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B_K on 2+1 flavor Iwasaki DWF lattices

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We present current results from an ongoing calculation of B_K on 2+1 flavor domain wall fermion lattices with $\beta = 2.13$ generated with the Iwasaki gauge action (inverse lattice spacing $a^{-1} \approx$ 1.6 GeV). Nonperturbative renormalization and chiral fits to the partially quenched 2+1 flavor form are discussed. A new kind of source for large lattice volumes is introduced.

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1. Introduction

The continuing drive of lattice QCD to provide a connection between the first principles of quantum chromodynamics and the experimental measurements of such hadronic observables as the kaon mixing rate requires the application of lattice techniques to ever larger and more sophisticated ensembles. The proceedings of the 2005 Lattice Symposium[1] described the RBC and UKQCD collaborations' preliminary calculation of the kaon bag parameter on $16^3 \times 32 \times 8$, 2+1 flavor domain-wall fermion lattices using the DBW2 gauge action[3].

In this proceeding, we will describe the next stage of the joint RBC and UKQCD collaborations' effort to determine B_K using domain-wall fermions. Our new calculation uses $16^3 \times 32 \times 16$, 2+1 flavor domain-wall fermions and the Iwasaki gauge action; these lattices also feature a greatly reduced residual mass, due to a doubling of the size of our fifth dimension. This discussion will also include a full partially quenched chiral perturbation theory fit to the data and a preliminary non-perturbative renormalization. We will also briefly introduce some preliminary results from even larger $24^3 \times 64 \times 16$ lattices.

2. The Kaon Bag Parameter

The kaon bag parameter is a crucial endeavour for lattice QCD due to its key role connecting experimental results for kaon mixing and CP violation to constraints of the unitarity triangle. At the persent time, theoretical uncertainty in B_K remains the largest source of error in the calculation of the indirect CP-violation parameter ε. (For a recent review of B_K, see the plenary in these proceedings by Lee: [2])

It is defined by the ratio of the kaon-mixing matrix element and its vacuum saturation value:

$$B_K = \frac{\langle \overline{K^0} | \mathscr{O}_{LL}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} f_K^2 M_K^2}, \tag{2.1}$$

where M_K is the mass of the neutral kaon, f_K is the decay constant of the kaon (given by its coupling to the axial current), and $O_{LL}^{\Delta S=2} = (\bar{s}(1-\gamma_5)\gamma_\mu d)(\bar{s}(1-\gamma_5)\gamma_\mu d)$ is a four-quark operator coupling to left-handed quarks that changes strangeness by 2.

In the chiral limit, B_K contains only the operator of the form $VV + AA$:

$$\mathscr{O}_{VV+AA}^{\Delta S=2} = (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu \gamma_5 d)(\bar{s}\gamma_\mu \gamma_5 d). \tag{2.2}$$

Thanks to the degree of control offered over chiral symmetry breaking by domain-wall fermions, we can ensure that this operator is not washed out by large off-chirality terms[4]. Since these unwanted terms will contribute as $O(m_{\text{res}}^2)$, we simply need to keep the residual mass small.

3. Lattices

Our new lattices are $16^3 \times 32 \times 16$ and have bare sea quark masses $m_s = 0.04$ and $m_l \in (0.01, 0.02, 0.03)$. These lattices were generated on QCDOC computers at Brookhaven National Laboratories and the University of Edinburgh using the RHMC with multiple integration timescales. Evolutions of length 4000 time units were generated for each sea quark mass, using trajectories of length $\tau = 1$, and the first 1000 time units were discarded for thermalization. We select configurations separated by 40 for analysis, giving us a total of 75 lattices on which we measure weak matrix elements for each light sea quark mass.

Each data set includes two- and three-point correlators from the 15 nondegenerate combinations of 5 valence quark masses $m_V \in \{0.01, 0.02, \ldots, 0.05\}$. The correlators are calculated by inversion of the Dirac operator using wall sources (after fixing to Coulomb gauge) located at $t = 5$ and 27. The available time-length on the lattice is doubled by summing the propagators found using periodic and antiperiodic boundary conditions at the t-boundary.

The lattice scale for this ensemble in the physical sea and valence quark mass limit is determined by a consensus of methods including the rho meson mass and static quark potential; these methods are in agreement to the precision indicated. We find $a^{-1} = 1.60(3)$ GeV[5].

The residual mass is determined from the pseudoscalar correlator having a sink at the midpoint of the fifth dimension, which agrees with the (negative) quark mass at which the pseudoscalar mass extrapolates to zero. We find $m_{\text{res}} = 0.00307(4) \approx 5$ MeV, which is fairly small, a third of our lightest quark mass.

We may then use a simple leading-order chiral fit to the pseudoscalar mass to determine the physical light and strange quark masses on our lattices. We find $m_l = 0.00176(7)$ and $m_s = 0.0427(17)$. Notice that since the physical strange quark mass turns out to be near our sea input $(0.04 + m_{\text{res}})$, we need not extrapolate in this quantity to take the physical limit.

4. $16^3 \times 32$ Results

It is possible to determine B_K in the most naive way by simply taking the appropriate three-point correlator and dividing by the wall-wall two-point correlator to remove the matrix elements associated with the wall sources. However, this method requires the introduction of the noisy two-point wall-wall correlator and multiplication by the pseudoscalar decay constant, pseudoscalar mass and axial current renormalization factor Z_A.

Rather than introducing these sources of error, we may determine B_K at once by a clever use of pseudoscalar-axial wall-point correlators:

$$B_P^{\text{bare}} = \frac{2V}{\frac{8}{3}} \frac{\mathscr{C}_{wpw}^{P\mathscr{O}P}(t_{\text{src}}, t, t_{\text{snk}})}{\mathscr{C}_{wp}^{PA^4}(t_{\text{src}}, t)\mathscr{C}_{pw}^{A^4P}(t, t_{\text{snk}})}, \tag{4.1}$$

where V is volume, and the $\mathscr{C}$'s are correlators with superscripts denoting the source, insertion (for three-point correlators) and sink operators (one of P pseudoscalar, A^4 time-component axial or the $\mathscr{O}_{VV+AA}^{\Delta S=2}$ mixing operator in this case) and subscripts denoting source, insertion and sink shapes (p point or w wall). For each three-point correlator, the point insertion is summed over all space in its timeslice.

We expect B_K to approach its asymptotic value far from the source and sink. Depicted in Fig. 1 is the plateau for a sample set of masses on our lightest sea mass ensemble. The plateau appears to be reached well outside our indicated fit range.

In Fig. 2, we show B_K fit to the partially quenched 2+1 flavor continuum form given by [6]:

$$\left(\frac{B_K}{B_0}\right)^{\text{PQ},2+1} = 1 + \frac{1}{48\pi^2 f^2 m_{xy}^2} \left[I_{\text{conn}} + I_{\text{disc}} + b m_{xy}^4 + c(m_X^2 - m_Y^2)^2 + d m_{xy}^2 (2m_D^2 + m_S^2) \right] \tag{4.2}$$

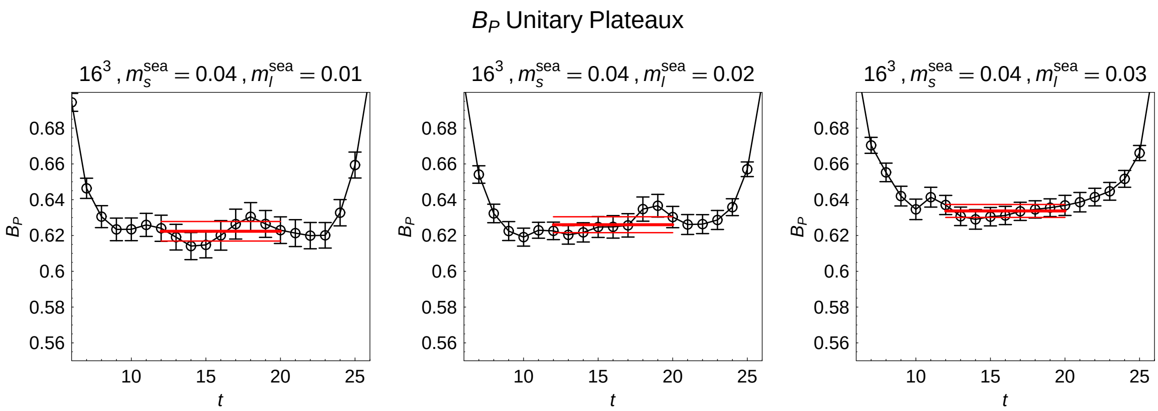

Figure 1: Plateau of B_P for valence masses equal to sea masses on $m_s = 0.04, m_l = 0.01$. Fitting range, value and uncertainty are indicated by the red band.

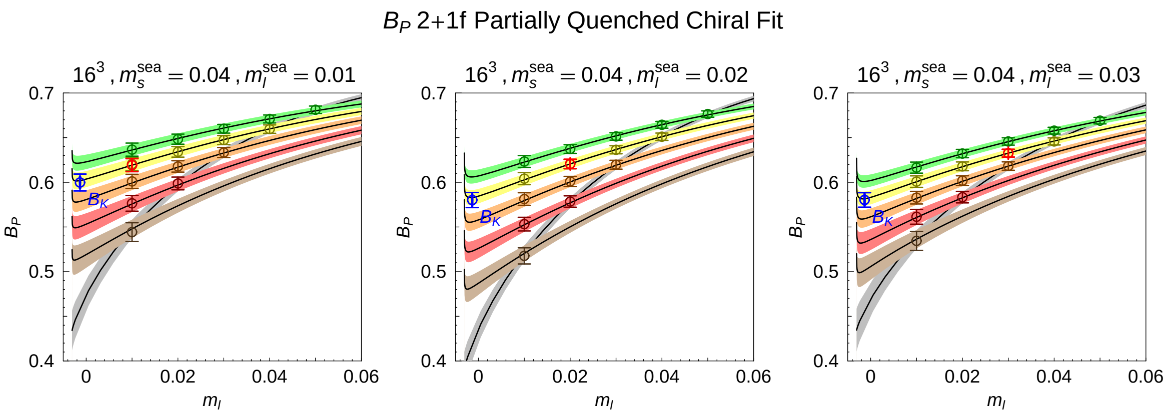

Figure 2: Pseudoscalar bag parameter as a function of light valence quark mass (where the chiral limit is at $m = -m_{\rm res}$). Each colored band indicates a different strange valence quark mass: brown = 0.01 through green = 0.05. The blue point marks valence extrapolation to physical B_K, and the red point marks the valence = sea point.

where B_0, b, c and d are fit parameters, f is the chiral limit of the pseudoscalar decay constant, m_Q is the mass of the pseudoscalar meson with the indicated composition, and the I's are known chiral logs. To use the continuum form with domain-wall fermions, we shift each quark mass by $m_{\rm res}$. The resulting chiral form should be accurate up to $O(a)$ uncertainties in $m_{\rm res}$, which are not included, although they could be modeled by NLO terms in the chiral expansion.

Taking the valence = sea point (red on Fig. 2) for each set of lattices, we may extrapolate to the chiral limit for the light quarks (leaving the strange quarks at the physical strange quark mass). This yields a value for the bag parameter of a kaon with nondegenerate quarks with physical masses (See Fig. 5.):

$$B_K^{\rm bare} = 0.607(10). \tag{4.3}$$

Alternatively, we may take the valence = physical limit first (blue on Fig. 2), taking advantage of the chiral fitting form, and then extrapolate linearly to the sea = physical limit. This alternate

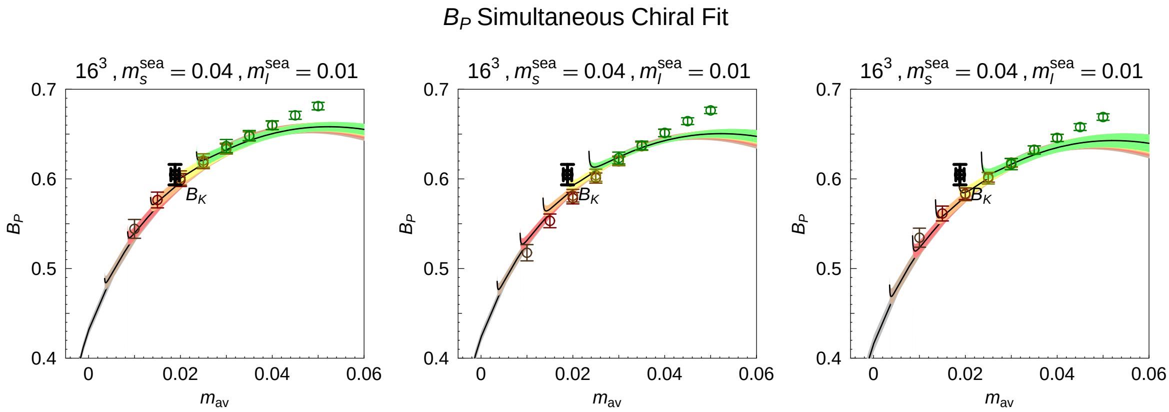

Figure 3: Pseudoscalar bag parameter as a function of average valence quark mass. The colored bands indicate the results of a simultaneous fit to all three data sets, excluding points with heavy average valence mass (above 0.03) where we do not expect chiral perturbation theory to be accurate. The black point marks the physical kaon with nondegenerate unquenched strange and light masses.

method yields a somewhat lower value of

$$B_K^{\text{bare}} = 0.606(14). \tag{4.4}$$

However, either of these two-step extrapolations could be avoided by fitting all three data sets from different sea quark masses simultaneously to the partially quenched chiral form; see Fig. 3. Such a simultaneous fit gives

$$B_K^{\text{bare}} = 0.605(11). \tag{4.5}$$

In order to compare our B_K result, we must determine the renormalization of its operator and the matching to $\overline{\text{MS}}$ scheme. Since domain-wall fermions are well suited to nonperturbative renormalization in the RI-MOM scheme, we avoid problems associated with lattice perturbation theory; see Ref. [4] for a full description of the method. We find $Z_{B_K} = 0.90(2)$, giving a renormalized value

$$B_K = 0.546(10)(11). \tag{4.6}$$

See Fig. 6 for a comparison of this value to other recent results.

5. Operator Source

One alternative to the technique outlined above would be to exchange the two wall sources for a single source located at the third-point operator insertion. By gauge fixing the lattice, we may determine the appropriate sink to use for the two walls. This method has a couple advantages: It reduces the number of inversions required, and it allows us to vary the length of the plateau.

Although a comparison shows (in Fig. 4 that these two methods agree, even after using four times as many configurations, the statistical error remains larger than the operator sink method. The increased noise in the operator source results (possibly due to the lack of a sum over space at the insertion) means that it is preferable to use the operator sink method.

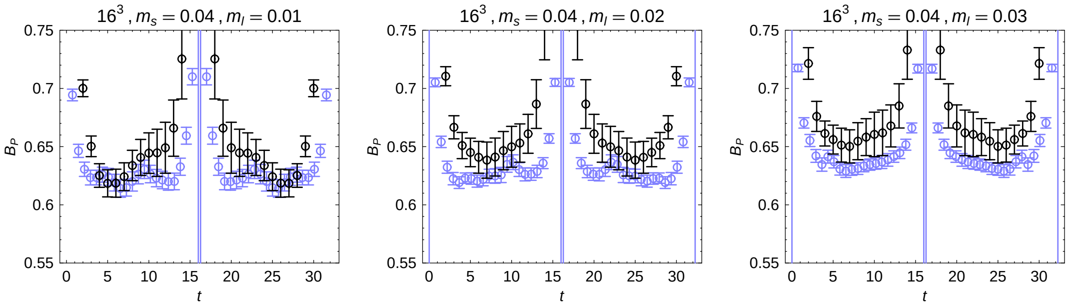

Figure 4: Comparison between the operator source (black) and operator sink (pale blue) methods. Due to the differences in the way the plateaux are calculated, the operator sink points are mirrored on either side of the operator source for easier comparison.

6. $24^3 \times 64$ and the Convolved Source

The RBC and UKQCD collatorations have now generated $24^3 \times 64 \times 16$, 2+1 flavor DWF lattices with the same sea quark masses as the 16^3 volumes described previously. The RBC/UKQCD B_K project is now focused on measuring B_K on these lattices. Since the wall sources will now be much larger, we have tried using a new kind of source that should have better overlap with physical meson states.

The convolved source is constructed to have two properties: a characteristic size on the order of a physical meson (around 16^3 at this lattice scale) and translational invariance to avoid contamination by higher-momentum states. The source is constructed by applying a convolution operator to a random wall source. Each spatial location in the source is assigned the sum of the randomly selected values inside a box with the desired characteristic size:

$$s(x) = \sum_{|\vec{x}-\vec{y}|_\infty < L_{\rm box}} \eta(y), \tag{6.1}$$

where η is a different random number at each spatial location, selected from a Gaussian distribution; $L_{\rm box}$ is the desired size (16 in this case); and the infinity norm will limit the summation to the shape of a box.

Since fluctuations between different Gaussian random numbers will average to zero, this source should contribute nothing for parts of the source that are separated by more than $L_{\rm box}$. Since both the convolution operator and the random wall source (in the limit of many sets of random numbers) are translationally invariant, the convolved source will be as well.

We have tested this convolved source extensively and find that although it does improve the approach to the plateau and maintains translational invariance, the noise introduced by the random source exceeds the gauge fluctuation noise, making it not viable for our purposes. Since standard wall sources have no such noise, we will merely rely on the very long time-lengths available on the lattice to allow a slower approach to the plateau to be sufficient.

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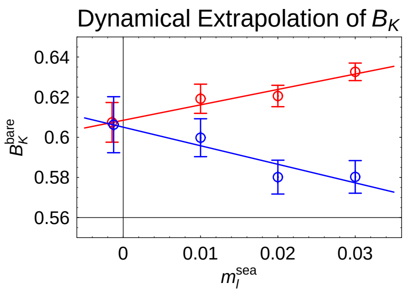

Figure 5: Two extrapolations of B_K to the physical point: **red:** extrapolates valence = sea to the physical point; **blue:** takes valence = physical, then extrapolates sea to the physical point

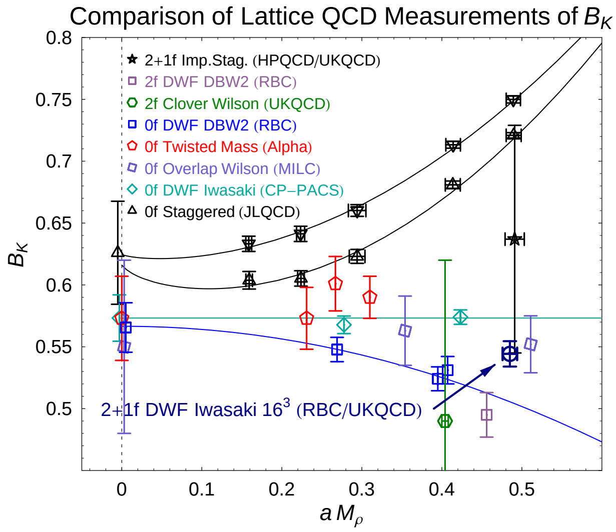

Figure 6: Dynamical extrapolation of renormalized B_K to the physical point of 2+1 flavor lattices at 1.6 GeV compared to other recent values

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