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Regge and Bjorken asymptotics in N=4 SUSY

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We review the BFKL approach to the Regge processes in QCD. In the multi-colour QCD the BKP equations for composite states of several Reggeized gluons turn out to be integrable. An effective gauge-invariant Lagrangian allows one to construct the Feynman rules for the interaction of particle and Reggeons in the momentum representation. It is shown, that the BFKL and DGLAP equations in N=4 SUSY have the property of the maximal transcedentality. With the use of the AdS/CFT correspondence we investigate relations between weak and strong coupling regimes in the framework of the Eden-Staudacher equation and the Pomeron - Graviton duality.

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1. Introduction

In the leading logarithmic approximation (LLA) the scattering amplitude in QCD has the form

$$M_{AB}^{A'B'}(s,t) = s\sum_{n=0}^{\infty}(\alpha_s \ln s)^n\, a_n(t)\,,\; \alpha_s = \frac{g^2}{4\pi}\to 0\,. \tag{1.1}$$

In the Born approximation it is factorized

$$M_{AB}^{A'B'}(s,t)|_{Born} = g\,T_{A'A}^{c}\,\delta_{\lambda_{A'}\lambda_A}\,\frac{2s}{t}\,g\,T_{B'B}^{c}\,\delta_{\lambda_{B'}\lambda_B}\,. \tag{1.2}$$

After summing radiative corrections we obtain for the amplitude the Regge-type expression [1]

$$M_{AB}^{A'B'}(s,t) = M_{AB}^{A'B'}(s,t)|_{Born}\,s^{\omega(t)}\,, \tag{1.3}$$

where the gluon Regge trajectory is

$$\omega(-|q|^2) = -\frac{\alpha_c}{4\pi^2}N_c\int d^2k\,\frac{|q|^2}{|k|^2|q-k|^2}\approx -\frac{\alpha_c}{2\pi}\ln\frac{|q^2|}{\lambda^2}\,. \tag{1.4}$$

The particles at high energies for the process $AB \to A'B'd_1...d_{n-1}$ in LLA are produced in the multi-Regge kinematics

$$s \gg s_r = (k_{r-1}+k_r)^2 \gg |q_r|^2\,,\; k_r = q_r - q_{r+1}\,. \tag{1.5}$$

The gluon production amplitude in this region has the multi-Regge form [1]

$$M_{2\to 1+n} \sim \frac{s_1^{\omega_1}}{|q_1|^2}\,gT_{c_2c_1}^{d_1}C(q_2,q_1)\,\frac{s_2^{\omega_2}}{|q_2|^2}...C(q_n,q_{n-1})\,\frac{s_n^{\omega_n}}{|q_n|^2}\,, \tag{1.6}$$

where the Reggeon-Reggeon-gluon vertex for the produced gluon with a definite helicity is

$$C(q_2,q_1) = \frac{q_2\,q_1^*}{q_2-q_1}\,. \tag{1.7}$$

It is convenient to introduce the complex variables for the gluon transverse coordinates and momenta

$$\rho_k = x_k + iy_k\,,\; \rho_k^* = x_k - iy_k\,,\; p_k = i\frac{\partial}{\partial\rho_k}\,,\; p_k^* = i\frac{\partial}{\partial\rho_k^*}\,. \tag{1.8}$$

Then in the coordinate representation the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation is [1]

$$E\,\Psi(\vec\rho_1,\vec\rho_2) = H_{12}\,\Psi(\vec\rho_1,\vec\rho_2)\,,\; \Delta = -\frac{\alpha_s N_c}{2\pi}\min E\,, \tag{1.9}$$

where Δ is the Pomeron intercept. In the operator form the BFKL Hamiltonian is [2]

$$H_{12} = \ln|p_1p_2|^2 + \frac{1}{p_1p_2^*}\ln|\rho_{12}|^2 p_1p_2^* + \frac{1}{p_1^*p_2}\ln|\rho_{12}|^2 p_1^*p_2 - 4\psi(1)\,, \tag{1.10}$$

where $\rho_{12} = \rho_1 - \rho_2$. The kinetic energy is proportional to the gluon Regge trajectories $\omega(-|p_{1,2}|^2)$ and the potential energy $\sim \ln|\rho_{12}|^2$ is related to the Fourier transformation of the product of two vertices $C(q_2,q_1)$.

The BFKL Hamiltonian is invariant under the Moebius transformation [3]

$$\rho_k \to \frac{a\rho_k + b}{c\rho_k + d}\,. \tag{1.11}$$

For the classification of its eigenstates one can introduce the Casimir operators of this group

$$M^2 = \left(\sum_{r=1}^{2} \vec{M}^{(r)}\right)^2 = \rho_{12}^2\; p_1\; p_2\,,\; M^{*2} = (M^2)^*\,. \tag{1.12}$$

Their eigenvalue equations

$$M^2 f_{m,\widetilde{m}} = m(m-1)\, f_{m,\widetilde{m}}\,,\; M^{*2} f_{m,\widetilde{m}} = \widetilde{m}(\widetilde{m}-1)\, f_{m,\widetilde{m}} \tag{1.13}$$

define the conformal weights

$$m = 1/2 + i\nu + n/2\,,\; \widetilde{m} = 1/2 + i\nu - n/2 \tag{1.14}$$

for the principal series of unitary representations.

The Hamiltonian has the property of the holomorphic separability [4]

$$H_{12} = h_{12} + h_{12}^*\,, \tag{1.15}$$

where the holomorphic Hamiltonian h_{12} equals

$$h_{12} = \ln(p_1 p_2) + \frac{1}{p_1}\ln\rho_{12}\; p_1 + \frac{1}{p_2}\ln\rho_{12}\; p_2 - 2\psi(1)\,. \tag{1.16}$$

2. Integrability of the multi-colour BFKL dynamics

One can write the Bartels-Kwiecinskii-Praszalowicz equation [5] for the n-gluon state as follows

$$E\Psi(\vec{\rho}_1,...,\vec{\rho}_n) = \sum_{k<l}\frac{T_k^a T_l^a}{(-N_c)} H_{k,l}\,\Psi(\vec{\rho}_1,...,\vec{\rho}_n)\,, \tag{2.1}$$

where T_k^a is the gauge group generator acting on the colour index of the gluon k

$$T_{bc}^a = -i\, f_{abc}\,,\; [T_k^a, T_l^b] = i\, f_{abc} T_k^c\, \delta_{kl}\,. \tag{2.2}$$

The BKP equation is especially simple in the multi-colour QCD. In particular, for the eigenfunction of the Hamiltonian $H = \frac{1}{2}\sum_k H_{k,k+1}$ we obtain the holomorphic factorization [4]

$$\Psi(\vec{\rho}_1,\vec{\rho}_2,...,\vec{\rho}_n) = \sum_{r,s} a_{r,s}\,\Psi_r(\rho_1,...,\rho_n)\,\Psi_s(\rho_1^*,...,\rho_n^*) \tag{2.3}$$

and the duality symmetry [6]

$$\rho_{r,r+1} \to p_r \to \rho_{r-1,r}\,. \tag{2.4}$$

The holomorphic Hamiltonian h has the integrals of motion [2, 7]

$$q_r = \sum_{k_1<k_2<...<k_r} \rho_{k_1k_2}\rho_{k_2k_3}...\rho_{k_{r-1}k_r}\, p_{k_1} p_{k_2}...p_{k_r}\,,\; [q_r, h] = 0\,. \tag{2.5}$$

The integrability of the BFKL dynamics [7] is related to the fact, that H coincides with the local Hamiltonian of the Heisenberg spin model [8].

In particular for the Pomeron ($n=2$) one can obtain the following wave function [3]

$$f_{m,\widetilde{m}}(\overrightarrow{\rho_1},\overrightarrow{\rho_2};\overrightarrow{\rho_0}) = \left(\frac{\rho_{12}}{\rho_{10}\,\rho_{20}}\right)^m \left(\frac{\rho_{12}^*}{\rho_{10}^*\,\rho_{20}^*}\right)^{\widetilde{m}} \tag{2.6}$$

with the corresponding energy having the holomorphic separability property

$$E_{m,\widetilde{m}} = \varepsilon_m + \varepsilon_{\widetilde{m}} \quad , \quad \varepsilon_m = \psi(m) + \psi(1-m) - 2\psi(1)\,,\;\; \psi(x) = \frac{d}{dx}\ln\Gamma(x)\,. \tag{2.7}$$

Thus, the intercept of the BFKL Pomeron is [1]

$$\Delta = 4\,\frac{\alpha_s}{\pi} N_c \ln 2 \tag{2.8}$$

and, as a result, one obtains the violation of the Froissart bound

$$\sigma \sim s^{\Delta} > c \ln^2 s\,. \tag{2.9}$$

Therefore there is a problem how to restore the s-channel unitarity for scattering amplitudes. The consistent way to solve this problem is to use the effective field theory for Reggeized gluons which is similar to the Gribov Reggeon calculus for Pomerons. For the multi-Regge kinematics of intermediate particles such effective model was constructed in Ref. [9].

3. Effective field theory for high energies

In the next-to-leading approximation one should consider more complicated processes - the production of particle clusters in the multi-Regge kinematics

$$P_A + P_B = Q_1 + Q_2 + \ldots + Q_n\,,\; s \gg s_i = 2Q_iQ_{i+1} \gg |q_i^2|\,, \tag{3.1}$$

$$Q_i^2 = M_i^2\,,\; Q_k = \sum_j p_j^{(k)}\,,\; k = 1,2,\ldots,n\,. \tag{3.2}$$

We define the parton rapidity y as follows

$$y = \frac{1}{2}\ln\frac{\varepsilon_k + |k|}{\varepsilon_k - |k|} \tag{3.3}$$

and investigate the interaction of particles belonging to the rapidity interval $|y-y_0| < \eta\,,\; \eta \ll \ln s$. The gluon and quark fields are transformed according the corresponding representations of $SU(N_c)$

$$v_\mu(x) = -iT^a v_\mu^a(x)\,,\; \psi(x)\,,\; \bar{\psi}(x)\,. \tag{3.4}$$

We introduce also the fields describing the production and annihilation of Reggeized gluons

$$A_\pm(x) = -iT^a A_\pm^a(x)\,. \tag{3.5}$$

Under the global colour group rotations the fields are transformed in the standard way

$$\delta v_\mu(x) = [v_\mu(x),\chi],\; \delta\psi(x) = -\chi\,\psi(x),\; \delta A_\pm(x) = [A_\pm(x),\chi]\,, \tag{3.6}$$

but under the local gauge transformations ($\chi(x) \to 0$ for $x \to \infty$) we have

$$\delta v_\mu(x) = \frac{1}{g}[D_\mu, \chi(x)],\ \delta\psi(x) = -\chi(x)\,\psi(x),\ \delta A_\pm(x) = 0\,. \tag{3.7}$$

In the quasi-multi-Regge kinematics the Reggeon fields satisfy the kinematical constraints

$$\partial_\mp A_\pm(x) = 0\,. \tag{3.8}$$

For the gauge invariance one should introduce new effective vertices with an arbitrary number of gluons. The effective gauge-invariant action local in the rapidity y has the form [10]

$$S = \int d^4x\left(L_0 + L_{ind}^{GR}\right)\,, \tag{3.9}$$

where the usual QCD Lagrangian is

$$L_0 = i\bar{\psi}\hat{D}\psi + \frac{1}{2}Tr\,G_{\mu\nu}^2,\, D_\mu = \partial_\mu + g v_\mu,\, G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu] \tag{3.10}$$

and the induced contribution for the gluon-Reggeon interactions is given below

$$L_{ind}^{GR} = -\frac{1}{g}\partial_+ P\exp\left(-\frac{1}{2}\int_{-\infty}^{x^+} v_+(x')dx'^+\right)\partial_\sigma^2 A_- - \frac{1}{g}\partial_- P\exp\left(-\frac{1}{2}\int_{-\infty}^{x^-} v_-(x')dx'^-\right)\partial_\sigma^2 A_+\,. \tag{3.11}$$

One can formulate the Feynman rules in the momentum space [11]. The effective vertices for the interaction of r gluons with a Reggeized gluon have the form [10]

$$\Delta^{\nu_0\nu_1\ldots\nu_r+}_{a_0a_1\ldots a_r c} = -\vec{q}_\perp^{\,2}\prod_{s=0}^{r}(n^+)^{\nu_s}\,2\,Tr\,(T^c G_{a_0a_1\ldots a_r}) \tag{3.12}$$

with the following representation for $G_{a_0a_1\ldots a_r}$ [11]

$$G_{a_0a_1\ldots a_r} = \sum_{\{i_0,i_1,\ldots,i_r\}}\frac{T^{a_{i_0}}T^{a_{i_1}}T^{a_{i_2}}\ldots T^{a_{i_r}}}{k_{i_0}^+(k_{i_0}^+ + k_{i_1}^+)\ldots(k_{i_0}^+ + k_{i_1}^+ + \ldots + k_{i_{r-1}}^+)}\,. \tag{3.13}$$

These vertices satisfy the recurrent relations (Ward identities) [10]

$$k_r^+\,\Delta^{\nu_0\nu_1\ldots\nu_r+}_{a_0a_1\ldots a_r}(k_0^+,\ldots,k_r^+) = \sum_{i=0}^{r-1} i\,f_{aa_ma_i}\Delta^{\nu_0\ldots\nu_r+}_{\ldots a_{i-1}aa_{i+1}\ldots a_r}(k_0^+,\ldots,k_{i-1}^+,k_i^+ + k_r^+,k_{i+1}^+,\ldots)\,. \tag{3.14}$$

4. DGLAP and BFKL equations in $N=4$ SUSY

Usual parton distributions are expressed in terms of the corresponding unintegrated quantities

$$f_a(x,Q^2) \ = \ \int_{k_\perp^2<Q^2} dk_\perp^2\,\varphi_a(x,k_\perp^2)\,. \tag{4.1}$$

With the use of the Mellin transformation

$$f_a(j,Q^2) = \int_0^1 dx\,x^{j-1}\,f_a(x,Q^2) \tag{4.2}$$

the kernel of the DGLAP equation is written in terms of the anomalous dimension matrix γ_{ab}

$$\frac{d}{d\ln Q^2}\, f_a(j,Q^2) = \sum_b \gamma_{ab}(j)\, f_b(j,Q^2)\,. \tag{4.3}$$

The momenta $f_a(j,Q^2)$ are proportional to matrix elements of the light-cone components of the local twist-2 operators being Lorentz tensors or pseudo-tensors

$$O^a = \tilde{n}^{\mu_1}...\tilde{n}^{\mu_j}\, O^a_{\mu_1,...,\mu_j}\,,\ \tilde{\mathscr{O}}^a = \tilde{n}^{\mu_1}...\tilde{n}^{\mu_j}\,\tilde{\mathscr{O}}^a_{\mu_1,...,\mu_j}\,. \tag{4.4}$$

Their anomalous dimensions do not depend on the different tensor projections

$$\tilde{n}^{\mu_1}...\tilde{n}^{\mu_{1+\omega}}\, O^a_{\mu_1,...,\mu_{1+\omega},\sigma_1,...,\sigma_{|n|}}\, l_\perp^{\sigma_1}...l_\perp^{\sigma_{|n|}}\,. \tag{4.5}$$

The solution of the BFKL equation due to its Möbius invariance is classified by the anomalous dimension $\gamma=\frac{1}{2}+i\nu$ and the conformal spin $|n|$ which coincides with the number of transverse indices of the tensor O^a.

In the next-to-leading approximation the eigenvalue of the BFKL kernel is written below

$$\omega = \omega_0(n,\,\gamma) + 4\,\hat{a}^2\,\Delta(n,\gamma)\,,\ \hat{a} = g^2N_c/(16\pi^2)\,. \tag{4.6}$$

In QCD $\Delta(n,\gamma)$ is a non-analytic function of the conformal spin $|n|$ [12, 13]

$$\Delta_{QCD}(n,\gamma) = c_0\delta_{n,0} + c_2\delta_{n,2} + \mathit{analytic\ terms}\,,$$

but in $N=4$ SUSY the Kronecker symbols are cancelled [13].

Moreover, in this model we obtain for $\Delta(n,\gamma)$ the Hermitian separability

$$\Delta(n,\gamma) = \phi(M) + \phi(M^*) - \frac{\rho(M)+\rho(M^*)}{2\hat{a}/\omega}\,,\, M = \gamma + \frac{|n|}{2}\,, \tag{4.7}$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2)\,,\, \beta'(z) = \frac{1}{4}\left[\Psi'\Big(\frac{z+1}{2}\Big) - \Psi'\Big(\frac{z}{2}\Big)\right]. \tag{4.8}$$

It is important, that here all special functions have the maximal trancedentality property [13].

$$\phi(M) = 3\zeta(3) + \Psi''(M) - 2\Phi(M) + 2\beta'(M)\Big(\Psi(1) - \Psi(M)\Big), \tag{4.9}$$

where

$$\Phi(M) = \sum_{k=0}^{\infty}\frac{\beta'(k+1)}{k+M} + \sum_{k=0}^{\infty}\frac{(-1)^k}{k+M}\left(\Psi'(k+1)\ -\ \frac{\Psi(k+1)-\Psi(1)}{k+M}\right). \tag{4.10}$$

For one loop anomalous dimension matrix in the case $N=4$ the calculations were performed in Ref. [14]. In this model one can introduce the following twist-2 Wilson operators

$$\mathscr{O}^g_{\mu_1,...,\mu_j} = \hat{S}G^a_{\rho\mu_1}D_{\mu_2}D_{\mu_3}...D_{\mu_{j-1}}G^a_{\rho\mu_j}\,, \tag{4.11}$$

$$\tilde{\mathscr{O}}^g_{\mu_1,...,\mu_j} = \hat{S}G^a_{\rho\mu_1}D_{\mu_2}D_{\mu_3}...D_{\mu_{j-1}}\tilde{G}^a_{\rho\mu_j}\,, \tag{4.12}$$

$$\mathscr{O}^q_{\mu_1,...,\mu_j} = \hat{S}\bar{\Psi}^a\gamma_{\mu_1}D_{\mu_2}...D_{\mu_j}\Psi^a\,, \tag{4.13}$$

$$\tilde{\mathscr{O}}^{q}_{\mu_1,...,\mu_j} = \hat{S}\bar{\Psi}^a \gamma_5 \gamma_{\mu_1} D_{\mu_2}...D_{\mu_j}\Psi^a \,, \tag{4.14}$$

$$\mathscr{O}^{\varphi}_{\mu_1,...,\mu_j} = \hat{S}\bar{\Phi}^a D_{\mu_1} D_{\mu_2}...D_{\mu_j}\Phi^a \,. \tag{4.15}$$

The diagonalization of the anomalous dimension matrix $U\gamma U^+$ gives the result

$$\begin{vmatrix} -4S_1(j-2) & 0 & 0 \\ 0 & -4S_1(j) & 0 \\ 0 & 0 & -4S_1(j+2) \end{vmatrix} \,, \; \begin{vmatrix} -4S_1(j-1) & 0 \\ 0 & -4S_1(j+1) \end{vmatrix} \tag{4.16}$$

containing the universal function γ_{uni} for the super-multiplet of all twist-2 operators

$$\gamma^{(0)}_{uni}(j) = -4S_1(j-2) \,,\; S_r(j) = \sum_{i=1}^{j} \frac{1}{i^r} \,. \tag{4.17}$$

Note, that this function has the maximal transcedentality property, which leads to an integrability of the evolution equations for matrix elements of quasi-partonic operators in $N=4$ SUSY [14].

5. Two- and three- loop universal anomalous dimension in $N=4$

In an accordance with the fact, that the eigenvalue of the BFKL equation is expressed in terms of the most complicated harmonic sums and using the hypothesis, that the anomalous dimension in $N=4$ theory can be obtained from the BFKL equation by the analytic continuation of its kernel to integer values of $|n|$ [13], one can argue, that the perturbative expansion of the universal anomalous dimension

$$\gamma_{uni}(j) = \hat{a}\gamma^{(0)}_{uni}(j) + \hat{a}^2\gamma^{(1)}_{uni}(j) + \hat{a}^3\gamma^{(2)}_{uni}(j) + ... \tag{5.1}$$

contains in each order of the perturbation theory only special functions with the highest transcedentality. With such assumption we obtain [13]

$$\frac{1}{8}\gamma^{(1)}_{uni}(j+2) = 2S_1(j)\left(S_2(j)+S_{-2}(j)\right) - 2S_{-2,1}(j) + S_3(j) + S_{-3}(j) \,, \tag{5.2}$$

where

$$S_{-r}(j) = \sum_{i=1}^{j} \frac{(-1)^i}{i^r} \,,\; S_{-2,1} = \sum_{m=1}^{j} \frac{(-1)^m}{m^2} S_1(m) \,. \tag{5.3}$$

This result was verified by the direct calculation of the anomalous dimension matrix in two loops [15].

On the other hand, recently the three-loop anomalous dimension matrix for QCD was calculated [16]. It gave us a possibility to find the universal anomalous dimension in three loops for $N=4$ SUSY using the hypothesis of the maximal transcedentality [17]

$$\begin{aligned} \frac{1}{32}\gamma^{(2)}_{uni}(j+2) &= 24S_{-2,1,1,1} - 12\left(S_{-3,1,1}+S_{-2,1,2}+S_{-2,2,1}\right) \\ &+6\left(S_{-4,1}+S_{-3,2}+S_{-2,3}\right) - 3S_{-5} - 2S_3S_{-2} - S_5 \\ &-2S_1^2\left(3S_{-3}+S_3-2S_{-2,1}\right) - S_2\left(S_{-3}+S_3-2S_{-2,1}\right) \\ &-S_1\left(8\bar{S}_{-4}+\bar{S}^2_{-2}+4S_2\bar{S}_{-2}+2S_2^2\right) \end{aligned}$$

$$-S_1\left(3S_4-12\overline{S}_{-3,1}-10\overline{S}_{-2,2}+16\overline{S}_{-2,1,1}\right)\,, \tag{5.4}$$

where the corresponding harmonic sums are given below

$$S_a(j)=\sum_{m=1}^{j}\frac{1}{m^a}\,,\;S_{a,b,c,\cdots}(j)=\sum_{m=1}^{j}\frac{1}{m^a}\,S_{b,c,\cdots}(m)\,, \tag{5.5}$$

$$S_{-a}(j)=\sum_{m=1}^{j}\frac{(-1)^m}{m^a},\,S_{-a,b,\cdots}(j)=\sum_{m=1}^{j}\frac{(-1)^m}{m^a}\,S_{b,\cdots}(m), \tag{5.6}$$

$$\overline{S}_{-a,b,c\cdots}(j)=(-1)^jS_{-a,b,\ldots}(j)+S_{-a,b,\cdots}(\infty)\Big(1-(-1)^j\Big). \tag{5.7}$$

6. Comparison with other approaches

The three-loop anomalous dimension for $N=4$ SUSY at $j=1+\omega\to 1$

$$\gamma_{uni}^{N=4}(j)=\hat{a}\,\frac{4}{\omega}-32\zeta_3\,\hat{a}^2+32\zeta_3\,\hat{a}^3\,\frac{1}{\omega}+\ldots \tag{6.1}$$

is in an agreement with the predictions of the BFKL equation [13].

Near the negative even points $j+2r=\omega\to 0$ one can verify, that the anomalous dimension satisfy the equation

$$\gamma_{uni}=4\,\frac{\hat{a}}{\omega}+\frac{\gamma_{uni}^2}{\omega} \tag{6.2}$$

corresponding to the resummation of the double logarithmic terms $\sim\alpha/\omega^2$.

Further, one can find the universal anomalous dimension at large j

$$\gamma_{uni}=a(z)\,\ln j\,,\;z=\frac{\alpha N_c}{\pi}=4\hat{a} \tag{6.3}$$

valid up to three loops

$$a(z)=-z+\frac{\pi^2}{12}z^2-\frac{11}{720}\pi^4z^3+\ldots. \tag{6.4}$$

On the other hand using the well-known AdS/CFT correspondence [18] between the superstring model on the anti-de-Sitter space and the $N=4$ supersymmetric Yang-Mills theory A. Polyakov with collaborators obtained the prediction for $a(z)$ in the strong coupling limit [19]

$$\lim_{z\to\infty}a(z)=-z^{1/2}+\frac{3\ln 2}{4\pi}+\ldots. \tag{6.5}$$

In Ref. [15] the resummation of the perturbation theory for $a(z)$ was suggested in the form

$$\widetilde{a}=-z+\frac{\pi^2}{12}\,\widetilde{a}^2. \tag{6.6}$$

The prediction of this equation for three loops is in a rather good agreement with the exact result

$$\widetilde{a}=-z+\frac{\pi^2}{12}z^2-\frac{1}{72}\pi^4z^3+\ldots \tag{6.7}$$

and with its asymptotic behaviour at $z\to\infty$.

Recently Eden and Staudacher obtained the following expression for a

$$a(z) = -2g\sqrt{2}\,f(0)\,,\ \sqrt{z} = \frac{1}{2\varepsilon\sqrt{2}}\,, \tag{6.8}$$

where $f(x)$ satisfies the integral equation [20]

$$\varepsilon\, f(x) = \frac{t}{e^t-1}\left(\frac{J_1(x)}{x} - \int_0^\infty dx'\,K(x,x')\,f(x')\right). \tag{6.9}$$

The integral kernel is expressed in terms of the Bessel functions

$$K(x,y) = \frac{J_1(x)\,J_0(y) - J_1(y)\,J_0(x)}{x-y}\,. \tag{6.10}$$

Using the Laplace transformation

$$f(x) = \int_{-i\infty}^{i\infty}\frac{d\,j}{2\pi\,i}\,e^{x\,j}\,\phi(j) \tag{6.11}$$

one can write the following anzatz for the solution of the ES equation

$$\phi(j) = \sum_{n=1}^{\infty}(\delta_{n,1} - a_{n,\varepsilon})\sum_{s=1}^{\infty}\frac{\left(\sqrt{(j+s\,\varepsilon)^2+1}+j+s\,\varepsilon\right)^{-n}}{\sqrt{(j+s\,\varepsilon)^2+1}}\,. \tag{6.12}$$

The coefficients $a_{n,\varepsilon}$ satisfy the set of algebraic equations

$$a_{n,\varepsilon} = \sum_{n'=1}^{\infty}K_{n,n'}(\varepsilon)\,\left(\delta_{n',1} - a_{n',\varepsilon}\right)\,, \tag{6.13}$$

where the integral kernel is [22]

$$K_{n,n'}(\varepsilon) = 2n\sum_{R=0}^{\infty}(-1)^R\,\frac{2^{-2R-n-n'}}{\varepsilon^{2R+n+n'}}\,\zeta(2R+n+n')\,\frac{(2R+n+n'-1)!\,(2R+n+n')!}{R!\,(R+n)!\,(R+n')!\,(R+n+n')!}\,. \tag{6.14}$$

One can verify from this expression, that in all orders of the perturbation theory for $a(z)$ the maximal transcedentality is valid and the coefficients in front of the products of ζ-functions are integer numbers. Note, that for the modified ES equation derived in Ref. [21] the kernel $K_{n,n'}(\varepsilon)$ should be multiplied by the factor $i=\sqrt{-1}$ for odd values of the sum $n+n'$.

Using the new variable $z = j+\sqrt{j^2+1}$ one can write the dispersion representation

$$\xi(z) = \int_L \frac{dz'}{2\,\pi\,i}\,\frac{\xi(z') - \xi(-1/z')}{z-z'} \tag{6.15}$$

for the function

$$\xi(z) = \frac{z^2+1}{2z}\,\left(\phi(j-\varepsilon) - \phi(j)\right). \tag{6.16}$$

The corresponding discontinuity satisfies the linearized "unitarity" constraint

$$\frac{\xi(z) - \xi(-1/z)}{2\sqrt{j^2+1}} = 1 - \sum_{s=1}^{\infty}\frac{\xi\left(j+s\,\varepsilon+\sqrt{(j+s\,\varepsilon)^2+1}\right)}{\sqrt{(j+s\,\varepsilon)^2+1}}\,. \tag{6.17}$$

7. BFKL Pomeron and graviton in N=4 SUSY

Let us calculate the Pomeron intercept in the $N = 4$ supersymmetric gauge theory at large coupling constants [17]. To begin with, one can simplify the eigenvalue for the BFKL kernel in the diffusion approximation as follows (see [12])

$$j = 2 - \Delta - D\nu^2\,,\ \gamma_{uni} = \frac{j}{2} + i\nu\,, \tag{7.1}$$

assuming, that the parameter Δ is small at large $z \sim \alpha$. Due to the energy-momentum conservation we have $\gamma|_{j=2} = 0$ and therefore γ can be expressed only in terms of the parameter Δ

$$\gamma = (j-2)\left(\frac{1}{2} - \frac{1/\Delta}{1+\sqrt{1+(j-2)/\Delta}}\right). \tag{7.2}$$

On the other hand with the use of the AdS/CFT correspondence [18] the above BFKL equation can be written as the graviton Regge trajectory

$$j = 2 + \frac{\alpha'}{2}\,t\,,\ t = E^2/R^2\,,\ \alpha' = \frac{R^2}{2}\,\Delta\,. \tag{7.3}$$

The behaviour of γ at $g \to \infty,\ j \to \infty$ is known from the paper of Polyakov with collaborators [19]

$$\gamma_{|z\to\infty} = -\sqrt{j-2}\,\Delta^{-1/2}_{|j\to\infty} = \sqrt{\pi j}\,z^{1/4} \tag{7.4}$$

As a result we obtain the following Pomeron intercept at large couplings [17] (see also Ref. [23])

$$j = 2 - \Delta,\ \Delta = \frac{1}{\pi}z^{-1/2} \tag{7.5}$$

To verify this result independently one can calculate the slope of the anomalous dimension at $j = 2$

$$\gamma'(2) = \frac{1}{2} - \frac{1}{2\Delta} = b = -\frac{\pi^2}{6}z + \frac{\pi^4}{72}z^2 - \frac{\pi^6}{540} + \dots. \tag{7.6}$$

Similar to the case $j \to \infty$ we use the following resummation procedure [15]

$$\frac{\pi^2}{6}z = -\widetilde{b} + \frac{1}{2}\widetilde{b}^2\,. \tag{7.7}$$

The weak and strong coupling asymptotics of the solution of this equation is given below

$$\widetilde{b} = -\frac{\pi^2}{6}z + \frac{\pi^4}{72}z^2 - \frac{\pi^6}{432}z^3 + \dots\,,\ \lim_{z\to\infty}\widetilde{\Delta} = \frac{\sqrt{3}}{2\pi}z^{-1/2}\,, \tag{7.8}$$

which is in a good agreement with the above results for Δ and b

$$b = -\frac{\pi^2}{6}z + \frac{\pi^4}{72}z^2 - \frac{\pi^6}{540}z^3 + \dots. \tag{7.9}$$

8. Discussion

It is important, that in QCD the gluons and quarks are reggeized. For solving the unitarization problem for the BFKL Pomeron one should use the effective action for interactions of Reggeons and particles in the quasi-multi-Regge kinematics. The Reggeon calculus in the form of a 2+1 field theory can be derived from the action. In the framework of this approach the t -channel unitarity is automatically fulfilled. The s-channel unitarity is incorporated in the Reggeon theory through the bootstrap equations (see [1]) and various relations among the effective vertices. The next-to-leading correction to the eigenvalue of the BFKL kernel in $N = 4$ SUSY does not contain the non-analytic terms. It is a sum of the most complicated functions which could appear in this order. Using the hypothesis of the maximal transcedentality for the universal anomalous dimension of the twist-2 operators we calculated this quantity up to the third order. We suggested a resummation procedure and verified the strong coupling predictions obtained from the AdS/CFT correspondence. In particular, we investigated the Eden-Staudacher equation for $\gamma(j)$ at $j \to \infty$ and calculated the intercept of the BFKL Pomeron in $N = 4$ SUSY at the strong coupling regime.

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