

## Azimuthal angle decorrelation of Mueller–Navelet jets at NLO

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In this contribution we study azimuthal angle decorrelation in inclusive dijet cross sections taking into account the next-to-leading (NLO) corrections to the BFKL kernel while keeping the jet vertices at leading order. We show how the angular decorrelation for jets with a wide relative separation in rapidity largely decreases when the NLO corrections are included.

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## 1. Introduction

One of the important questions still open in Quantum Chromodynamics is how to describe scattering amplitudes in the Regge limit where the center-of-mass energy,  $s$ , is much larger than all other Mandelstam invariants and mass scales. In this region it is needed to resum logarithmically enhanced contributions of the form  $(\alpha_s \ln s)^n$  to all orders. This is achieved using the leading-logarithmic (LL) Balitsky–Fadin–Kuraev–Lipatov (BFKL) evolution equation [1].

Observables where BFKL effects should be dominant require of a large energy to build up the parton evolution and the presence of two large and similar transverse scales. An example is the inclusive hadroproduction of two jets with large and similar transverse momenta and a big relative separation in rapidity, the so-called Mueller–Navelet jets. When  $Y$ , the distance in rapidity between the most forward and backward jets, is not large a fixed order perturbative analysis should be enough to describe the cross section but when it increases a BFKL resummation of  $(\alpha_s Y)^n$  terms is needed. This observable was proposed in Ref. [2] as a clean configuration to look for BFKL effects at hadron colliders. A power-like rise for the partonic cross section was predicted. At a more exclusive level one can study the azimuthal angle decorrelation of the pair of jets. BFKL enhances soft real emission as  $Y$  increases reducing the angular correlation in transverse plane originally present in the back-to-back Born configuration. The LO prediction for this azimuthal dependence was first investigated in Ref. [3] where it was shown that it overestimates the rate of decorrelation when compared with the Tevatron data [4]. In the present contribution we summarize the work of Ref. [5] where  $\alpha_s (\alpha_s Y)^n$  next-to-leading logarithmic (NLL) corrections to the BFKL kernel [6] were included. Previous numerical studies of this kernel were performed in Ref. [7] using an implementation of the NLL iterative solution proposed in Ref. [8]. In coming works we would like to include the next-to-leading order (NLO) jet vertex [9], investigate more convergent versions of the kernel [10] and include parton distributions effects. It will be also interesting to study if the jet definition in the NLO kernel proposed in Ref. [11] has sizable phenomenological implications. We believe Mueller–Navelet jets should be an important test of our understanding of small  $x$  resummation to be performed at the Large Hadron Collider at CERN.

## 2. The dijet partonic cross section

We are interested in the calculation of the partonic cross section  $\text{parton} + \text{parton} \rightarrow \text{jet} + \text{jet} + \text{soft emission}$ , with the two jets having transverse momenta  $\vec{q}_1$  and  $\vec{q}_2$  and being produced at a large relative rapidity separation  $Y$ . In the particular case of gluon–gluon scattering we have

$$\frac{d\hat{\sigma}}{d^2\vec{q}_1 d^2\vec{q}_2} = \frac{\pi^2 \bar{\alpha}_s^2}{2} \frac{f(\vec{q}_1, \vec{q}_2, Y)}{q_1^2 q_2^2}, \quad (2.1)$$

with  $\bar{\alpha}_s = \alpha_s N_c / \pi$  and  $f$  the gluon Green's function which is the solution to the BFKL equation.

The partonic cross section is obtained by integration over the phase space of the two emitted gluons together with the jet vertices:

$$\hat{\sigma}(\alpha_s, Y, p_{1,2}^2) = \int d^2\vec{q}_1 \int d^2\vec{q}_2 \Phi_{\text{jet}_1}(\vec{q}_1, p_1^2) \Phi_{\text{jet}_2}(\vec{q}_2, p_2^2) \frac{d\hat{\sigma}}{d^2\vec{q}_1 d^2\vec{q}_2}. \quad (2.2)$$

For the jet vertices we use the LO ones  $\Phi_{\text{jet}_i}^{(0)}(\vec{q}, p_i^2) = \theta(q^2 - p_i^2)$ , where  $p_i^2$  corresponds to a resolution scale for the transverse momentum of the gluon jet. In this way a full NLO accuracy is not achieved but it is possible to pin down those effects stemming from the gluon Green's function. To extend this analysis it would be needed to calculate the Mellin transform of the NLO jet vertices in Ref. [9] where the definition of a jet is much more involved than here. To proceed further it is very convenient to recall the work of Ref. [12] and use the operator representation  $\hat{q}|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle$  with normalization  $\langle\vec{q}_1|\hat{1}|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2)$ . In this notation the BFKL equation simply reads  $(\omega - \hat{K})\hat{f}_\omega = \hat{1}$  where we have performed a Mellin transform in rapidity space:

$$f(\vec{q}_1, \vec{q}_2, Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y} f_\omega(\vec{q}_1, \vec{q}_2). \quad (2.3)$$

The kernel has the expansion  $\hat{K} = \bar{\alpha}_s \hat{K}_0 + \bar{\alpha}_s^2 \hat{K}_1 + \dots$ . To NLO accuracy this implies that the solution can be written as

$$\hat{f}_\omega = (\omega - \bar{\alpha}_s \hat{K}_0)^{-1} + \bar{\alpha}_s^2 (\omega - \bar{\alpha}_s \hat{K}_0)^{-1} \hat{K}_1 (\omega - \bar{\alpha}_s \hat{K}_0)^{-1} + \mathcal{O}(\bar{\alpha}_s^3). \quad (2.4)$$

The basis on which to express the cross section reads

$$\langle\vec{q}|v, n\rangle = \frac{1}{\pi\sqrt{2}} (q^2)^{iv - \frac{1}{2}} e^{in\theta}. \quad (2.5)$$

The action of the NLO kernel on this basis, which was calculated in Ref. [13], contains non-diagonal terms and can be written as

$$\begin{aligned} \hat{K}|v, n\rangle = & \left\{ \bar{\alpha}_s \chi_0(|n|, v) + \bar{\alpha}_s^2 \chi_1(|n|, v) \right. \\ & \left. + \bar{\alpha}_s^2 \frac{\beta_0}{8N_c} \left[ 2\chi_0(|n|, v) \left( i \frac{\partial}{\partial v} + \log \mu^2 \right) + \left( i \frac{\partial}{\partial v} \chi_0(|n|, v) \right) \right] \right\} |v, n\rangle, \end{aligned} \quad (2.6)$$

where, from now on,  $\bar{\alpha}_s$  stands for  $\bar{\alpha}_s(\mu^2)$ , the coupling evaluated at the renormalization point  $\mu$  in the  $\overline{\text{MS}}$  scheme. The first line of Eq. (2.6) corresponds to the scale invariant sector of the kernel. Both functions  $\chi_0$  and  $\chi_1$  can be found in Ref. [5].

Using the notation

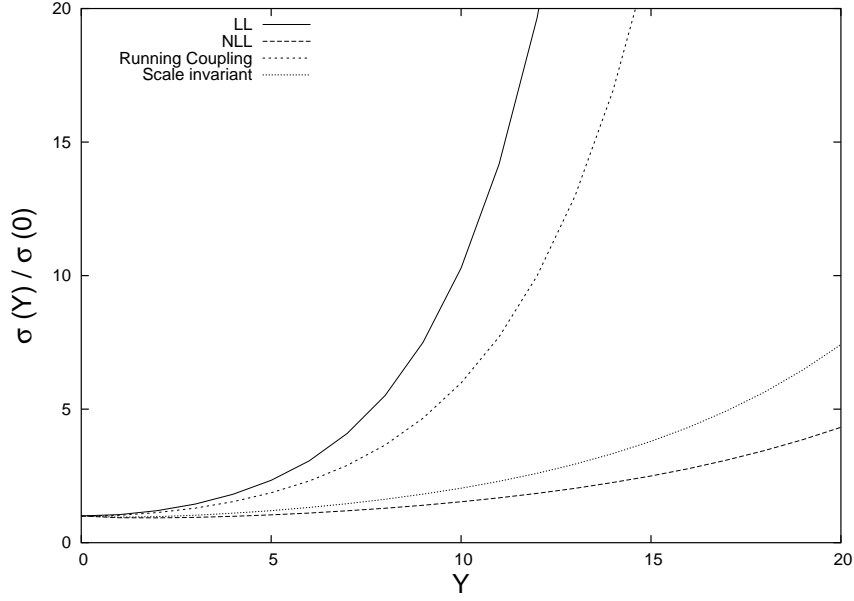
$$c_1(v) \equiv \frac{1}{\sqrt{2}} \frac{1}{(\frac{1}{2} - iv)} (p_1^2)^{iv - \frac{1}{2}}, \quad (2.7)$$

and  $c_2$  being the complex conjugate of this expression with  $p_1^2$  replaced by  $p_2^2$ , we can then write the cross section as

$$\begin{aligned} \hat{\sigma}(\alpha_s, Y, p_{1,2}^2) = & \frac{\pi^2 \bar{\alpha}_s^2}{2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dv e^{\bar{\alpha}_s \chi_0(|n|, v) Y} c_1(v) c_2(v) \delta_{n,0} \\ & \times \left\{ 1 + \bar{\alpha}_s^2 Y \left[ \chi_1(|n|, v) + \frac{\beta_0}{4N_c} \left( \log(\mu^2) + \frac{i}{2} \frac{\partial}{\partial v} \log \left( \frac{c_1(v)}{c_2(v)} \right) + \frac{i}{2} \frac{\partial}{\partial v} \right) \chi_0(|n|, v) \right] \right\}. \end{aligned} \quad (2.8)$$

The angular differential cross section in the case where the two resolution momenta are equal,  $p_1^2 = p_2^2 \equiv p^2$ , and using the notation  $\phi = \theta_1 - \theta_2 - \pi$  can be expressed as

$$\begin{aligned} \frac{d\hat{\sigma}(\alpha_s, Y, p^2)}{d\phi} = & \frac{\pi^2 \bar{\alpha}_s^2}{4p^2} \sum_{n=-\infty}^{\infty} \frac{1}{2\pi} e^{in\phi} \int_{-\infty}^{\infty} dv e^{\bar{\alpha}_s \chi_0(|n|, v) Y} \frac{1}{(\frac{1}{4} + v^2)} \\ & \times \left\{ 1 + \bar{\alpha}_s^2 Y \left[ \chi_1(|n|, v) - \frac{\beta_0}{8N_c} \chi_0(|n|, v) \left( 2 \log \left( \frac{p^2}{\mu^2} \right) + \frac{1}{(\frac{1}{4} + v^2)} \right) \right] \right\}. \end{aligned} \quad (2.9)$$



**Figure 1:** Evolution of the partonic cross section with the rapidity separation of the dijets.

It can be conveniently rewritten as

$$\frac{d\hat{\sigma}(\alpha_s, Y, p^2)}{d\phi} = \frac{\pi^3 \bar{\alpha}_s^2}{2p^2} \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{in\phi} \mathcal{C}_n(Y), \quad (2.10)$$

with

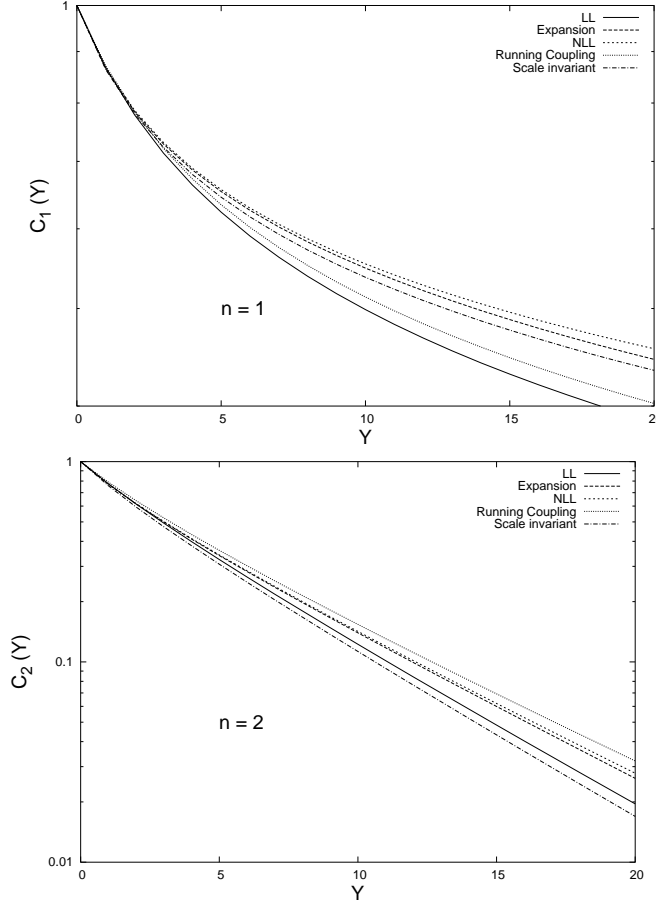
$$\mathcal{C}_n(Y) = \int_{-\infty}^{\infty} \frac{dv}{2\pi} e^{\frac{\bar{\alpha}_s(p^2)Y \left( \chi_0(|n|, v) + \bar{\alpha}_s(p^2) \left( \chi_1(|n|, v) - \frac{\beta_0}{8N_c} \frac{\chi_0(|n|, v)}{\left(\frac{1}{4} + v^2\right)} \right) \right)}{\left(\frac{1}{4} + v^2\right)}}. \quad (2.11)$$

The coefficient governing the energy dependence of the cross section corresponds to  $n = 0$ :

$$\hat{\sigma}(\alpha_s, Y, p^2) = \frac{\pi^3 \bar{\alpha}_s^2}{2p^2} \mathcal{C}_0(Y). \quad (2.12)$$

The rise in rapidity of this observable, with  $p = 30 \text{ GeV}$ ,  $n_f = 4$  and  $\Lambda_{\text{QCD}} = 0.1416 \text{ GeV}$ , is shown in Fig. 1. Clearly the NLL intercept is very much reduced with respect to the LL case. In our plots we show the results for a LO version of the kernel, which has the largest intercept. Then we plot the effects of the running of the coupling alone, which tends to reduce the growth with rapidity. And finally we show the contribution from the scale invariant pieces which provide the most negative pieces. The remaining coefficients with  $n \geq 1$  all decrease with  $Y$ . This can be seen in the plots of Fig. 2. The consequence of this decrease is that the angular correlations also diminish as the rapidity interval between the jets gets larger. This point can be studied in detail using the mean values

$$\langle \cos(m\phi) \rangle = \frac{\mathcal{C}_m(Y)}{\mathcal{C}_0(Y)}. \quad (2.13)$$



**Figure 2:** Evolution in  $Y$  of the  $\mathcal{C}_n(Y)$  coefficients for  $n = 1, 2, 3$ .

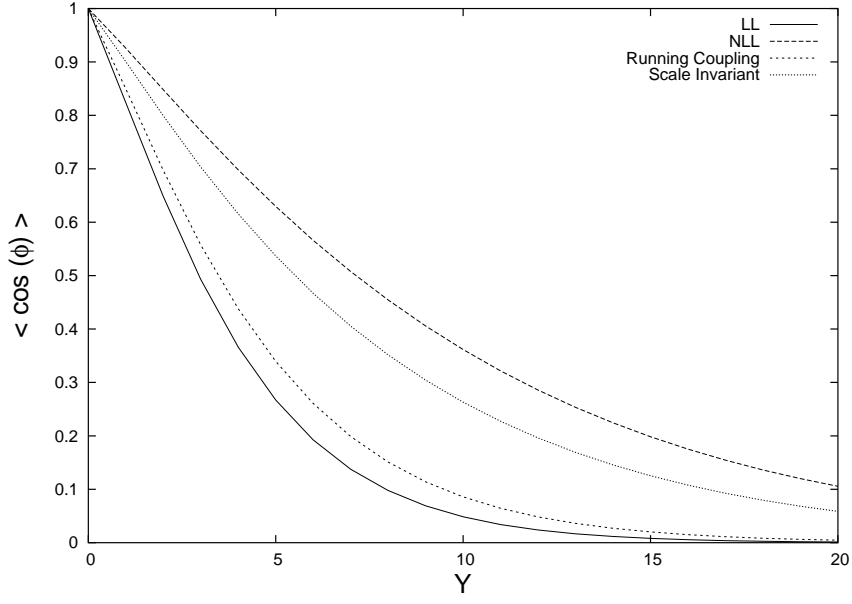
$\langle \cos(\phi) \rangle$  is calculated in Fig. 3. The most important consequence of this plot is that the NLL effects dramatically decrease the azimuthal angle decorrelation. This is already the case when only the running of the coupling is introduced but the scale invariant terms make this effect much bigger. This is encouraging from the phenomenological point of view given that the data at the Tevatron typically have lower decorrelation than predicted by LL BFKL or LL with running coupling. It is worth noting that the difference in the prediction for decorrelation between LL and NLL is mostly driven by the  $n = 0$  conformal spin. This can be understood looking at the ratio

$$\frac{\langle \cos(\phi) \rangle^{\text{NLL}}}{\langle \cos(\phi) \rangle^{\text{LL}}} = \frac{\mathcal{C}_1^{\text{NLL}}(Y) \mathcal{C}_0^{\text{LL}}(Y)}{\mathcal{C}_0^{\text{NLL}}(Y) \mathcal{C}_1^{\text{LL}}(Y)}, \quad (2.14)$$

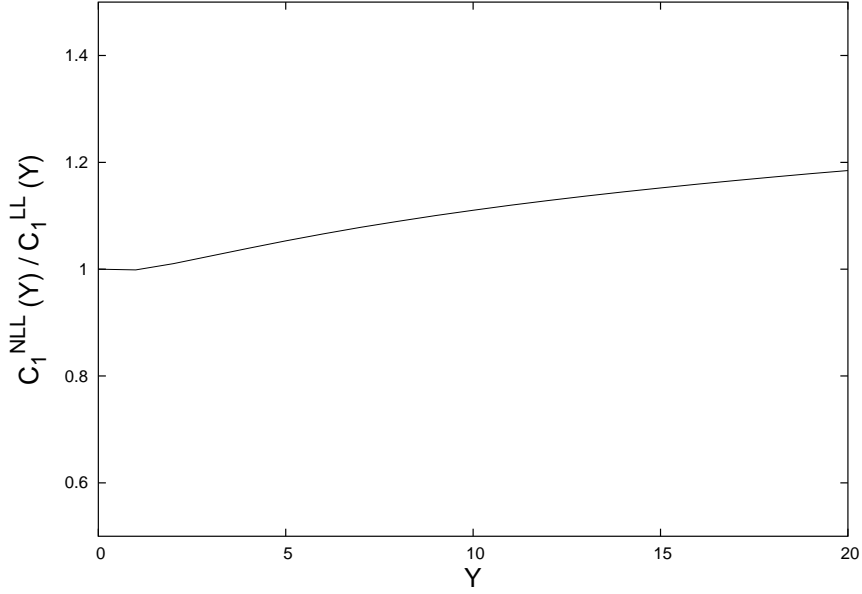
and noticing that

$$1.2 > \frac{\mathcal{C}_1^{\text{NLL}}(Y)}{\mathcal{C}_1^{\text{LL}}(Y)} > 1. \quad (2.15)$$

This ratio is calculated in Fig. 4. This point is a consequence of the good convergence in terms of asymptotic intercepts of the NLL BFKL calculation for conformal spins larger than zero. In particular the  $n = 1$  case is special in that the property of zero intercept at LL,  $\chi_0(1, 1/2) = 0$ , is



**Figure 3:** Dijet azimuthal angle decorrelation as a function of their separation in rapidity.



**Figure 4:** Comparative ratio between the NLL and LL coefficients for  $n = 1$  conformal spin.

preserved under radiative corrections since

$$\chi_1 \left( 1, \frac{1}{2} \right) = \mathcal{S} \chi_0 \left( 1, \frac{1}{2} \right) + \frac{3}{2} \zeta(3) - \frac{\beta_0}{8N_c} \chi_0^2 \left( 1, \frac{1}{2} \right) + \frac{\psi''(1)}{2} - \phi \left( 1, \frac{1}{2} \right) \quad (2.16)$$

is also zero.

### 3. Conclusions

An analytic procedure has been presented to calculate the effect of higher order corrections in

the description of Mueller–Navelet jets where two jets with moderately high and similar transverse momentum are produced at a large relative rapidity separation in hadron–hadron collisions. This is a promising observable to study small  $x$  physics at the Large Hadron Collider at CERN given its large energy range. The focus of the analysis has been on those effects with direct origin in the NLO BFKL kernel, while the jet vertices have been considered at LO accuracy. It has been shown how the growth with energy of the cross section is reduced when going from a LL to a NLL approximation, and how the azimuthal angle decorrelations largely decrease due to the higher order effects. The present study has been performed at partonic level while the implementation of a full analysis, including parton distribution functions, NLO jet vertices and the investigation of collinearly improved kernels, will be published elsewhere [14].

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