PROCEEDINGS OF SCIENCE

NLO Photon Impact Factor: Present Status and Outlook

Grigorios Chachamis[∗†]
Lehrstuhl für Theoretische Physik II, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany
E-mail: chachamis@physik.uni-wuerzburg.de

Jochen Bartels
II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, D-22761 Hamburg, Germany
E-mail: bartels@mail.desy.de

The numerical calculation of the full NLO photon impact factor and its implementation into a physical cross section are the remaining steps which are required for testing the NLO BFKL resummation against data of, for example, $\gamma^*\gamma^*$ collisions. We have performed the numerical integration over phase space for the virtual corrections to the NLO photon impact factor: along with the previously calculated real corrections, this completes the computation of the NLO corrections. We present first numerical results. The NLO corrections for the photon impact factor are sizeable and negative.

DIFFRACTION 2006 - International Workshop on Diffraction in High-Energy Physics
September 5-10 2006
Adamantas, Milos island, Greece

[∗]Speaker.

[†]The participation of the speaker in the conference was supported by the Sofja Kovalevskaja Award of the Alexander von Humboldt Foundation sponsored by the German Federal Ministry of Education and Research.

PoS(DIFF2006)026

1. Introduction

$\gamma^*\gamma^*$ collisions are an excellent probe of BFKL dynamics because they do not involve any non-perturbative target. The off-shell photons fluctuate into colour dipoles that can further interact strongly. If the transverse size of the dipoles is small (high virtuality), then any soft effects are suppressed. One can tune at will the virtualities of the projectiles, Q_1^2 and Q_2^2, such that perturbation theory will be applicable. Data from LEP [1] for virtual photon photon total cross section suggest indeed a steep rise with the scattering energy but not as steep as it is suggested by LO BFKL. Actually, the data are more in favour of a rise with a power of ~ 0.3 comparing to the $\omega_{\rm BFKL} \approx 0.5$. One is bound to wonder what the corrections beyond the LO are, and whether they alone could lower the intercept closer to the experimental value.

Regge factorisation implies that the total cross section will be a convolution between a process dependent part and a process independent part accounting for the energy dependence. The latter is the BFKL Green's function f_ω , namely the amplitude for the interaction between the two reggeized gluons exchanged in the t-channel, whereas the former consists of the so called *impact factors*: the coupling of the Green's function to the external projectiles. In our case of $\gamma^*\gamma^* \to hadrons$ scattering, we deal with the virtual photon impact factors. Higher order corrections in the process under consideration can enter in two ways, either through the impact factors or through the Green's function.

The calculation of the NLO corrections to the BFKL kernel [2, 3] proved the corrections to be very large and negative, lowering the BFKL Pomeron intercept down to even to negative values. Various studies [4] have shown that it is needed to take into account renormalisation group constraints, and collinear contributions to the BFKL kernel have to be resumed consistently. In such an improved approximation the behaviour of the intercept is tamed, and its value is about 0.3, compatible with the data.

However, as already mentioned, these are only one part of the corrections to the $\sigma_{tot}^{\gamma^*\gamma^*}$. The NLO corrections to the Born impact factor have to be computed as well for a complete analysis if one wants really to test the NLO BFKL Pomeron against experimental data.

2. The NLO Corrections

The NLO corrections to the photon impact factor is a long project divided into distinct steps[1]. Firstly, analytic results were obtained for the one loop corrections to the coupling of the reggeized gluon to the $\gamma^* \to q\bar{q}$ vertex. The process used for that purpose was $\gamma^* + q \to q\bar{q} + q$ [6]. The next step was the calculation of the cross section of the process $\gamma^* + q \to q\bar{q}g + q$ with a large rapidity gap between the fragmentation system $q\bar{q}g$ and the other quark. From this calculation, the real corrections of the virtual photon impact factor in the next-to leading order were obtained [7, 8]. The cancellation of infrared divergencies when combining the real and virtual parts was demonstrated in Ref. [8], while the renormalisation of the ultraviolet divergencies took place in Ref. [6]. The latest step so far, involved analytic manipulation and numerical integration over phase space for the real corrections [9]. The final step which will be presented in this contribution is the numerical integration over phase space for the virtual corrections which, for the case of

[1] A different approach has been outlined in Ref. [5]

longitudinaly polarized photons, completes the full numerical calculation of the NLO impact factor. The phase space integration involves integration over the fraction of the longitudinal momentum carried by the quark in the dipole, α, and over the transverse momentum that runs along the fermion loop, $\mathbf{k}$. The total cross section for $\gamma^*\gamma^*$ scattering is given by:

$$\begin{aligned}\sigma_{\gamma^*\gamma^*} &= \frac{1}{s}\mathrm{Im}\,T_{\gamma^*\gamma^*}(s,t=0)\\ &= \int \frac{d^{D-2}\mathbf{r}}{(2\pi)^{D-2}}\int \frac{d^{D-2}\mathbf{r}'}{(2\pi)^{D-2}}\,\Phi_A(Q_1^2,\mathbf{r},s_0)\Phi_B(Q_2^2,\mathbf{r}',s_0)\frac{f(s,\mathbf{r},\mathbf{r}',s_0)}{\mathbf{r}^2\mathbf{r}'^2}\,,\end{aligned} \tag{2.1}$$

where $f(s,\mathbf{r},\mathbf{r}',s_0)$ is the gluon Green's function:

$$f(s,\mathbf{r},\mathbf{r}',s_0) = \int \frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} f_\omega(\mathbf{r},\mathbf{r}')\,. \tag{2.2}$$

In the following we will suppress the Q^2 dependence of the impact factors. We can express the total NLO corrections ($\mathscr{O}(\alpha_s^3)$) to the photonic cross section as:

$$\begin{aligned}\sigma^{(1)}_{\gamma^*\gamma^*} &= \frac{1}{s}\mathrm{Im}\,T^{(1)}_{\gamma^*\gamma^*}(s,t=0)\\ &= \int \frac{d^{D-2}\mathbf{r}}{(2\pi)^{D-2}}\Phi_A^{(1)}(\mathbf{r},s_0)\frac{1}{\mathbf{r^4}}\Phi_B^{(0)}(\mathbf{r}) + \int \frac{d^{D-2}\mathbf{r}}{(2\pi)^{D-2}}\Phi_A^{(0)}(\mathbf{r})\frac{1}{\mathbf{r^4}}\Phi_B^{(1)}(\mathbf{r},s_0)\\ &+ \int \frac{d^{D-2}\mathbf{r}}{(2\pi)^{D-2}}\Phi_A^{(0)}(\mathbf{r})\ln(\frac{s}{s_0})2\omega^{(1)}(\mathbf{r}^2)\frac{1}{\mathbf{r^4}}\Phi_B^{(0)}(\mathbf{r})\\ &+ \int \frac{d^{D-2}\mathbf{r}}{(2\pi)^{D-2}}\frac{d^{D-2}\mathbf{r}'}{(2\pi)^{D-2}}\Phi_A^{(0)}(\mathbf{r})\frac{\mathbf{1}}{\mathbf{r^4}}\mathscr{K}_{\mathbf{real}}(\mathbf{r},\mathbf{r}')\frac{\mathbf{1}}{\mathbf{r'^4}}\phi_{\mathbf{B}}^{(\mathbf{0})}(\mathbf{r}')\ln(\frac{\mathbf{s}}{\mathbf{s_0}}).\end{aligned} \tag{2.3}$$

where the Born impact factor is

$$\Phi^{(0)}_{\gamma^*} = \int d\mathbf{k}\,d\alpha\, I_2(\alpha,\mathbf{k}), \tag{2.4}$$

and the full NLO corrections, $\Phi^{(1)}_{\gamma^*}$ are:

$$\begin{aligned}\Phi^{(1)}_{\gamma^*} &= \Phi^{(1,virtual)}_{\gamma^*}\Big|^{\text{finite}} - \frac{2\Phi^{(0)}_{\gamma^*}}{(4\pi)^2}\left\{\beta_0\ln\frac{\mathbf{r^2}}{\mu^2} + C_F\ln(\mathbf{r^2})\right\}\\ &+ \int_0^1 d\alpha\int\frac{d\mathbf{k}}{(4\pi)^2}I_2(\alpha,\mathbf{k})\Big\{C_A\left[\ln^2\alpha(1-\alpha)s_0 - \ln^2 M^2\right]\\ &+ C_A\left[-2\ln(\mathbf{r}^2)\,\ln(\frac{s_0}{\mathbf{r}^2})\right]\\ &+ 2C_F\left[8-3\ln\alpha(1-\alpha)\Lambda^2 + \ln^2 M^2 + \ln^2\frac{\alpha}{1-\alpha}\right]\Big\}\\ &+ C_A\,\Phi^{(1,real)}_{\gamma^*}\Big|^{finite}_{C_A} + C_F\,\Phi^{(1,real)}_{\gamma^*}\Big|^{finite}_{C_F}\,.\end{aligned} \tag{2.5}$$

The energy scale s_0, which in the BFKL equation scales the arguments of the logarithms, appears naturally in the real NLO corrections to the impact factor as an energy cutoff: gluons with rapidities above s_0 belong in the fragmentation region of the photon. Thus, in the NLO fixed order calculation

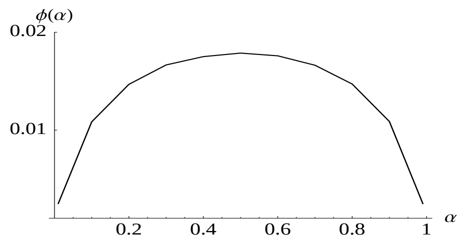

Figure 1: $\alpha \leftrightarrow 1-\alpha$ symmetry. $\phi(\alpha)$ is the α-unintegrated impact factor.

of the $\gamma^*\gamma^*$ total cross section, s_0 enters through both the impact factor and the LO BFKL kernel. However, at the end, the total cross section, at fixed order, has to be independent of it. Λ is a cutoff parameter which has been introduced to separate the collinear singularities of the real diagrams; our numerical result has to be independent of Λ. The very first term in Eq. (2.5) accounts for the finite pieces of the amplitudes from the virtual diagrams, convoluted with the Born ampitude and integrated over α and $\mathbf{k}$. In other words, to perform the phase space integration we need to convolute the amplitude from the one loop virtual corrections to the $\gamma^* g \to q\bar{q}$ vertex with the Born one, sum over helicities and colour indices and finally integrate over the loop momentum. For that purpose we have written a MATHEMATICA program that generates the code that serves as the integrand for the loop momentum integration. The numerical integration was performed using the Monte Carlo routine VEGAS. For the following sections, we will redefine the first four lines in Eq. (2.5) as virtual corrections whereas the fifth line accounts for what in the following we will call real corrections.

3. The Result: Checks and Plots

The photon impact factor at Born level has certain properties which also have to be present in the NLO corrections. Before we present first numerical results of the NLO corrections we mention a few checks that we have run through to ensure the numerical consistence of our results.

$\alpha \leftrightarrow 1-\alpha$ symmetry

The photon impact factor is symmetric under the exchange quark antiquark, and the integrand (before the α-integration, but after the integration over $\mathbf{k}$) is symmetric under $\alpha \leftrightarrow 1-\alpha$ and vanishes as $\alpha \to 0$, $\alpha \to 1$ (Fig. 1).

$\Phi^{(1)}_{\gamma^*}|_{\mathbf{r}\to 0} \sim \mathbf{r}^2$

The NLO impact factor at the $\mathbf{r} \to 0$ limit has to vanish like $\mathbf{r}^2$ (modulo logarithms). We can safely state that all parts of the corrections vanish as $\mathbf{r} \to 0$, although our numerical accuracy of the virtual part does not yet allow to determine its exact functional form.

Scale invariance

The impact factor exhibits a scaling property, namely it is not a function of Q^2 and $\mathbf{r}^2$ independently

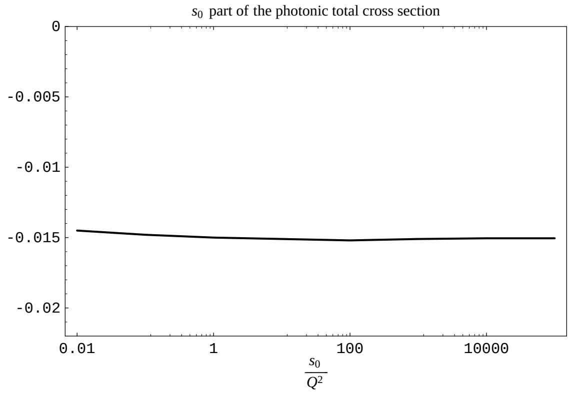

Figure 2: The part of the total $\gamma^*\gamma^*$ cross section that depends on s_0, plotted as a function of s_0/Q^2.

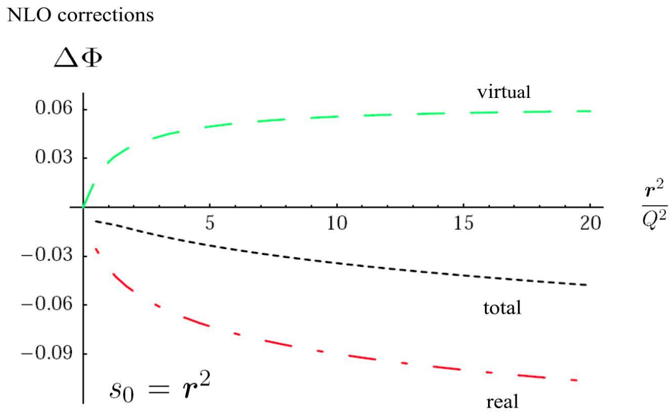

Figure 3: Virtual, real and total NLO corrections to the photon impact factor for $s_0 = \mathbf{r}^2$.

but rather a function of their ratio, it is also therefore dimensionless. Dividing Eq. (2.5) into two parts (first four lines and the fifth one), and redefining what we call real and what virtual corrections had an aditional motivation. The two parts exhibit individually the scaling property. We have numerically verified the validity of this scaling property for the virtual corrections.

s_0-independence of the NLO fixed order $\sigma_{tot}^{\gamma^*\gamma^*}$

Here we numerically check whether, in Eq. (2.5), the sum of all those pieces which explicitely or implicitly depend on s_0 are, at the end of the day, independent of it. That is indeed the case as we see in Fig. 2, demonstrating the s_0-independence of the NLO fixed order $\sigma_{tot}^{\gamma^*\gamma^*}$.

Numerical results for the NLO corrections

We finally present plots for the NLO corrections[2] and for the full NLO impact factor, chosing the energy scale either $s_0 = \mathbf{r}^2$ (Figs. 3 and 4) or fixed $s_0 = 10$ GeV^2 (Figs. 5 and 6). We have chosen to use $Q^2 = 15$ GeV^2 as the scale that sets the running of the coupling α_s. As we can see from the figures, the real corrections are large and negative. This is because they do not correspond to the

[2]We thank A. Kyrieleis for providing us with the data for the curves of the real corrections [9].

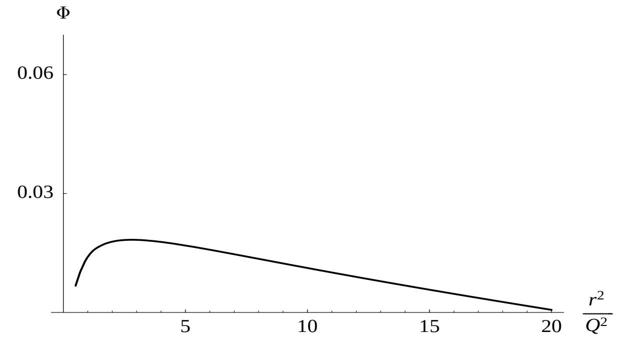

Figure 4: The full NLO impact factor for $s_0 = \mathbf{r}^2$.

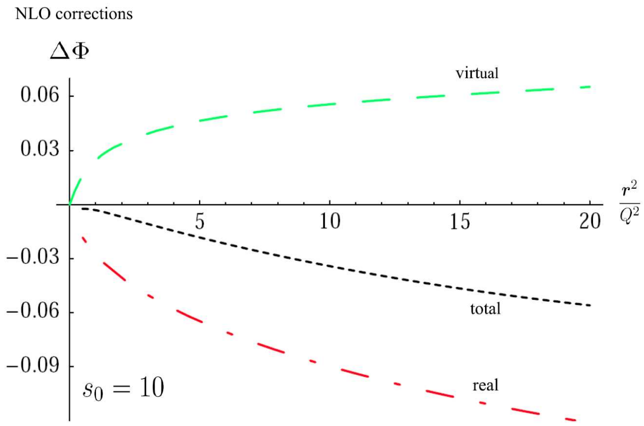

Figure 5: Virtual, real and total NLO corrections to the photon impact factor for $s_0 = 10$ GeV^2.

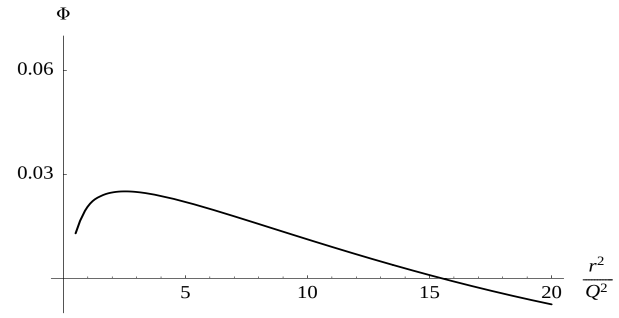

Figure 6: The full NLO impact factor for $s_0 = 10$ GeV^2.

whole phase space. The central region is subtracted. On the other hand, the virtual part is also large but comes with a positive sign. The combination of the two is finally negative and large, bringing the NLO impact factor, for fixed scale s_0, even to negative values. This illustrates that the shape of the full NLO impact factor depends critically on the choice of s_0. If we select a fixed value for s_0, then the corrections grow when s_0 becomes smaller. A large fixed s_0, on the other hand, has exactly the opposite effect. A final answer will be obtained once we compute the convolution of the NLO impact factor with the NLO gluon Green's function [10]. Another factor that affects the functional form of the NLO corrections is the choice of the scale that regulates the running of α_s. A recent NLO analysis for the electroproduction of two light vector mesons [11] demonstrates the importance of that point.

4. Conclusions

We have presented first numerical results of the full NLO corrections to the photon impact factor. As expected, the corrections are sizeable and negative, and they tend to decrease the value of the impact factors.

We are thus ready to proceed to the next step, namely to convolute it with the NLO BFKL Green's function and produce estimates for the total cross section of the scattering of virtual photons. Our results for the photon impact factor can also be used to compute, in NLO accuracy, the cross section for the production of forward jets in deep inelastic ep scattering.

References

[1] http://l3.web.cern.ch/l3/

[2] V. S. Fadin, L. N. Lipatov, Phys. Lett. **B429** (1998) 127.

[3] M. Ciafaloni, G. Camici, Phys. Lett. B **430** (1998) 349.

[4] G.P. Salam, JHEP**8907** (1998) 19; M. Ciafaloni and D. Colferai, Phys. Lett.**B452** (1999) 372; M. Ciafaloni, D. Colferai and G.P. Salam, Phys. Rev. **D60** (1999) 114036; R.S. Thorne, Phys. Rev. **D60** (1999) 054031; J. R. Forshaw, D. A. Ross and A. Sabio Vera, Phys. Lett. **B498** (2001) 149; G. Altarelli, R. D. Ball and S. Forte, Nucl. Phys. **B575** (2000) 313, Nucl. Phys. **B621** (2002) 359, Nucl. Phys. **B674** (2003) 459; M. Ciafaloni, D. Colferai, G. P. Salam and A. M. Stasto, Phys. Lett. **B576** (2003) 143, Phys. Rev. **D68** (2003) 114003, Phys. Lett. **B587** (2004) 87; J. R. Andersen and A. Sabio Vera, Phys. Lett. **B567** (2003) 116, Nucl. Phys. **B679** (2004) 345, `hep-th/0406009`.

[5] V. S. Fadin, D. Y. Ivanov and M. I. Kotsky, Nucl. Phys. **B658** (2003) 156.

[6] J. Bartels, S. Gieseke and C. F. Qiao, Phys. Rev. D **63** (2001) 056014 [Erratum-ibid. D **65** (2002) 079902].

[7] J. Bartels, S. Gieseke and A. Kyrieleis, Phys. Rev. D **65** (2002) 014006.

[8] J. Bartels, D. Colferai, S. Gieseke and A. Kyrieleis, Phys. Rev. D **66** (2002) 094017.

[9] J. Bartels and A. Kyrieleis, Phys. Rev. D **70** (2004) 114003.

[10] J. R. Andersen, A. Sabio Vera, Nucl. Phys. **B679** 345 (2004), Nucl. Phys. **B699** (2004) 90 Phys. Lett. **B567** (2003) 116

[11] D. Y. Ivanov, A. Papa, Nucl. Phys. **B732** (2006) 183.