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Double hairpin diagrams and the planar equivalence of N=1 supersymmetric Yang-Mills theory and one-flavor QCD

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Recent work by Armoni, Shifman, and Veneziano suggests a large-N equivalence between supersymmetric Yang-Mills Theory and one-flavor QCD. One consequence of this "orientifold projection" is that scalar and pseudoscalar mesons in one-flavor QCD should have degenerate mass since they lie within the same Wess-Zumino supermultiplet. We use lattice calculations to investigate the mass shifts caused by "double-hairpin" annihilation diagrams in quenched QCD to test for this degeneracy. Similar quark-antiquark annihilation processes are studied in the 2-dimensional $CP^{(N-1)}$ model with quenched fermions.

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1. Introduction

The theoretical connections between QCD and string theory which have emerged in recent years have provided a new framework for the study of low energy hadron dynamics. While these connections are, for the most part, only convincingly argued in the limit of large N_c, it is of great interest to test the predictions of string/gauge correspondences against real-world $N_c = 3$ QCD using both phenomenology and lattice results. A particularly interesting correspondence in this regard is the recently discovered orientifold planar equivalence between N = 1 SUSY Yang-Mills theory and ordinary nonsupersymmetric one-flavor QCD [1, 2, 3]. Although this is a direct field theory to field theory equivalence, it has its roots in string/gauge duality. To the extent that this equivalence is reliable at $N_c = 3$, it provides a powerful new source of insight into low energy quark dynamics. What is truly remarkable about this connection is that the quark of the daughter theory (one-flavor QCD) is the orientifold projection of a gluino in the parent theory (N = 1 SUSY YM). If this equivalence is even qualitatively valid at $N_c = 3$, it would expose a deep and surprising role of supersymmetry in low energy hadron physics. The prediction of orientifold equivalence for the size of the quark condensate in one-flavor QCD has been compared with lattice results in [4]. In our work, we consider another very interesting prediction of orientifold equivalence which, if valid, would be an unmistakable relic of the supersymmetry of the parent theory. We refer to the prediction that the lowest-lying scalar and pseudoscalar mesons of one-flavor QCD (hereafter referred to as σ and η' respectively) should be degenerate. In the parent SUSY theory, this degeneracy results from the simple fact that σ and η' belong to the same Wess-Zumino supermultiplet and that supersymmetry is unbroken. This is conveniently described in terms of the supersymmetric chiral Lagrangian constructed by Veneziano and Yankielovich (VY) [5]. In this formalism, the scalar and pseudoscalar mesons are associated with the real and imaginary parts of the complex scalar Wess-Zumino field representing gluino-antigluino bound states. Although the scalar-pseudoscalar meson degeneracy is natural in the supersymmetric parent theory, it is much more mysterious from the point of view of one-flavor QCD. In that theory, we are accustomed to thinking of the pseudoscalar meson η' as a would-be Goldstone boson which acquires a mass via the axial anomaly. In lattice calculations, the role of the anomaly in generating the η' mass has been verified in detail by the study of the quark-line-disconnected "hairpin diagram" contributions to the pseudoscalar correlator. In contrast to the would-be Goldstone boson interpretation of the η', the scalar meson σ is usually presumed to be in most respects a typical P-wave quark-antiquark meson. Phenomenology and lattice calculations suggest that the corresponding flavor nonsinglet mesons are quite heavy, in the 1300-1500 MeV range. The flavor singlet σ correlator includes hairpin diagrams along with the valence correlator which results in a large negative mass shift of the flavor singlet relative to the nonsinglet. The scalar hairpin correlator was calculated in quenched lattice QCD in [6] as part of a study of the spin-parity structure of the OZI rule. Within the quenched approximation, [6] and [7] contain all of the Monte Carlo results necessary for testing the orientifold prediction. Although errors on the scalar hairpin mass shift are sizable, we show that within these errors, the combination of the upward shift of the pseudoscalar mass and the downward shift of the scalar mass, when evaluated in the one-flavor theory, brings the two mesons into approximate degeneracy. In order to try to explore the mechanism responsible for the degeneracy, we also present the results of similar pseudoscalar and scalar meson flavor singlet mass calculations in two-dimensional CP^3. In this model it is relatively easy to use the overlap operator in high-statistics calculations to explore the chiral limit. This permits us to compare the scalar and pseudoscalar masses with much smaller statistical error.

2. Orientifold planar equivalence

The orientifold planar equivalence of SUSY Yang-Mills theory (SYM) and one-flavor QCD states that SYM is equivalent to one-flavor QCD to the extent that $N_c = 3$ can be considered large. This equivalence can most easily be seen by proceeding in two steps:

1. Define the orientifold-A projection of SYM and show that this theory is precisely equal to one-flavor QCD for the special case of $N_c = 3$.
2. Demonstrate that for large N_c the orientifold-A projection is equivalent to SUSY Yang-Mills.

The orientifold-A projection can be understood by examining the SYM Lagrangian.

$$L_{SYM} = -\frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \frac{i}{g^2} \overline{\lambda}^a \gamma_\mu (D_\mu \lambda)^a \tag{2.1}$$

Here $\lambda^a = \begin{pmatrix} \eta^a_\alpha \\ \overline{\eta}^{\dot{\alpha} a} \end{pmatrix}$ is the gluino, the fermionic super-partner of the gluon, written as a four-component Majorana spinor in the adjoint representation, and $D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c$ is the adjoint covariant derivative. Note that the SYM Lagrangian contains $(N^2 - 1)$ Weyl fermionic (gluino) degrees of freedom. The orientifold-A projection organizes these Weyl degrees of freedom in Dirac spinors associated with the $\frac{N(N-1)}{2}$ anti-symmetric generators of SU(N).

$$\lambda^i_j = \lambda^a (t^a)^i_j \to \psi^{[ij]} = \psi^{\tilde{a}} (t^{\tilde{a}})^{ij} = \begin{pmatrix} \eta^a_\alpha \\ \overline{\eta}^{\dot{\alpha} b} \end{pmatrix} (t^{\tilde{a}})^{ij} \tag{2.2}$$

Where:

- $\tilde{a}$ is an index associated with the $\frac{N(N-1)}{2}$ anti-symmetric generators of SU(N)
- a,b are indices associated with the $(N^2 - 1)$ generators of SU(N)
- $(t^{\tilde{a}})^{ij}$ are the anti-symmetric generators of SU(N)

The gauge field and couplings are unchanged. For $N_c = 3$ there are three antisymmetric generators, and the antisymmetric representation is the antitriplet. Thus, the orientifold-A projection is identical to one-flavor QCD. Proof that the orientifold-A projection is equivalent to SYM for large N_c is provided in [1, 2, 3].

3. The effect of hairpin diagrams

The propagator for a flavor singlet meson in full QCD includes an arbitrary number of annihilation vertices. In the quenched approximation all fermion lines must be attached to a creation/annihilation operator so only the valence and single-annihilation diagrams are present. If the coupling of fermion loops in the hairpin diagram is treated as a pure mass insertion m_0^2, then by geometric summation:

$$m^2_{full\ QCD} = m^2_{valence} \pm m^2_{hairpin} \tag{3.1}$$

The sign of the mass insertion is *positive* for pseudoscalars because the operator $\overline{\psi}\gamma^5\psi$ is anti-hermitian and thus the valence correlator has a negative sign. The sign of the mass insertion is *negative* for scalars since $\overline{\psi}\psi$ is hermitian.

4. QCD monte carlo results

We have tested the prediction of scalar-pseudoscalar degeneracy in one-flavor QCD using one of the lightest quark masses from the studies [6, 7], corresponding to clover improved Wilson fermions with $C_{SW} = 1.57$ with hopping parameter $\kappa = .1427$. The calculation used 300 quenched gauge configurations generated with the Wilson action at $\beta = 5.7$ on a $12^3 \times 24$ lattice. The modified quenched approximation (MQA) [8] was used to resolve the problem of exceptional configurations. Scalar and pseudoscalar hairpin diagrams were calculated using the "allsource" method [9]. In this technique the quark propagator is calculated from a sum of identical unit color-spin sources located at all space-time points on the lattice. When this propagator is contracted over color indices at a particular space-time point, the result is a gauge-invariant term corresponding to a closed quark loop originating at that point, plus a large number of gauge-dependent terms corresponding to open quark loops. The gauge-dependent terms tend to cancel on average due to their random phases, permitting the calculation of closed quark loops and loop-loop correlators (hairpins). The results of these studies for $\kappa = .1427$ are presented in Table 1. The most striking finding is the large negative contribution of the hairpin to the mass of the scalar meson. Although the statistical errors in the measurement of the scalar disconnected diagram are large, the size and sign of the scalar mass shift make scalar-pseudoscalar mass degeneracy plausible. The analysis assumes the absence of excited states in both the scalar and pseudoscalar hairpin diagrams. This assumption has been confirmed for the pseudoscalar in [7].

J^{PC}	$ValenceMass^2$	$MassShift^2$	$TotalMass^2$
0^{-+}	$+[315(6)]^2$	$+[407(11)]^2$	$+[515+12-13]^2$
0^{++}	$+[1416(14)]^2$	$-[1350(90)]^2$	$+[427+249-756]^2$

Table 1: One-flavor QCD results in Mev^2

5. $CP^{(N-1)}$ monte carlo results

Although it is a 1+1 dimensional model, $CP^{(N-1)}$ shares several characteristics with QCD that make it an interesting testing ground for QCD concepts. In addition to confinement and asymptotic freedom, $CP^{(N-1)}$ has a topological charge structure similar to QCD [10]. A major computational advantage is that we are able to use the overlap Dirac operator to calculate fermion propagators for light quarks with relative ease. $CP^{(N-1)}$ is a model of N complex scalar fields z subject to the

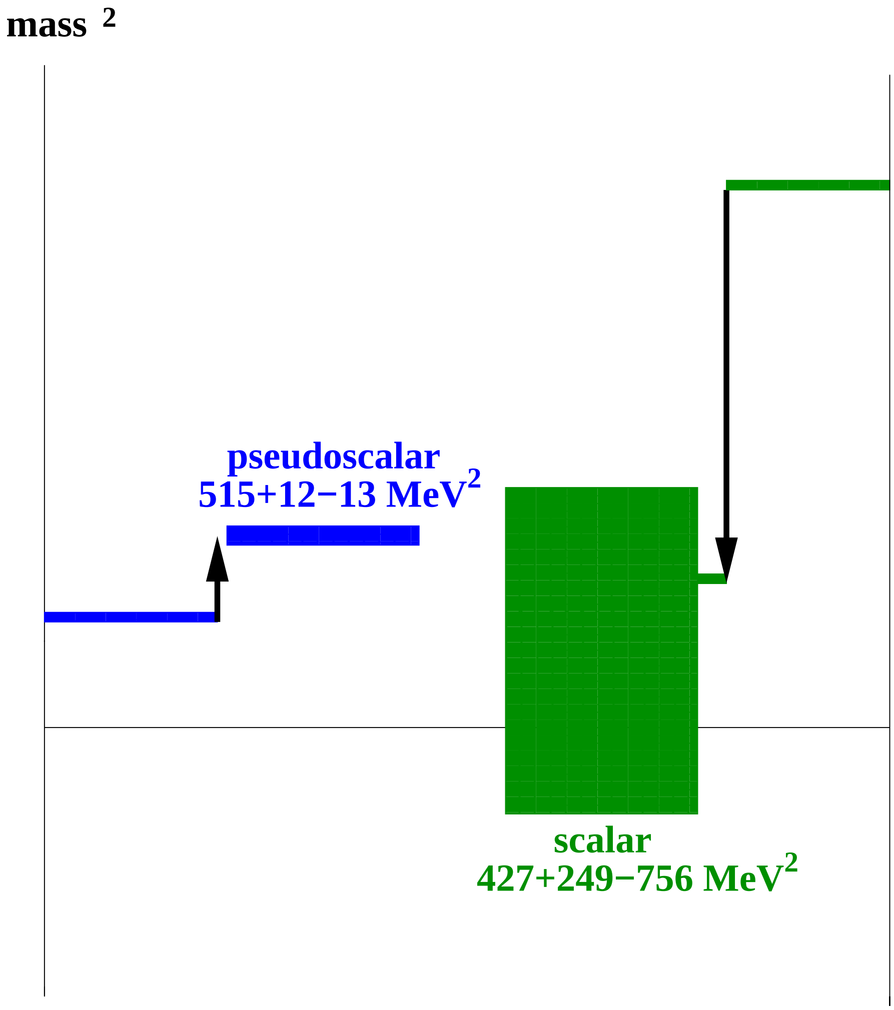

Figure 1: Scalar and pseudoscalar $mass^2$ for one-flavor QCD

constraint z*z=1. The Lagrangian for $CP^{(N-1)}$ can be written in a form that includes a U(1) gauge field:

$$L = \beta(\partial_\mu - iA_\mu)z_i^*(\partial_\mu + iA_\mu)z_i \tag{5.1}$$

$$A_\mu = \frac{i}{2}(z_i^*\partial_\mu z^i - z^i\partial_\mu z_i^*) \tag{5.2}$$

This gauge field can be used to write a Lagrangian for $CP^{(N-1)}$ with fermions:

$$L = \overline{\psi^a}(i\gamma^\mu\partial_\mu - \gamma^\mu A_\mu)\psi^a - m\overline{\psi^a}\psi^a = \overline{\psi^a}(i\gamma^\mu D_\mu - m)\psi^a \tag{5.3}$$

Following the approach of the QCD studies, we treat the fermions in the quenched approximation, which is the proper venue for studying hairpin vertices.

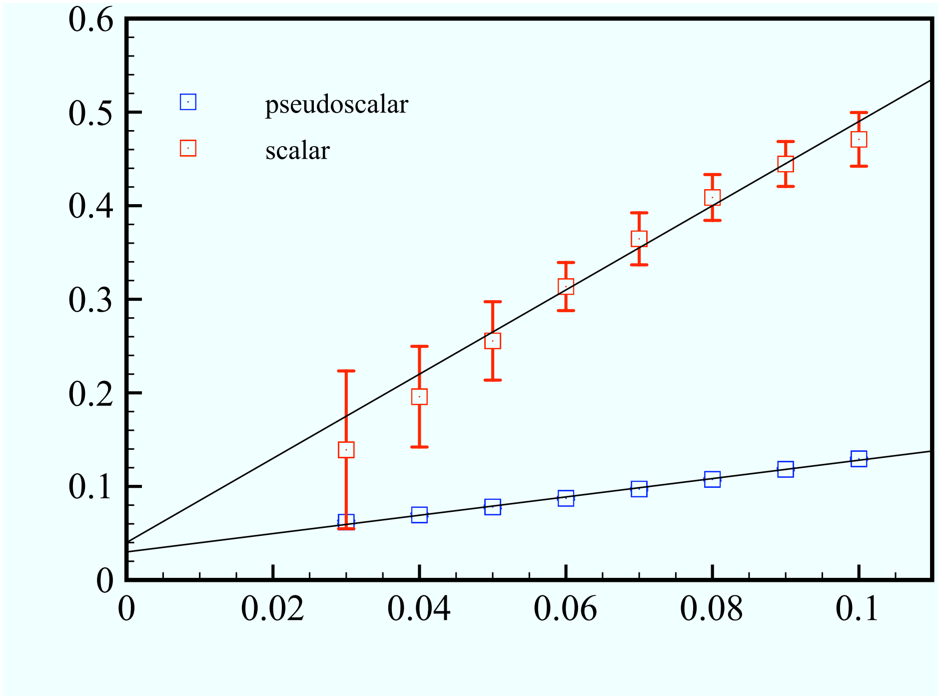

Figure 2: $CP^{(N-1)}$ meson $mass^2$ vs quark mass

We calculated scalar and pseudoscalar correlators using 1000 gauge configurations on a 30×30 lattice with overlap fermions and $\beta = 1.0$. By using smeared operators we were able to demonstrate that the hairpin diagrams did not contain excited states. The pseudoscalar behavior is similar to QCD except that the mass measured from the valence correlator approaches a non-zero constant in the chiral limit due to a quenched chiral loop effect. As in QCD, fitting the quenched scalar valence correlator is complicated by the presence of an η'-π intermediate state. Much of the statistical error in the scalar mass can be traced to the removal of this quenched artifact. The scalar is also heavier than the pseudoscalar, making it more difficult to extract the mass accurately. A smeared operator analysis shows that the scalar hairpin correlator lacks excited states but that the vertex exhibits p^2 dependence. Figure 2 is a plot of the quantities $m_p^2 + m_{0p}^2$ and $m_s^2 - m_{0s}^2$ in the range M_{quark} = 0.03-0.10 along with a linear chiral extrapolation.

6. Discussion

Although the statistical and systematic errors in our QCD calculations are significant, the important qualitative result we have shown here is that the hairpin insertion diagram in the flavor singlet channel corresponds to a positive mass shift for the pseudoscalar meson and a negative mass shift for the scalar meson, and that the values obtained on the lattice for these mass shifts are of roughly the right magnitude to bring an otherwise light pseudoscalar and an otherwise heavy scalar into approximate degeneracy in 1-flavor QCD. In the case of $CP^{(3)}$ our calculations also indicate approximate scalar-pseudoscalar degeneracy in the chiral limit. The $CP^{(3)}$ data presented here include statistical errors only. The chiral extrapolation of the full scalar mass is clearly sensitive to the linear fit parameters. Further study is required to quantify systematic errors in meson masses, particularly in the intermediate state subtraction in the scalar valence correlator.

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PoS(LAT2006)044