give the ratio arising from the mass difference determined by the full fit.

this determination to estimate the consequences of smaller lattice spacing, for instance less than 20% pion splitting for $m(\pi^+) = 200$ MeV provided $r_0/a > 8.2$.

4. Flavour singlet PS meson: η_2

In QCD, the flavour singlet pseudoscalar meson acquires a mass through the anomaly, so is not a goldstone boson. It is important to check that this feature, which is linked to topological charge fluctuations, is reproduced in lattice evaluations. With $N_f = 2$ degenerate quarks, the flavour singlet pseudoscalar meson (called η_2) is related to the experimental $\eta'(958)$ and is expected [9] to have a mass around 800 MeV. We fit the η_2 correlators (2×2 matrix with local and non-local operator $\bar{\psi}\gamma_5\psi \to \bar{\chi}\tau_3\chi$ at maximal twist) for t-range 3-10 with 2 states. We compare results from TMQCD [3] with older results (see ref. [10] for a review) in fig. 3. Note that the ETMC results are at substantially smaller quark masses. We see that the η_2 mass is consistent with a constant behaviour in the chiral limit with $m(\eta_2) \approx .88$ GeV ($r_0 m(\eta_2) = 2$).

We now discuss why the errors are so large for the η_2, despite the fact that we measure all t and t', we use many gauge configurations and stochastic errors are small. The origin of the problem is that the signal for the disconnected part of the correlator comes from only a small part of the total data sample. For instance (at $\mu = 0.004$ with 48 t-values for 888 gauge configurations) with $|t'-t| = 10$, 2.1% of the data contributes 26% of the signal. Thus the statistical impact of the data set is smaller than expected since parts of the data have big fluctuations (in a fermionic loop related to topological charge density). So even more configurations are needed to get reliable and small errors in the case of disconnected contributions.

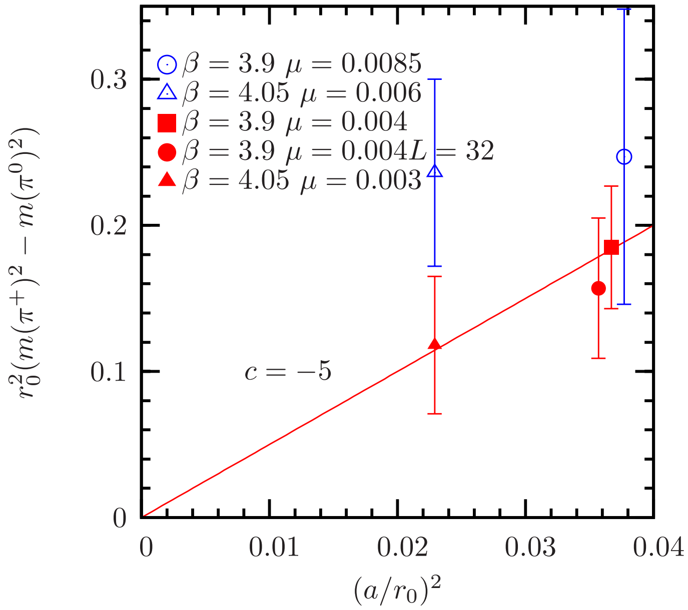

Figure 2: Pion charge splitting

5. Vector mesons

We compare the local-local correlators (including disconnected parts for the neutral meson) for vector mesons in fig. 4. Note that the disconnected contribution is negligible for the vector coupling to neutral ρ-mesons. We find that the ratio is consistent with constant (ie no mass splitting) and the value of that constant can be related to renormalisation constants. For the vector coupling to neutral ρ-mesons this implies $(Z_A/Z_V)^2 \approx 1.5$ at the finer lattice spacing, consistent with [12]. We find agreement with a ratio of 1.0 for the tensor couplings, as expected since there is only one tensor renormalisation which then cancels.

From fits to a 2×2 matrix of correlators (from the connected neutral contribution only here in t-range 8-18) we obtain ρ^0 masses (in lattice units). We also report values for the ρ^+ masses [3] (from fits to a 4×4 matrix of correlators). These values are consistent with no flavour splitting for vector mesons as expected [11].

β	3.9	3.9	3.9	4.05	4.05
μ	0.004	0.004	0.0085	0.003	0.006
L	32	24	24	32	32
$am(\rho^0)$	.400(25)	.395(17)	.419(17)	.372(29)	.346(12)
$am(\rho^+)$	.416(12)	.404(22)	.428(8)	.337(20)	.337(12)

We now consider decay transitions from the vector meson to two pions following the methods used in ref. [13]. The transition $\rho^0(0) \to \pi^+(1)\pi^-(-1)$, where $q(\pi) = \pm 2\pi/L$, even for the lightest pion ($M(\pi^+) \approx 300$ MeV, $L = 24a$, $a = 0.086$ fm) is not open (note, however, that decay for a ρ^0 meson with non-zero momentum is open). For $\rho^0(0)$, there is an energy splitting of $\Delta ma = 0.19$ assuming that the two pion state has twice the energy of a π^+ with appropriate momentum.

We show the normalised result in fig. 5 where it is compared to a two-state model [13]. The value of the transition amplitude x is consistent with the empirical ρ decay width. One can also estimate the effect of this mixing transition on the ρ mass using the two-state model. This gives a downward shift of ma of .02 (eg from .41 to .39 for the ρ mass). This shift, induced by the proximity of the lightest two pion level, is comparable to our statistical error in determining the ρ mass. This suggests that we do not yet see major modifications of our ρ meson masses from mixing with the decay channel.

6. Flavour singlet scalar mesons

There is considerable confusion in allocating the experimental flavour-singlet scalar meson (f_0) spectrum to specific content: since scalar glueball, $\bar{u}u + \bar{d}d$, $\bar{s}s$, $\pi\pi$ and/or KK in an S-wave, etc., can all contribute. Lattice QCD can help considerably here, but it will be difficult as we now illustrate.

We took a first look in TMQCD (here $M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm) and made a 2 state fit (t-range 6 to 23) to the 6×6 correlator matrix (P, S, A_4 both local and fuzzed at sink and source, including disconnected contributions). We find states $ma = .103(5)$ (π^0) and $m'a = .227(28)$ (f_0, energy consistent with $2m(\pi^0)$).

Thus in the scalar channel we find a clear signal - but at the mass of two pions. This is not unexpected - but emphasises the problems of studying scalar mesons with light quarks in dynamical lattice gauge theory, where the light two-body state will dominate the correlators.

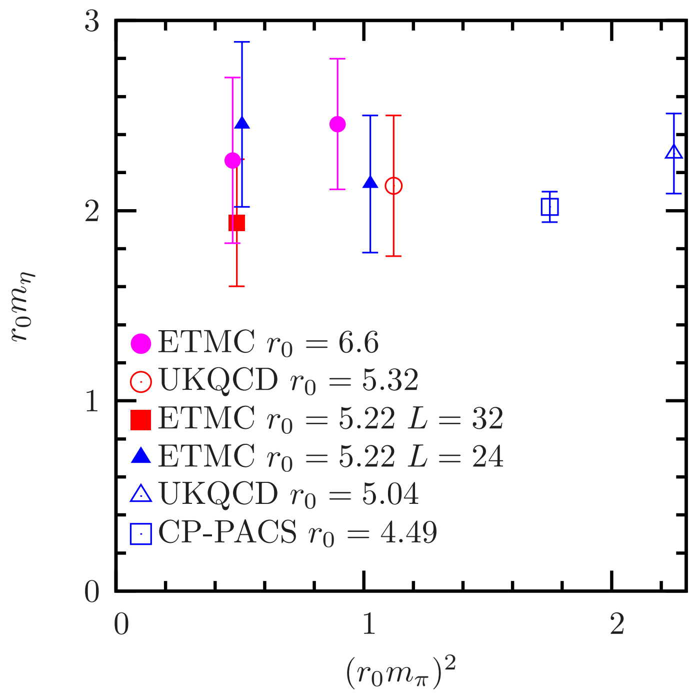

Figure 3: η_2 mass versus quark mass. Here we summarise results with light pions and $a < 0.1$ fm ($r_0/a > 4.5$).

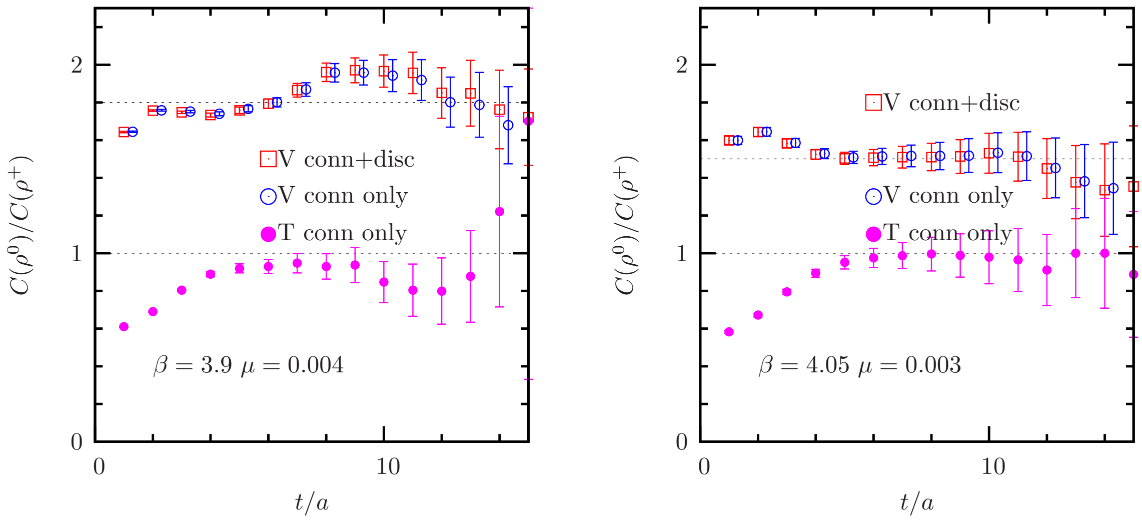

Figure 4: Ratios of correlator for neutral ρ-mesons to charged. The dotted lines guide the eye in the case that there is no mass splitting.

7. Summary

TMQCD allows efficient evaluation of disconnected contributions using a powerful variance reduction method.

The π charge splitting goes to zero like a^2 as expected, and the sign is consistent with the nature of the phase transition.

The flavour singlet pseudoscalar meson (η_2) has been studied to lighter quarks than previously and is consistent with a mass of around 800 MeV in chiral limit.

For vector mesons we find negligible charge splitting and the decay (transition to π π) is accessible to study.

Our study of the flavour singlet scalar meson (f_0) with light dynamical quarks, finds the expected π π contribution which will obscure further study of heavier states.

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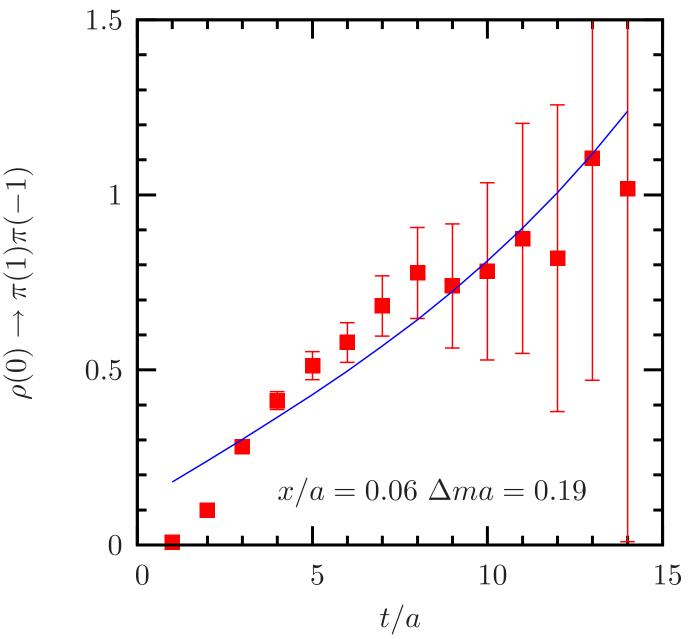

Figure 5: Transition $\rho^0 \to \pi^+\pi^-$ from the lattice (with $M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm). The solid line is from a two state model (ρ at rest and two pions with momentum $q = \pm 2\pi/L$) with energy gap $\Delta ma = 2aE(\pi) - am(\rho) = 0.19$ and transition amplitude x/a.

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