

## Exotic and higher spin mesons in charmonium

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Exotic and higher spin ( $> 1$ ) mesons are still not thoroughly investigated in lattice QCD. Using a set of derivative based operators we report our exploratory study of these mesons in charmonium region. We use a  $12^3 \times 48$  anisotropic ( $\xi = 3$ ) clover lattice with temporal lattice spacing  $a^{-1} = 6.05$  GeV. Techniques developed in this exploratory study will be utilized in our future comprehensive study of light hybrid mesons that are to be explored in the 12 GeV GlueX experiment at Jefferson Laboratory.

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## 1. Introduction

Recently there has been a considerable resurgence of interest in charmonium physics, motivated by the observation of a number of puzzling resonances at  $B$  factories, and by the advent of CLEO-c as well as by the BES upgrade. The QCD spectrum is potentially very rich, admitting states with high angular momenta, glueballs, four-quark states, and hybrid mesons with manifest gluonic degrees of freedom, and therefore numerous ideas have been advanced [1] to interpret these newly observed states. To resolve these ideas, a comprehensive, first-principles calculation of the charmonium spectrum using lattice QCD is therefore crucial. Moreover, a detailed lattice study of hybrid mesons in the charmonium region will help us to develop techniques that can be used in the light-quark sector, whose exploration is the goal of the GlueX experiment as part of the 12 GeV upgrade at Jefferson Laboratory.

To date, the lattice investigations of the spectrum of exotics and states with higher angular momentum are quite limited. In the light-quark sector, though several calculations have been performed to study the exotic  $1^{-+}$  [2], a firm conclusion from lattice QCD about the existence of this state at low pion mass region is yet to be drawn. In the heavy quark sector only a handful of calculations have been performed to study hybrids and higher spin states [3, 4]. The main difficulty encountered when studying these states is the poor overlap of these states with the chosen interpolating operators. Since it is not possible to construct an exotic-meson interpolating field from local quark bilinears alone, one needs to include link variables directly in the interpolating field, introducing additional statistical noise. Similarly, states with higher angular momentum require operators constructed from displaced fields, likewise introducing statistical noise. To efficiently tackle such states requires the use of appropriate smearing of the quark and gluon fields, and a large statistical ensemble.

Besides the spectrum, it is only recently that there has been a lattice study of transition form factors between charmonium states [5]; in two exploratory studies it has been demonstrated that radiative decays [5] as well as two-photon-decay form factors [6] can be studied on the lattice. We intend to extend those studies from form factors of conventional, low-lying charmonium states, to hybrids and other excited states. A subsequent study in the light quark sector will provide much needed information on the photocouplings to the light hybrid mesons relevant for the GlueX experiment.

In these proceedings we report the preliminary results of a comprehensive study of exotics and higher spin mesons in the charmonium region, in the quenched approximation to QCD. We employ as a basis of operators those introduced in Ref. [3], together with additional operators missing in their construction. In forthcoming publications, we will report in more detail the results on the masses of ground and excited states, as well as on transition form factors between various states, including hybrids. These results in the charm sector will facilitate our similar study in the light-quark sector, using dynamical fermions.

## 2. Numerical details

### 2.1 Anisotropic clover action

The relatively large value of the charm-quark mass,  $m_c \sim 1.1 - 1.5$  GeV, poses challenges to lattice QCD at the currently employed inverse lattice spacings of  $a^{-1} \simeq 2 - 3$  GeV, since  $m_c a$  is then

large introducing substantial discretization uncertainties. Whilst a non-relativistic action might be appropriate for the  $b$  quark, in the charmonium sector it is more accurate to use a fully relativistic formulation. We employ an anisotropic lattice, with finer lattice spacing in the temporal than spatial directions, using a Wilson gauge action and clover fermion action. The use of an anisotropic lattice introduces additional parameters beyond those of the isotropic formulation, which must be tuned to yield the required anisotropy. We will now outline this tuning.

In the quenched approximation to QCD, the anisotropy in the gauge sector can be tuned before, and independently of, the anisotropy in the fermion sector. We follow the approach of Ref. [7], tuning the action to yield a renormalized anisotropy  $\xi \equiv a/a_0 = 3$ , where  $a$  and  $a_0$  are the spatial and temporal lattice spacings respectively.

We write the fermion action in the form  $\sum \bar{\psi} Q \psi$ , where

$$Q = m_0 + v_0 \nabla_0 \gamma_0 - \frac{1}{2} r_0 a_0 \Delta_0 + \sum_k \left( v \nabla_k \gamma_k - \frac{1}{2} r a \Delta_k \right) - \frac{a}{2} \left[ \omega_0 \sum_k \sigma_{0k} F_{0k} + \omega \sum_{k<l} \sigma_{kl} F_{kl} \right], \quad (2.1)$$

Detail about this action and its implementation is given in Ref. [7]. Classical values of action parameters are given by [3]

$$m_0 = m_q \left( 1 + \frac{1}{2} m_s m_q \right), \quad (2.2)$$

$$v_t = v_s \frac{1 + \frac{1}{2} a_s m_q}{1 + \frac{1}{2} a_s m_q}, \quad (2.3)$$

$$c_s = v_s, \quad (2.4)$$

$$c_t = \frac{1}{2} \left( v_s + v_t \frac{a_t}{a_s} \right). \quad (2.5)$$

Following Ref. [3], classical values of the clover coefficients, Eqs. 2.4 and 2.5, are improved by tadpole factors,  $u_s$  and  $u_t$ , for spatial and temporal links, respectively, to yield

$$c_s \longrightarrow c_s / u_s^3, \quad (2.6)$$

$$c_t \longrightarrow c_t / (u_t u_s^2). \quad (2.7)$$

The charm mass is determined by tuning the bare quark mass  $m_0$  non-perturbatively such that the spin average of the lowest S-wave mesons (1S) coincides with its experimental value, i.e. :  $(3m_{J/\Psi} + m_{\eta_c})/4 = 3.067$  GeV. The parameters  $v_t$  and  $v_s$  are not independent; we choose  $v_t = 1$  and then tune  $v_s$  non-perturbatively to satisfy the lattice dispersion relation

$$c(p)^2 = \frac{E(p)^2 - E(0)^2}{p^2} = \xi^2 \frac{a_0^2 E(p)^2 - a_0^2 E(0)^2}{a^2 p^2} = 1. \quad (2.8)$$

Keeping all other parameters fixed, we tune  $v_s$  to satisfy the above relation to within  $\sim 1\%$ .

## 2.2 Interpolating operators for exotic and higher spin mesons

Following Ref. [3], we employ a set of operators for exotic and higher spin mesons using covariant derivatives on the quark fields. In Ref. [3], such operators have forward directional

derivatives,  $\vec{\nabla}$ , i.e., acting only on the quark field. For states at rest, this can create required quantum numbers. However, for non-zero momentum, the forward directional derivative alone fails to yield operators of definite charge conjugation. One needs a backward-forward derivative  $\overleftrightarrow{\nabla}$  for this purpose. Hence we change the operator table in Ref. [3] by replacing  $\vec{\nabla} \rightarrow \overleftrightarrow{\nabla} = \overrightarrow{\nabla} - \overleftarrow{\nabla}$ . Backward-forward derivatives were also utilized to study exotics in Ref. [8].

We use gauge-invariant quark smearing [5], along with stout-link smearing [10], to obtain better overlap of the operators to the ground states. Smearred-source point-sink (SP), as well as smearred-source smearred-sink (SS), correlators are used to obtain better signal. Smearing parameters are tuned separately for the various channels. We observe that displacement lengths of one lattice site and two lattice sites yield largely indistinguishable results in effective masses of these operators, and therefore for this calculation we choose displacement length one.

### 3. Results

We employ 1996 quenched configurations on a  $12^3 \times 48$  lattice (with inverse temporal lattice spacing  $6.05 \text{ GeV}^{-1}$ , obtained from the static quark-antiquark potential. To generate meson correlators we used Dirichlet boundary condition with source at 5.

In Figs. 1 - 4, we present representative effective masses for the four different combinations of  $PC$ , for various lattice irreducible representations  $A_1(0)$ ,  $A_2(3)$ ,  $E(2)$ ,  $T_1(1)$  and  $T_2(2)$ ; the numbers in brackets denote the lowest continuum spin lying in each irrep.. To facilitate a comparison of the quality of effective masses, we employ the same scale in each figure, except for the case  $A_1^{--}(0^{--})$  where the signal is noisy and the mass larger. In each channel, including exotics but with the exception of the  $A_1^{--}$ , we obtain a very good signal for several operators. Any of these operators would yield a good ground-state mass with reasonable statistical errors. The quality of the data is better than that of Ref. [3], since we adopted tuned smearred operator in part to reduce noise. A detailed analysis of this data using the variational method, employing multi-exponential fits, and using Bayesian statistics to yield ground and excited-state masses will be reported in forthcoming publications.

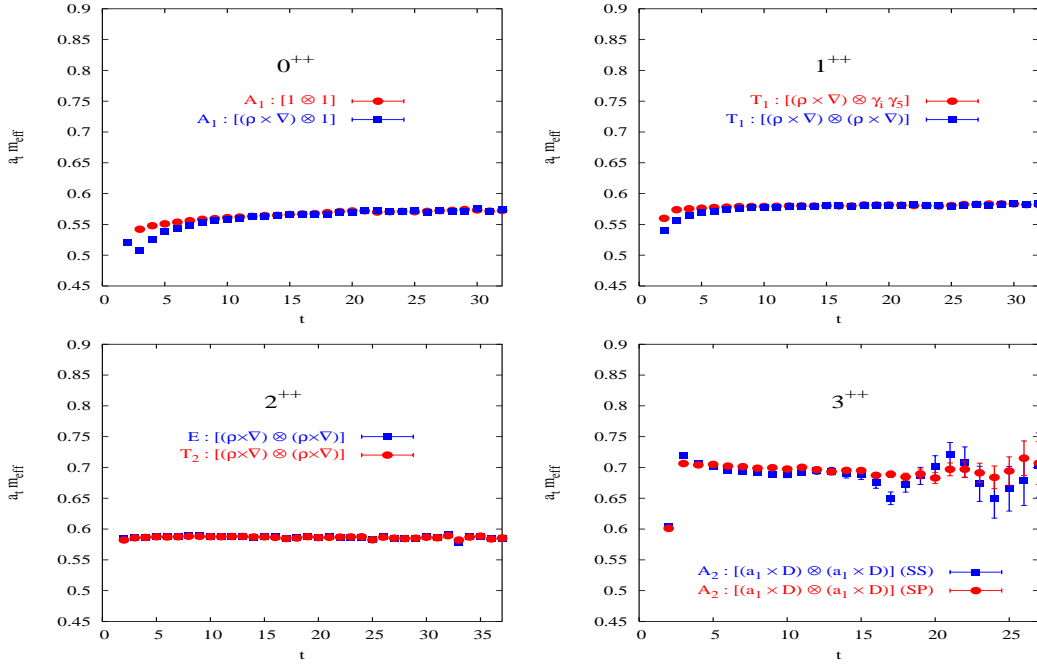
Even our preliminary analysis enables us to make two observations. Firstly, the mass of the exotic  $1^{-+}$  observed in this study is lower than that found in Ref. [3], where the effective mass is poorer<sup>1</sup>. We also observe a good signal for the other exotics,  $0^{+-}$  and  $2^{+-}$ . Secondly, we observe in the  $T_2^{+-}$  channel, the bottom left in Fig. 2, that one operator in this channel produces a lower effective mass than the others. We believe this is an overlap to the  $3^{+-}$  state, as  $T_2^{+-}$  operators have an overlap to both the  $2^{+-}$  and  $3^{+-}$  states<sup>2</sup>. In the bottom right figure in Fig. 4, we plot effective masses from the  $T_2^{+-}$  and  $A_2^{+-}$  correlators simultaneously, and observe that they coincide within statistics. Finally, Fig. 5 shows the relative position of the various channels, together with the observed experimental ordering.

### 4. Conclusions

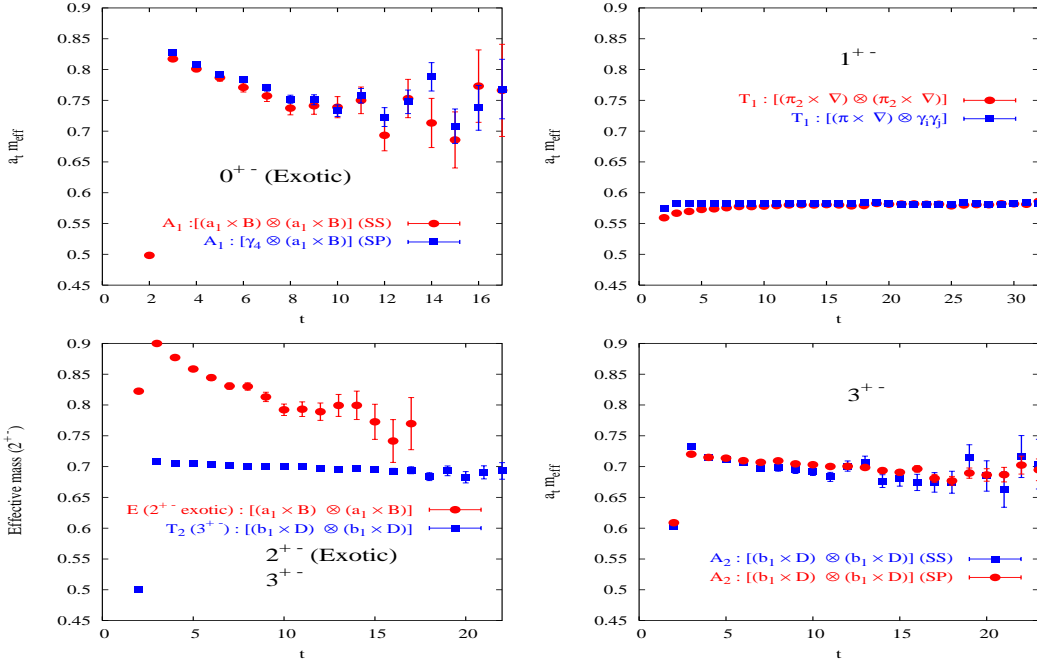
We present here our preliminary results on exotic and exotic state mesons in the charmonium

<sup>1</sup>although the operators  $(\rho \times B)_{T_1}$  and  $(a_0 \times \nabla)_{T_1}$  can, at finite  $a$ , overlap with  $4^{-+}$  (a non-exotic), as well as with  $1^{-+}$ , in the continuum no such  $4^{-+}$  can remain. As such, we are confident in assigning a  $1^{-+}$  status to the observed ground state.

<sup>2</sup>An operator of the type  $(b_1 \times D)_{T_2}$  retains an overlap with  $3^{+-}$  in the continuum.

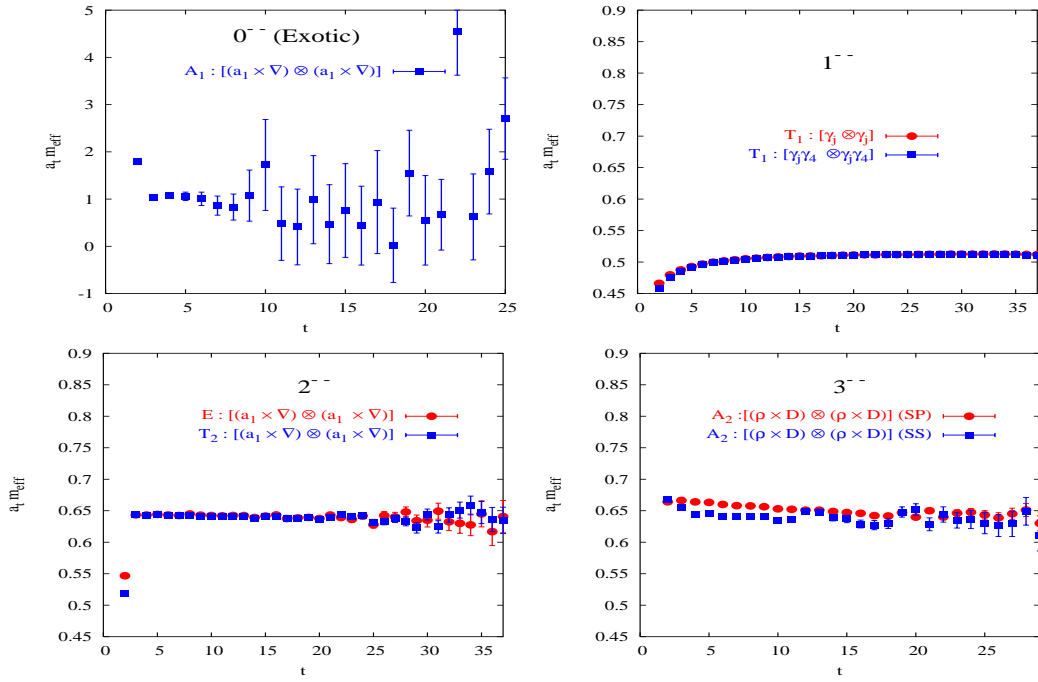
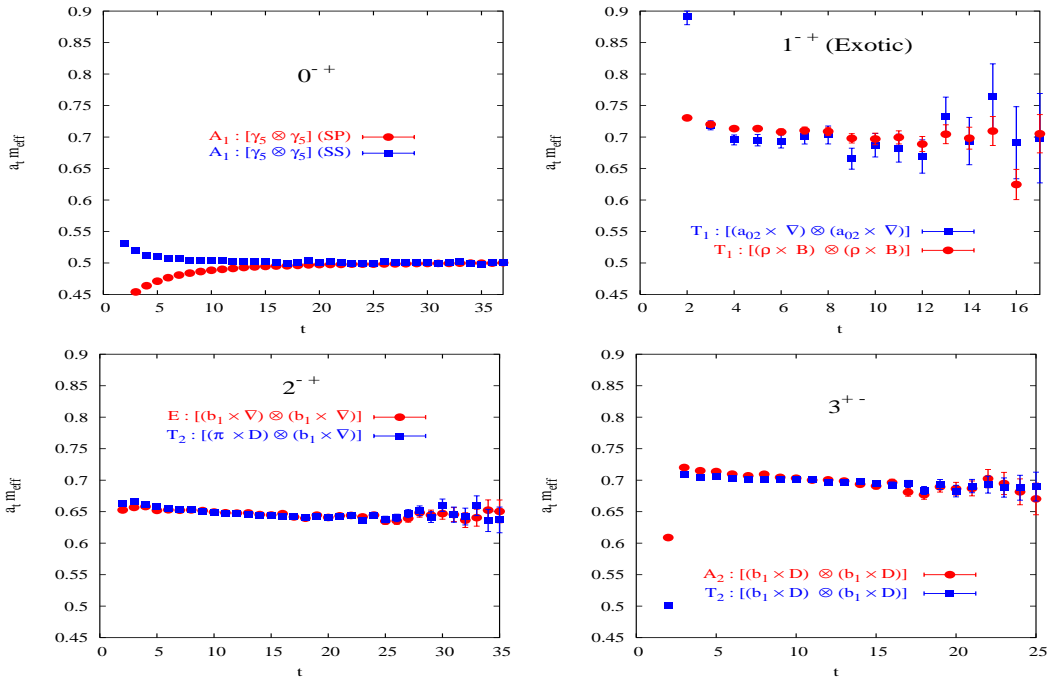


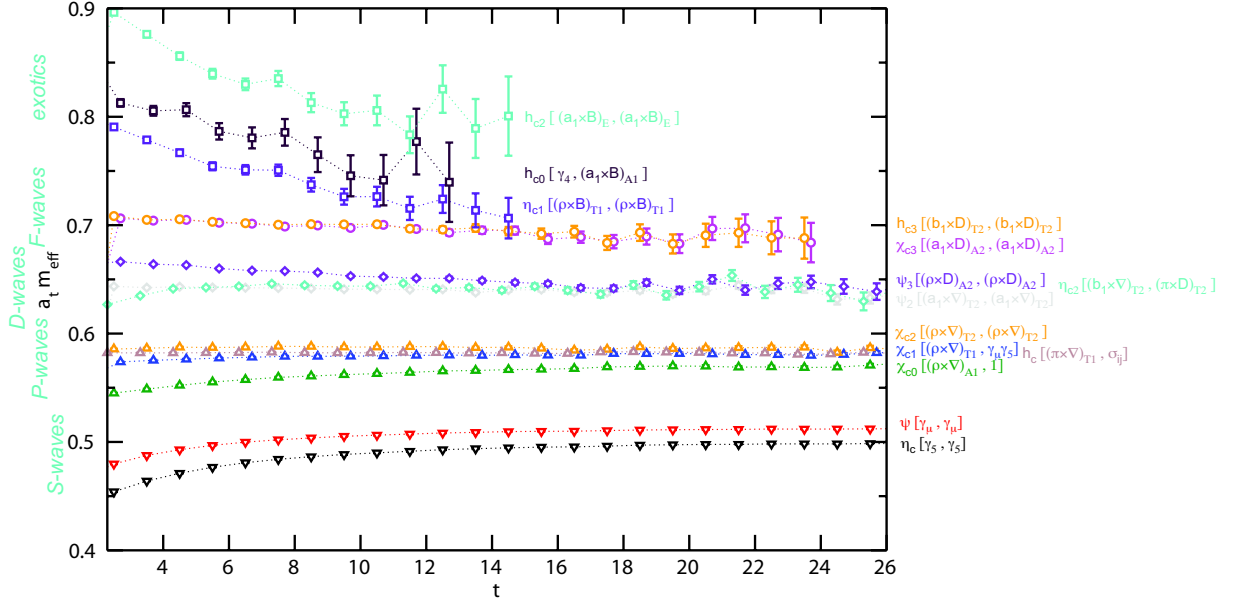
**Figure 1:** Effective masses for  $J^{++}$  states with various local and displaced operators.



**Figure 2:** Effective masses for  $J^{+-}$  states.

sector. We observe excellent signals for most channels, including exotics, and a comprehensive analysis with results for ground and excited states is in preparation. Our preliminary observation suggests the presence of the  $1^{-+}$  exotic at around 4.2 GeV, lower than that was observed in Ref. [3]. The methodology developed here will be utilized in the light quark sector with dynamical clover fermions. Prediction of masses and particularly photo-couplings in the light quark sector will be quite helpful to the GlueX experiments at JLab.


 Figure 3: Effective masses for  $J^{--}$  states.

 Figure 4: Effective masses for  $J^{+-}$  states. Bottom right figure is a comparison of  $3^{+-}$  obtained from  $A_2$  and  $T_2$  representations.



**Figure 5:** Effective masses for various channels showing their relative positions.

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