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Bose-Einstein Condensates and a Lorentz-Symmetry Violating Background

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Recently, the proposal of the remnant effects of Lorentz symmetry and *CPT* breaking in non-relativistic Quantum Mechanics has been investigated in the presence of the non-minimal couplings. Such a possibility opens up the way to study new type of phase generation and we can investigate the influences of this background on a variety of phase transitions. In the present work, we propose to reasses the Bose-Einstein Condensates (BEC) starting from a relativistic theory with non-minimal coupling to a Lorentz and *CPT* breaking background to compute its non-relativistic limit and study the contribution of the background to the Gross-Pitevskii equation for the BEC. The non-minimal coupling is the one that generates an Aharonov-Casher phase, motivated by the fact that we would like to study circular states in this system.

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1. Introduction

Studies about consequences of spontaneous symmetry breaking in context of a fundamental theory have received special attention over the past years. The immediate consequence is the non-equivalence between particle and observer Lorentz transformations [1] . In the last decade a line of works [2] explored this breaking in the context of string theories. Models with Lorentz symmetry and CPT breaking have been used as a low-energy limit of an extension of the Standard Model, valid at the Plank scale [3]. An effective action that incorporates CPT and Lorentz symmetry violation is obtained and it keeps unaffected the $SU(3)\times SU(2)\times U(1)$ gauge structure of the underlying theory.

Concerning the gauge sector of the Standard Model Extension (SME), many studies has been developed that focuses on many different aspects [4]-[5]. The fermion sector has been investigated as well, initially by considering general features (dispersion relations, plane-wave solutions, and energy eigenvalues) [3], and later by scrutinizing CTP-violating probing experiments [6] conceived to set up upper bounds on the breaking parameters.

Our approach to the Lorentz symmetry breaking consists in adopting the 4-dimensional version of a Chern-Simons topological term, namely $\varepsilon_{\mu\nu\kappa\lambda}v^{\mu}A^{\nu}F^{\kappa\lambda}$, where $\varepsilon_{\mu\nu\kappa\lambda}$ is the 4-dimensional Levi-Civita symbol and v^{μ} is a fixed four-vector acting as a background. This idea was first settled down in the context of QED in [17]. A study of the consequences of such breaking in QED is extensively analyzed in [7], [8]. An extension of the Carroll-Field-Jackiw model in $(1+3)$ dimensions, including a scalar sector that yields spontaneous symmetry breaking (Higgs sector), was recently developed and analyzed, yielding in an Abelian-Higgs gauge model with violation of Lorentz symmetry [9].

On the other hand, the study of a vortex nature in a Bose-Einstein Condensates (BEC) was a subject of great deal of papers and the most relevant results have been reported in an interesting review work by Fetter [12]. Its nature is closely related to the existence of an order parameter of the kind $\Phi(\mathbf{r},t)=\sqrt{n(\mathbf{r},t)}\exp(iS(\mathbf{r},t))$, which represents an irrotational flow from a hydrodynamic viewpoint; actually, from the Gross-Pitaevskii Equation (GPE) for the order parameter, one can derive the equations of motion that resemble the equation of irrotational hydrodynamics, from which the phases $S(\mathbf{r},t)$ plays the role of a velocity field potential, i.e., $\overrightarrow{v}=\frac{\hbar}{m}\nabla S(\mathbf{r},t)$. The existence of the vortex is related to the fact that $\overrightarrow{v}$ is nothing but the gradient of the phase $S(\mathbf{r},t)$. Without any discontinuity in this quantity, one should observe divergence lines for $\overrightarrow{v}$; but, with second derivative discontinuity the violation of the Bianchi identity for that phase, $[\partial_i,\partial_j]S(\mathbf{r},t)\neq 0$, a kind of singularity emerges, which implies that the velocity field has a non-zero circularity, yielding $\nabla\times\overrightarrow{v}=\frac{\hbar}{m}\delta^2(x)$, and the vortex charge is defined as $\frac{\hbar}{m}$.

Recently, in a paper by Petrosyan and You [13], it has been shown that the quantum topological Aharanov-Casher phases [14] may be used to create circulating states of magnetically trapped atomic BEC's, after associating these states with vortex-states. This has been shown [15]. In another context, out of the atomic condensates systems, it was explored that a kind of Lorentz breaking implication in Quantum Mechanics [16] is that topological phases may be induced by the coupling of an external electric and magnetic field with a vector background responsible for the breaking Lorentz Symmetry. This breaking may be implemented by a non-minimal coupling in the covariant derivative. A special situation pointed out in [16] is that some configurations could produce AC phase, important in the study of vortex of BEC systems as discussed above.

In this paper, we propose to reassess this discussion in the light of the Lorentz breaking in Quantum Mechanics.

2. Effective Vector Potential in the Gross-Pitaevskii Equation.

The Hamiltonian that describes the system follows from the Dirac Equation for a spin-$\frac{1}{2}$ particle and linear in the fields. The applicability for neutral bosons was stressed in [15] for the standard case that matter field couple to the electromagnetic fields in a non-minimal way. In this work, it is proposed to write the Hamiltonian with the same procedure, but introducing another kind of non-minimal coupling that breaks the Lorentz symmetry. The motivation for this kind of coupling in the problem of confined Bose-Einstein Condensates is that, as it is explored in [16], the background responsible for breaking the Lorentz symmetry generates a kind of Aharanov-Casher phase even in the case of neutral particles even in the spin-0 case, which makes of the BEC system a very interesting example for this application.

The starting point is the Dirac equation below

$$(i\gamma^{\mu}D_{\mu}-m)\Psi=0, \tag{2.1}$$

where the covariant derivative is chosen to be

$$D_{\mu}=\partial_{\mu}+eA_{\mu}+igv^{\nu}\widetilde{F}_{\mu\nu}, \tag{2.2}$$

where v^{μ} is a fixed four-vector acting as a background which breaks the Lorentz symmetry [17]. The Hamiltonian obtained after the non-relativistic limit is worked out read as follows:

$$H=\frac{1}{2m}\overrightarrow{\Pi}^{2}+e\varphi-\frac{e}{2m}\overrightarrow{\sigma}\cdot(\overrightarrow{\nabla}\times\overrightarrow{A})+\frac{1}{2m}gv^{0}\overrightarrow{\sigma}\cdot(\overrightarrow{\nabla}\times\overrightarrow{B})+\frac{g}{2m}\overrightarrow{\sigma}\cdot\overrightarrow{\nabla}\times(\overrightarrow{v}\times\overrightarrow{E}). \tag{2.3}$$

The generalized canonical moment is

$$\overrightarrow{\Pi}=\left(\overrightarrow{p}-e\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E}\right). \tag{2.4}$$

One can also add to (2.2) a torsion-like term, $ig_{a}\gamma_{5}v^{\nu}\widetilde{F}_{\mu\nu}$, which will contribute to the Hamiltonian with an extra term

$$H_{g_A}=\overrightarrow{\sigma}\cdot\left(g_{a}v^{0}\overrightarrow{B}-g_{a}\overrightarrow{v}\times\overrightarrow{E}\right)+g_{a}\overrightarrow{v}\cdot\overrightarrow{B}, \tag{2.5}$$

so that, the total Hamiltonian becomes

$$H_{t}=H+H_{g_A}. \tag{2.6}$$

The second quantized description of a collection of such fully polarized neutral bosons with spin-s is given, from (2.6), by a Lagrangian, enriched by a contact interaction term with coupling constant λ, as below:

$$\mathscr{L}=\psi^{\dagger}\left(i\hbar\partial_{t}+\mu B\mathbf{S}\right)\psi-\frac{1}{2m}\psi^{\dagger}\left(-i\hbar\nabla-a_{eff}\right)^{2}\psi-\frac{\lambda}{4}\left(\psi^{\dagger}\psi\right)^{2}+\mathscr{L}_{nm}, \tag{2.7}$$

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and $\mathscr{L}_{nm}$ is the contribution of the non-minimal term in the lagrangian;

$$\begin{aligned}\mathscr{L}_{nm} &= \frac{g}{2m}\psi^{\dagger}\left(\mathbf{S}.v^{0}\overrightarrow{\nabla}\times\overrightarrow{B}+\mathbf{S}.\overrightarrow{\nabla}\times(\overrightarrow{v}\times\overrightarrow{E})\right)\psi \\ &+g_{A}\psi^{\dagger}\left(\mathbf{S}.v^{0}\overrightarrow{B}-\mathbf{S}.(\overrightarrow{v}\times\overrightarrow{E})\right)\psi+g_{A}\psi^{\dagger}\left(\overrightarrow{v}.\overrightarrow{B})\right)\psi. \end{aligned} \tag{2.8}$$

Carrying out the adiabatic approximation to "freeze out " the spin degree of freedom in a state with quantum number m_s, in the same way as in the work of ref.[15], eq. 2.8 can be cast as

$$\begin{aligned}\mathscr{L}_{nm} &= \frac{g}{2m}m_{s}\psi^{\dagger}\left(v^{0}\overrightarrow{\nabla}\times\overrightarrow{B}+\overrightarrow{\nabla}\times(\overrightarrow{v}\times\overrightarrow{E})\right)\psi \\ &+m_{s}g_{A}\psi^{\dagger}\left(v^{0}\overrightarrow{B}-(\overrightarrow{v}\times\overrightarrow{E})\right)\psi+g_{A}\psi^{\dagger}\left(\overrightarrow{v}.\overrightarrow{B})\right)\psi. \end{aligned} \tag{2.9}$$

Finally, in order to obtain the Gross-Pitevskii Equation for the order parameter, we should take the variational principle in the total Lagrangian of the system. The result is:

$$\begin{aligned} i\hbar\partial_{t}\Phi = [&\frac{1}{2m}\left(-i\hbar\nabla-a_{eff}\right)^{2}-m_{s}\mu B+\frac{\lambda}{4}\Phi^{*}\Phi+ \\ &-\frac{g}{2m}m_{s}\left(v^{0}\overrightarrow{\nabla}\times\overrightarrow{B}+\overrightarrow{\nabla}\times(\overrightarrow{v}\times\overrightarrow{E})\right)+ \\ &+m_{s}g_{A}v^{0}\overrightarrow{B}-(\overrightarrow{v}\times\overrightarrow{E})+g_{A}\left(\overrightarrow{v}.\overrightarrow{B})\right)]\Phi \end{aligned} \tag{2.10}$$

and

$$a_{eff}=-e^{*}\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E}. \tag{2.11}$$

This term contains the Aharanov-Bohm, the Aharanov-Casher and an extra phase due to the $gv^{0}\overrightarrow{B}$ -term.

We first whish to verify the stability of the state of minimum energy. For that we adopt the following parametrization:

$$\Phi=\chi(r)e^{in\theta}. \tag{2.12}$$

Notice that the effective potential to be taken into account is

$$\begin{aligned}\mathscr{V}(\Phi^{*}\Phi) &= V(\Phi^{*}\Phi)-\Phi^{*}\left(\frac{g}{2m}m_{s}\left(v^{0}\overrightarrow{\nabla}\times\overrightarrow{B}+\overrightarrow{\nabla}\times(\overrightarrow{v}\times\overrightarrow{E})\right)\right)\Phi+ \\ &+\Phi^{*}\left(m_{s}g_{A}\left(v^{0}\overrightarrow{B}-(\overrightarrow{v}\times\overrightarrow{E})\right)+g_{A}\left(\overrightarrow{v}.\overrightarrow{B})\right)\right)\Phi, \end{aligned} \tag{2.13}$$

with,

$$V(\Phi^{*}\Phi)=\Phi^{*}\left(-m_{s}\mu B+\frac{\lambda}{4}\Phi^{*}\Phi\right)\Phi. \tag{2.14}$$

If we consider only the term above $V(\Phi^{*}\Phi)$, the minimum is $\chi(r)=2\sqrt{\frac{m_{s}\mu B}{\lambda}}$. The analysis of the complete potential $\mathscr{V}(\Phi^{*}\Phi)$ does not change the sign of the term $\Phi^{*}\Phi$, and the conditions for the minimum are still ensured. Notice that the non-minimal coupling contributes to an increasing in the mass term and the condensate becomes more stable.

We next address the question of the transition to the condensate phase.

3. Transition from normal to Condensate State

The influence of the non-minimal coupling in the phase transition is a relevant aspect to be investigated. In the previous section we have obtained a contribution that increases the effective mass term, so the non-minimal coupling could contribute to the behavior in the inertia of the condensate. To investigate this point, we take the eq. (2.10) in the stationary limit, without gauge field and background contributions, so that we have:

$$0 = \left(\frac{1}{2m} (-i\hbar\nabla)^2 - m_s \mu B + \frac{\lambda}{4} \Phi^* \Phi \right) \Phi. \tag{3.1}$$

The transition from the normal to the condensate state initiates with the order parameter intensity starting from zero. Close to the transition, we can neglect the $m_s \mu B$ -term. The equation above becomes

$$0 = \left(\frac{1}{2m} (-i\hbar\nabla)^2 - \frac{\lambda}{4} \Phi^* \Phi \right) \Phi, \tag{3.2}$$

by making $\alpha = \frac{\lambda}{4} \Phi^* \Phi$, we get,

$$0 = \left(\frac{1}{2m} (-i\hbar\nabla)^2 - \alpha \right) \Phi, \tag{3.3}$$

$$0 = \left(\nabla^2 - \frac{2m}{\hbar^2} \alpha \right) \Phi \tag{3.4}$$

Solving the equation above for one spatial dimension, we obtain $\Phi = \exp(-\frac{x}{\xi})$, with $\xi = \frac{\hbar}{\sqrt{2m|\alpha|}}$. The ξ parameter, in a BEC, is called healing length. α, which parametrizes the two-body interactions, is given by $\alpha = \frac{4\pi\hbar^2 an}{m}$, where n is the density of the condensate, m is the mass term and a is the s- wave scattering length, and $\xi = (8\pi na)^{-1/2}$. If we take into account the gauge contribution, $a_{eff} = -e^* \overrightarrow{A} + gv^0 \overrightarrow{B}$, to the eq. (2.10),

$$0 = \left(\frac{1}{2m} \left(-i\hbar\nabla + e^* \overrightarrow{A} - gv^0 \overrightarrow{B} \right)^2 - m_s \mu B + \alpha \right) \Phi. \tag{3.5}$$

The contribution of $m_s \mu B$ in a trapped condensate is usually the harmonic trap, given in the most general form by $V_{ext} = \frac{m}{2} \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right)$. In the Landau gauge $\vec{A} = (0, Hx, 0)$, the equation above becomes,

$$\left(\frac{\partial}{\partial x} \right)^2 \Phi + \left(\frac{\partial}{\partial y} - \frac{e^* H}{i\hbar} x \right)^2 \Phi + \left(\frac{\partial}{\partial z} - \frac{gv^0 \overrightarrow{B}_z}{i\hbar} \right)^2 \Phi + \xi^{-2} \Phi + V_{ext} \Phi = 0. \tag{3.6}$$

It is clear, from the equation above that the non-minimal coupling yields a harmonic-type oscillator, in addition to the one coming from the trap. When both the oscillators are in resonance, we claim that they take the condensate to an excited state.

The order parameter depends on x, y, and z. The problem turns into a Schroedinger equation for a charge in an external magnetic field. We put

$$\Phi = \varphi(x)\exp(ik_2 y + ik_3 z), \tag{3.7}$$

and we obtain thereby that,

$$\left(\frac{\partial}{\partial x}\right)^2 \varphi - \left(k_2 + \frac{e^* H}{\hbar} x\right)^2 \varphi - \left(k_3 + \frac{g v^0 H}{\hbar}\right)^2 \varphi + V_{ext}\Phi + \xi^{-2}\varphi = 0. \tag{3.8}$$

$\omega_0 = \frac{e^* H}{m}$ and ξ^{-2} is the healing length fixed by the Gross-Pitaevskii equation

$$\xi^{-2} = \frac{2mE}{\hbar^2}. \tag{3.9}$$

This yield the equation:

$$\left(\frac{\partial}{\partial x}\right)^2 \varphi - \left(k_2 + \frac{m\omega_0}{\hbar} x\right)^2 \varphi - \left(k_3 + \frac{g v^0 H}{\hbar}\right)^2 \varphi + V_{ext}\Phi + \frac{2mE}{\hbar^2}\varphi = 0. \tag{3.10}$$

We set $k_3 = \frac{g v^0 H}{\hbar}$, and the oscillator equation comes about:

$$\left(\frac{\partial}{\partial x}\right)^2 \varphi - \left(k_2 + \frac{m\omega_0}{\hbar} x\right)^2 \varphi + V_{ext}\Phi + \frac{2mE}{\hbar^2}\varphi = 0, \tag{3.11}$$

with the oscillator frequency ω_0 and energy E. If we assume that the confining potential V_{ext} do not change the energy spectrum of the oscillator above, therefore we have

$E = \hbar\omega_0(n + \frac{1}{2})$. Taking into account (3.9), we get the relationship

$$\frac{\hbar^2}{2m\xi^2} = \frac{\hbar e^* H}{m}\left(n + \frac{1}{2}\right) \tag{3.12}$$

For $n = 0$, we have the minimum that corresponds to the condensate without vortices ($H = H_{c2}$):

$$H_{c2} = \frac{\hbar}{e^* \xi^2} = \frac{\phi_0}{\pi\xi^2}, \tag{3.13}$$

where $\phi_0 = \frac{\pi\hbar}{e}$ is the magnetic flux quantum. The solution for the lowest energy level is a Gaussian function,

$$\varphi = C\exp\left[-\frac{1}{2\xi^2}\left(x - \frac{\hbar k_2}{2eH_{c2}}\right)^2\right] \tag{3.14}$$

The complete solution can be finally written down:

$$\Phi = C\exp\left[-\frac{1}{2\xi^2}\left(x - \frac{\hbar k_2}{2eH_{c2}}\right)^2\right]\exp(ik_2 y + i\frac{g v^0 H}{\hbar} z) \tag{3.15}$$

then we can observe that the Lorentz breaking contribution is in the appearance of a phase in the expression of the condensate order parameter. Then the velocity field assume the form,

$\vec{v} = \frac{\hbar}{m}\nabla S(\mathbf{r}, t) = \frac{g v^0 H}{m} + k_2$, therefore this contribution must affect the vortex solution.

3.1 Concluding Comments and Prospects

The main idea we have tried to convey in this paper concerns the possible relation we may establish between the formation of the Bose-Einstein Condensates and a scenario defined by the presence of a background that parametrizes the breaking of Lorentz symmetry form the particle transformations point of view.

The Lorentz symmetry violation background is non-minimally coupled to the Gross-Pitaevskii equation and through a number of steps discussed above, we can keep track of its effect to the phase that yields the vorticity of the condensate fluid. The ground state solution is worked out under some assumptions. It would be a natural step further to get the excited states and to really demonstrate our conjecture that the resonance among the trap and the Lorentz-symmetry violating background take the condensates to their excited states.

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