PoS(LAT2006)093

B_s mixing parameters in $N_f = 2+1$ full QCD

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We present preliminary results for B_s meson mixing parameters relevant for the mass and width differences ΔM_s and $\Delta\Gamma_s$. We use the MILC collaboration $N_f = 2+1$ gauge configurations, NRQCD heavy quarks and AsqTad light quarks. Operator matching is carried out through $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\Lambda_{QCD}/M)$ and $\mathcal{O}(\alpha_s/(aM))$. Comparisons are made with the recent measurement of ΔM_s at the Tevatron.

XXIVth International Symposium on Lattice Field Theory
July 23-28, 2006
Tucson, Arizona, USA.

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1. Introduction

A major achievement at the Tevatron in 2006 has been the observation of mixing in the $B_s^0 - \overline{B_s^0}$ system and the precision measurement of the mass difference ΔM_s. A two-sided bound from the DØ collaboration [1] was followed quickly by a $\sim 2\%$ precise measurement by the CDF collaboration [2]. $B_s^0 - \overline{B_s^0}$ mixing is a $\Delta B = 2$ process and sensitive to possible beyond the Standard Model physics. Hence there is much at stake in comparing the Tevatron ΔM_s value with Standard Model (SM) predictions and one needs to evaluate the Standard Model formula [3],

$$\Delta M_s = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \hat{B}_{B_s}, \tag{1.1}$$

as accurately as possible. $S_0(x_t)$ is the Inami-Lim function with $x_t \equiv m_t^2/M_W^2$ and η_2^B is a perturbative QCD correction factor. The nonperturbative QCD ingredient in (1.1) is the combination of mixing parameters $f_{B_s}^2 \hat{B}_{B_s}$, where f_{B_s} is the B_s meson decay constant and $\hat{B}_{B_s}$ the RG invariant bag parameter. We report here on unquenched lattice QCD calculations of the hadronic matrix elements that determine $f_{B_s}^2 \hat{B}_{B_s}$ and the analogous B_s mixing parameters relevant for the width difference $\Delta\Gamma_s$.

We work with two of the $20^3 \times 64$ coarse MILC ensembles with lattice spacings around 0.123fm and light sea quark masses of $m_f/m_s = 0.5$ and $m_f/m_s = 0.25$ respectively, where m_s is the physical s quark mass and m_f the light (u/d) quark mass. We use the AsqTad action for the valence s quark and NRQCD valence b quarks. We find,

$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 0.281(21)\,\mathrm{GeV}. \tag{1.2}$$

Inserting this into (1.1) leads to

$$\Delta M_s(SM\ theory) = 20.3(3.0)(0.8)\,ps^{-1}, \tag{1.3}$$

which should be compared with the CDF value of [2],

$$\Delta M_s(experiment) = 17.31^{+0.33}_{-0.18} \pm 0.07\,ps^{-1}. \tag{1.4}$$

The first error in (1.3) is the total lattice error and the second is due to uncertainties in $|V_{ts}^* V_{tb}|$ and m_t. The errors in (1.4) are statistical and systematic errors respectively. One sees that within errors (which are currently dominated by theory errors) there is agreement between experiment and the Standard Model prediction. This places nontrivial constraints on the size of any beyond the Standard Model effects in B_s mixing.

In the rest of this article we present further details of our hadronic matrix element calculations. We introduce the four-fermion operators contributing to ΔM_s and $\Delta\Gamma_s$, we discuss operator matching between continuum QCD and the lattice theory and also present some details of fitting the numerical data.

2. Matching of Operators

We have studied the following four-fermion operators that enter into calculations of ΔM_s and $\Delta\Gamma_s$ in the Standard Model ("i" and "j" are color indices).

$$OL \equiv [\overline{b^i} s^i]_{V-A} [\overline{b^j} s^j]_{V-A} \tag{2.1}$$

$$OS \equiv [\overline{b^i} s^i]_{S-P}[\overline{b^j} s^j]_{S-P} \tag{2.2}$$

$$O3 \equiv [\overline{b^i} s^j]_{S-P}[\overline{b^j} s^i]_{S-P} \tag{2.3}$$

One is interested in the hadronic matrix elements of these operators between the B_s^0 and $\overline{B_s^0}$ states. Such matrix elements are parametrized in terms of the B_s meson decay constant f_{B_s} and so-called "bag" parameters, B_{B_s} for operator OL, B_S for OS and $\tilde{B}_S$ for $O3$. One has,

$$\langle OL\rangle^{\overline{MS}}_{(\mu)} \equiv \langle \overline{B}_s|OL|B_s\rangle^{\overline{MS}}_{(\mu)} \equiv \frac{8}{3} f_{B_s}^2 B_{B_s}(\mu) M_{B_s}^2, \tag{2.4}$$

and similarly

$$\langle OS\rangle^{\overline{MS}}_{(\mu)} \equiv -\frac{5}{3} f_{B_s}^2 \frac{B_S(\mu)}{R^2} M_{B_s}^2, \qquad \langle O3\rangle^{\overline{MS}}_{(\mu)} \equiv \frac{1}{3} f_{B_s}^2 \frac{\tilde{B}_S(\mu)}{R^2} M_{B_s}^2, \tag{2.5}$$

with

$$\frac{1}{R^2} \equiv \frac{M_{B_s}^2}{(\overline{m}_b + \overline{m}_s)^2}. \tag{2.6}$$

In order to relate the matrix elements $\langle OX\rangle^{\overline{\mathrm{MS}}}$, $X = L, S$ or 3, to matrix elements evaluated via lattice simulations, one must match the continuum QCD four-fermion operators to operators written in terms of lattice heavy and light quark fields. At lowest order in $1/M$ lattice operators are the same as in (2.1) - (2.3) with the b fields replaced by NRQCD heavy quark or heavy anti-quark fields and the q fields by four component AsqTad fields [4, 5]. At $\mathscr{O}(\Lambda_{QCD}/M)$ one finds that additional dimension seven operators are required such as,

$$OLj1 \equiv \frac{1}{2M}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}s^i]_{V-A}[\overline{b^j} s^j]_{V-A} + [\overline{b^i} s^i]_{V-A}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}s^j]_{V-A}\right\} \tag{2.7}$$

and similar $1/M$ corrections $OSj1$ and $O3j1$ for the four-fermion operators OS and $O3$. Through $\mathscr{O}(\alpha_s)$, $\mathscr{O}(\Lambda_{QCD}/M)$ and $\mathscr{O}(\alpha_s/(aM))$ one then has,

$$\frac{a^3}{2M_{B_s}}\langle OX\rangle^{\overline{\mathrm{MS}}} = [1+\alpha_s\cdot\rho_{XX}]\langle OX\rangle + \alpha_s\cdot\rho_{XY}\langle OY\rangle + \left[\langle OXj1\rangle - \alpha_s(\zeta_{10}^{XX}\langle OX\rangle + \zeta_{10}^{XY}\langle OY\rangle)\right] \tag{2.8}$$

where $\langle OX\rangle$ without the superscript $\overline{\mathrm{MS}}$ stands for the matrix element in the lattice theory. The factor of $\frac{a^3}{2M_{B_s}}$ on the LHS of (2.8) takes into account the different normalization of states in QCD and the lattice theory and also makes the lattice matrix elements $\langle OX\rangle$ dimensionless. One sees that there is mixing between the different four-fermion operators already at lowest order in $1/M$. At $\mathscr{O}(\alpha_s)$ mixing occurs between $X,Y = L$ and S for $\langle OL\rangle$ and $\langle OS\rangle$ and between $X,Y = 3$ and L for $\langle O3\rangle$. The $\alpha_s\cdot\zeta_{10}^{XX}$ and $\alpha_s\cdot\zeta_{10}^{XY}$ terms in (2.8) are necessary to subtract $\mathscr{O}(\frac{\alpha_s}{aM})$ power law contributions from the matrix elements $\langle OXj1\rangle$.

3. Simulations and Fitting

The hadronic matrix elements $\langle\hat{O}\rangle$, $\hat{O} = OX$ or $OXj1$, are obtained by numerically evaluating three-point correlators

$$C^{(4f)}(t_1,t_2) = \sum_{\vec{x}_1,\vec{x}_2}\langle 0|\Phi_{\overline{B}_s}(\vec{x}_1,t_1)\,[\hat{O}]_{(0)}\,\Phi^\dagger_{B_s}(\vec{x}_2,-t_2)|0\rangle. \tag{3.1}$$

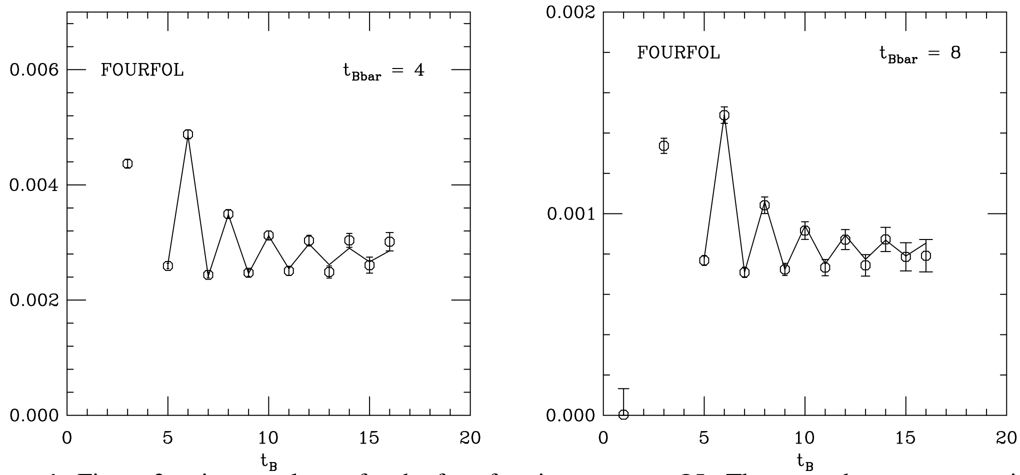

Figure 1: Fits to 3-point correlators for the four-fermion operator OL. The ground state exponential decay $e^{-E_B^{(0)}\cdot(t_B+t_{Bbar})}$ has been factored out.

Φ_{B_s} is an interpolating operator for the B_s meson. One fits $C^{(4f)}$ together with the B_s meson two-point correlator, $C^B(t)$, to the following forms.

$$C^{(4f)}(t_1,t_2) = \sum_{j,k=0}^{N_{exp}-1} A_{jk}\,(-1)^{j\cdot t_1}\,(-1)^{k\cdot t_2}\,e^{-E_B^{(j)}(t_1-1)}\,e^{-E_B^{(k)}(t_2-1)}, \tag{3.2}$$

$$C^B(t) = \sum_{\vec{x}} \langle 0|\Phi_{B_s}(\vec{x},t)\,\Phi^{\dagger}_{B_s}(0)|0\rangle = \sum_{j=0}^{N_{exp}-1} \xi_j\,(-1)^{j\cdot t}\,e^{-E_B^{(j)}(t-1)}. \tag{3.3}$$

The hadronic matrix elements entering the RHS of (2.8) are then given by,

$$\langle \hat{O}\rangle \equiv \langle \overline{B}_s|\hat{O}|B_s\rangle = \frac{A_{00}}{\xi_0}. \tag{3.4}$$

We accumulated data for $1 \leq t_1,t_2 \leq 16$ and carried out Bayesian fits. This introduces priors and prior widths for each of the fit parameters, A_{jk}, $E_B^{(j)}$ and ξ_j [6]. One tries to increase the number of exponentials until the fit values, the fit errors and the χ^2/dof stabilizes. We have found fits of the form (3.2) more challenging than in previous calculations of B meson decay constants [7, 8] and semileptonic form factors [9]. It was sometimes not possible to have stable fits as one continued to increase N_{exp}. Very good fits were interlaced with fits with worse χ^2 values. In order to get around this problem we fixed two of the parameters, $E_B^{(0)}$ and $E_B^{(1)}$, to their known values coming from the B two-point correlator (this can be accomplished by using very narrow prior widths for just these two parameters). Fit values for A_{00} and ξ_0 were then stable with respect to changes in the number of exponentials once $N_{exp} > 4 \sim 5$. We have inflated the fitting errors to cover any differences between fits with narrow widths for $E_B^{(0)}$ and $E_B^{(1)}$ and previous fits with broad widths that were successful, i.e. had good χ^2/dof. In Figs.1&2 we compare our fits with the data. Plots are given for effective amplitudes with the ground state exponential decay factored out. We show $C^{(4f)}(t_1,t_2)\times e^{E_B^{(0)}\cdot(t_1+t_2)}$ versus $t_1 \equiv t_B$ for two fixed values of $t_2 \equiv t_{Bbar}$.

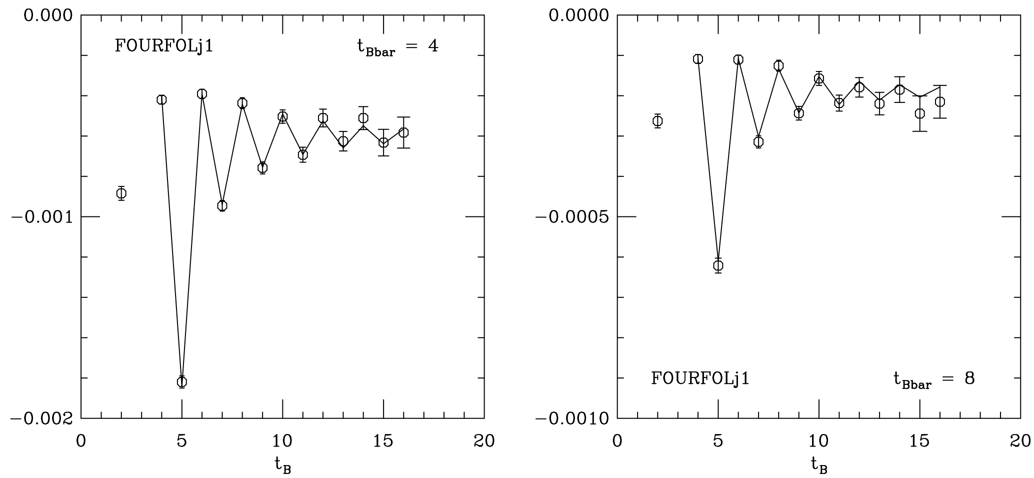

Figure 2: Same as Fig.1 for the $1/M$ correction $OLj1$.

Table 1: Error budget for quantities listed in (4.1).

Statistical + Fitting	9 %
Higher Order matching	9 %
Discretization	4 %
Relativistic	3 %
Scale (a^{-3})	5 %
Total	15 %

4. Results

Using the matrix elements $\langle OX \rangle$ and $\langle OXj1 \rangle$ determined from the fits we evaluate the RHS of (2.8) to obtain $\langle OL \rangle^{\overline{\rm MS}}$, $\langle OS \rangle^{\overline{\rm MS}}$ and $\langle O3 \rangle^{\overline{\rm MS}}$. Combining this with the definitions (2.4) and (2.5) gives us,

$$f^2_{B_s} B_{B_s}, \quad f^2_{B_s} \frac{B_S}{R^2}, \quad f^2_{B_s} \frac{\tilde{B}_S}{R^2}. \tag{4.1}$$

In Table 1 we list the main errors in these quantities. The perturbative error is a significant component. We take this to be $1 \times \alpha_s^2$ since the matching is done directly for the combination $f^2_{B_s} B_{B_s}$ the quantity needed for ΔM_s. We remark parenthetically that attempting to naively separate $f^2_{B_s}$ and B_{B_s} will increase the error because the matching error in f_{B_s} is usually also taken as $1 \times \alpha_s^2$.

Table 2 gives our final results for the square root of the quantities in (4.1) evaluated at scale $\mu = m_b$ together with the scale invariant combination $f_{B_s} \sqrt{\hat{B}_{B_s}}$. One sees that the light sea quark mass dependence is small compared to our other errors. Hence, we take the $m_f/m_s = 0.25$ result as our best determination. This leads to one of our main results given in (1.2), which provides the crucial nonperturbative QCD ingredient in the Standard Model formula for ΔM_s (1.1). For the other ingredients in this formula we use $\eta_2^B = 0.551(7)$, $\overline{m}_t(m_t) = 162.3(2.2)$GeV and $|V^*_{ts} V_{tb}| = 4.1(1) \times 10^{-2}$ to obtain the $\Delta M_s(theory)$ of (1.3). A consistencey check on the Standard Model can be carried out in a slightly different manner if one uses the experimental value for ΔM_s given in

Table 2: Results for the square root of quantities listed in (4.1).

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ [GeV]	0.281(21)	0.289(22)
$f_{B_s}\sqrt{B_{B_s}(m_b)}$ [GeV]	0.227(17)	0.233(17)
$f_{B_s}\frac{\sqrt{B_S(m_b)}}{R}$ [GeV]	0.295(22)	0.301(23)
$f_{B_s}\frac{\sqrt{\tilde{B}_S(m_b)}}{R}$ [GeV]	0.305(23)	0.310(23)

(1.4), combines it with the formula (1.1) plus the lattice result (1.2) and extracts a value for $|V_{ts}^* V_{tb}|$. One finds

$$|V_{ts}^* V_{tb}| = 3.8(3)(1) \times 10^{-2} \tag{4.2}$$

which is consistent with the standard value $4.1(1) \times 10^{-2}$ used above. The latter number follows from the measured value for $|V_{cb}|$ plus unitarity constraints.

In order to compare with previous lattice studies of B_s meson mixing that focused on bag parameters one can extract the latter parameters from our results in Table 2. We use $f_{B_s} = 0.260(29)$GeV [7], $\overline{m}_b = 4.25$GeV and $\overline{m}_s = 85$MeV and the results are summarized in Table 3. For B_{B_s} we also present results without the $1/M$ correction (i.e. dropping the second square bracket on the RHS of (2.8)). This is the more appropriate quantitity to compare against the JLQCD [10] result which did not include dimension seven operator corrections. For the other two bag parameters B_S and $\tilde{B}_S$ the $1/M$ corrections are a smaller effect and we only show our results using the full expression (2.8).

5. Summary

We have completed a calculation of hadronic matrix elements of heavy-light four-fermion operators relevant for $B_s^0 - \overline{B_S^0}$ mixing using MILC $N_f = 2+1$ unquenched configurations, AsqTad valence s quarks and NRQCD b quarks. Using our nonperturbative QCD results for $f_{B_s}^2 \hat{B}_{B_s}$ one finds agreement between Standard Model predictions for ΔM_s and recent precision measurement of this quantity at the Tevatron. We also present results for other hadronic matrix elements, $\langle OS \rangle^{\overline{MS}}$ and $\langle O3 \rangle^{\overline{MS}}$ relevant for the width difference $\Delta\Gamma_s$. These nonperturbative QCD inputs will play an important role in further tests of the Standard Model once accurate experimental values for $\Delta\Gamma_s$ become available. The HPQCD collaboration is focusing on reducing errors listed in Table 1. Work on $B_d^0 - \overline{B_d^0}$ mixing and the important ratio $f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$ is also underway.

Table 3: Bag parameters and comparison with previous work

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$	JLQCD [10]
			($N_f = 2$)
B_{B_s}	0.76(11)	0.80(12)	
B_{B_s}	0.88(13) (no 1/M)	0.92(14) (no 1/M)	0.85(6) (no 1/M)
$\hat{B}_{B_s}$	1.17(17)	1.23(18)	1.30(9) (no 1/M)
			Hashimoto et al. [11]
			(quenched)
$\frac{B_S}{R^2}$	1.29(19)	1.34(20)	1.24(16) (no 1/M)
$\frac{\tilde{B}_S}{R^2}$	1.38(21)	1.42(21)	
			Becirevic et al. [12]
			(quenched)
B_S	0.84(13)	0.87(13)	0.84(2)(4)
$\tilde{B}_S$	0.90(14)	0.93(14)	0.91(3)(8)

Acknowledgments

This work was supported by the DOE and NSF (USA) and by PPARC (UK). The numerical simulations were carried out at NERSC. We thank the MILC collaboration for use of their unquenched gauge configurations.

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